Risky Rents*

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Abstract

In this paper, we consider a symmetric contest game in which agents compete to increase their share of a risky rent. We show that a symmetric equilibrium always exists, and that it is unique under constant or decreasing absolute risk aversion. We then exhibit interpretable conditions so that increases in risk and risk aversion decrease equilibrium efforts in this strategic game.

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1 Introduction

Many economic, political and social situations can be described as contests in which agents compete by expending valuable resources to win a rent (or a prize, a privilege etc.).[1] The manner in which the rent is distributed among the agents can vary depending on the contest. The classical example of political lobbying is a good illustration of this variability. Consider the case of firms exerting lobbying efforts to influence government decisions. In some situations, the rent is indivisible and the winner takes all, for instance when the goal is to obtain a monopoly position. In other situations, for instance when firms compete for free licences or subsidies provided by the government, the rent is often shared between the competing firms.[2]

Moreover, the rent is risky in many situations. Typically, the lobbyists' benefit is imperfectly known because it could depend on future economic and political conditions. Consider the rent-seeking activity of polluting firms aiming for free pollution emission permits (Hanley and MacKenzie 2010, Rode 2014). The future value of these permits is generally unknown at the time of the grandfathering allocation process by the government, and hence it is risky. Similarly, contests in the form of advertising or marketing campaigns designed to increase market share are affected by future market uncertainty. As further illustrations, consider agents within an organization competing for a fixed budget, the allocation of import quota licenses among competing importers or the division of rents between the members of a cartel. All these situations may be viewed as contests for a share of a risky rent (Long and Vousden 1987).

Motivated by these examples, we analyze a strategic game in which risk-averse agents contest for shared and risky rents. We consider only the symmetric game. We first derive sufficient conditions ensuring that this game does not have asymmetric equilibria. We also prove that a symmetric equilibrium always exists, and that it is unique if the agents' utility function

[1]The theory of contests and its applications have attracted considerable attention since the pionneering works of Tullock (1967) and Krueger (1974). For a recent comprehensive review of this literature, see Konrad (2009). For a more technical survey of the theory of contests and its various applications, see Corchón (2007) and Long (2013). For a collection of papers on contests, see Congleton and Hillman (2015). For a review of the growing experimental literature, see Dechenaux et al. (2015).

[2]We notice that share contests go back to market share attraction models that were prominent in the old marketing and operations research literature (Konrad 2009, page 5).

displays constant or decreasing absolute risk aversion. We then show that, compared to a riskless situation, the introduction of a risk on rent always leads to lower equilibrium effort. Finally, we show that the equilibrium effort decreases i) if the agents' common level of risk aversion increases or ii) if the riskiness of the rent increases and a certain regularity condition is met.

The rest of the paper is organized as follows. The remainder of the Introduction briefly relates our work to the existing literature on risky contests. Section 2 introduces the model and notations. Section 3 studies the equilibrium properties of the model. Section 4 examines the effects of risk aversion and risk on equilibrium efforts. Finally, Section 5 provides a short conclusion.

1.1 Related literature

Only a few papers have examined the effect of risk and risk aversion in strategic contest models (Hillman and Katz 1984, Skaperdas and Gan 1995, Konrad and Schlesinger 1997, Treich 2010, Cornes and Hartley 2003 and 2012). However, in all these papers, risk stems from the probabilistic nature of the contest, not from the rent itself (see Remark 1 below). Indeed, in these models, agents compete to increase their respective probabilities of winning the rent through a lottery mechanism. In other words, the rent is riskless, but the rent allocation process is risky. In contrast, in our model, agents compete to increase their respective share of a risky rent. That is, the risk arises only from the risky nature of the rent, not from the rent allocation process.[3]

As far as we know, Long and Vousden (1987) is the only paper to have introduced risk on a shared rent in a contest with risk-averse agents. However, their model differs from ours on two main issues. First, they do not consider a risky rent, but instead a risky share of the rent. In this sense, they still consider a probabilistic allocation rule of a risk-free rent, as in the rest of the literature. Second, Long and Vousden assume that the efforts are separable from the utility function (see, e.g., their equation (2)). This assumption essentially means that the effort and the rent are not commensurable.[4] While

[3] A few papers (Harstad 1995, Wärneryd 2003) consider a risky rent. However, they assume that agents are risk neutral agents and instead focus on asymmetric information about the value of the rent.

[4] See also Öncüler and Croson (2005) who make a similar separability assumption. Relatedly, Schroyen and Treich (2016) examine various types of contests which differ depending on whether the effort or the rent are separable within the utility function.

technically convenient, this assumption is unusual in the literature (see the references above) since most strategic contest models assume that the utility function of risk-averse contestants is nonseparable in the rent and the cost of effort.

2 Model

We consider a symmetric contest game in which n identical risk-averse agents compete for a share of a divisible and risky rent. Agents expend simultaneously and independently efforts at a constant and equal unitary cost, normalized to 1 for simplicity. Let x_i denote agent i's effort level, and let x_{-i} denote the strategy profile of all agents but i. The share of the rent awarded to agent i, p_i, is given by the following contest success function (CSF):

$$p_i(x_i, x_{-i}) = \begin{cases} \frac{\phi(x_i)}{\phi(x_i)+\sum_{j\neq i}^{n}\phi(x_j)} & \text{if } (x_i, x_{-i}) \neq (0,0) \\ 1/n & \text{if } (x_i, x_{-i}) = (0,0), \end{cases} \tag{1}$$

where $\phi(.)$ is the impact function, common to all agents, which measures the impact of an agent's effort in the contest. We assume that the impact function has the following properties:

Assumption 1 (A.1) $\phi(.)$ is twice differentiable and satisfies $\phi(0) = 0$, $\phi'(x) > 0$ and $\phi''(x) \leq 0$ for $x > 0$.

The functional form in (1) with the associated assumptions in (A.1) is common in the literature on contests. A special case is $\phi(x) = x^m$, with $0 < m \leq 1$, which was first introduced by Tullock (1980).

We also assume that all agents are risk-averse expected utility maximizers with a common concave utility function, $u(.)$, which satisfies the following properties:

Assumption 2 (A.2) $u(.)$ is thrice differentiable and satisfies $u'(w) > 0$ and $u''(w) \leq 0$ for all w.

Several utility functions possess these properties. An example is the negative exponential utility function $u(w) = -\exp(-\lambda w)$ with $\lambda > 0$. This utility function displays constant absolute risk aversion (CARA), that is, $A(w) \equiv \frac{-u''(w)}{u'(w)} = \lambda$. Another example is the power utility function $u(w) = \frac{w^{1-\gamma}}{1-\gamma}$ with $\gamma \neq 1$ and $\gamma > 0$. This utility function has constant relative risk aversion (CRRA) and displays decreasing absolute risk aversion (DARA), namely, $A'(w) < 0$.

Finally, we assume that the contested rent is risky and its value $\widetilde{v}$ has the following properties:

Assumption 3 (A.3) $\widetilde{v}$ is a random variable with support contained in the interval $[\underline{v}, \overline{v}]$, where $0 < \underline{v} < \overline{v}$.

Therefore, given a strategy profile (x_i, x_{-i}), the payoff of agent i is given by

$$U_i(x_i, x_{-i}) = E[u(p_i(x_i, x_{-i})\widetilde{v} - x_i)], \tag{2}$$

where $E[.]$ is the expectation with respect to the risky rent $\widetilde{v}$, the only random variable in our model.

Let $X_i = [0, \infty)$ denote the strategy set of agent i, where $i \in I = \{1, 2, ..., n\}$. For a given quadruple $\Omega = \{n, \phi, u, \widetilde{v}\}$, the so-called "risky contest" described by (1) and (2) together with (A.1), (A.2) and (A.3) defines an n-player simultaneous-move game $\Gamma(X_i, \Omega)_{i \in I}$. All parameters are common knowledge. The equilibrium concept we use throughout is that of pure-strategy equilibrium.

Remark 1: Contest games under risk aversion The contest game under risk aversion commonly considered in the literature (see the references in Section 1.1) has the following payoff function:

$$U_i(x_i, x_{-i}) = p_i(x_i, x_{-i})u(v - x_i) + (1 - p_i(x_i, x_{-i}))u(-x_i). \tag{3}$$

In (3), $p_i(x_i, x_{-i})$ is interpreted as the probability that agent i obtains the rent v. In that model, the rent is riskless, but the rent allocation process is risky. In contrast, in our model, $p_i(x_i, x_{-i})$ is interpreted as the share of a risky rent $\widetilde{v}$ obtained by agent i. Therefore, for given agents' efforts, the rent allocation process is deterministic. Risk stems only from the uncertainty over the realization of the random variable $\widetilde{v}$. Note that both models are identical under risk neutrality (that is, $u''(w) = 0$) and when $v = E[\widetilde{v}]$.

Remark 2: Cournot games Consider a standard Cournot oligopoly game with (uncertain) inverse demand $D(\sum_i q_i, \widetilde{v})$, output q_i and cost function $c(q_i)$. The risk-averse firm thus maximizes:

$$E[u(D(\sum_i q_i, \widetilde{v})q_i - c(q_i))].$$

The study of this game dates back to Leland (1972). Now, under $D(Q, v) = \frac{v}{Q}$ and $c(.) = \phi^{-1}(.)$, observe that this game is strategically equivalent to our risky contest game. The isomorphism between Cournot and contest games has been noted by Menezes and Quiggin (2010).

3 Equilibrium properties

Given the specification of the CSF in (1), we can rewrite agent i's payoff as follows:

$$U_i(x_i, x_{-i}) = E\left[u\left(\frac{\phi(x_i)}{\phi(x_i)+b_{-i}}\widetilde{v} - x_i\right)\right] \quad \text{if } (x_i, x_{-i}) \neq (0,0), \tag{4}$$

where $b_{-i} = \sum_{j\neq i}^{n} \phi(x_j)$, while $U_i(0,0) = E\left[u\left(\widetilde{v}/n\right)\right]$. First, note that $(x_i, x_{-i}) = (0,0)$ cannot be an equilibrium. Indeed, $x_i = 0$ is not the best response to $x_{-i} = 0$ since $U_i(x_i, 0) = E[u(\widetilde{v} - x_i)] > U_i(0,0)$ for any $x_i > 0$ small enough.[5] Therefore, we can concentrate on the case where $x_{-i} \neq 0$ for all $i \in I$.

For a given $b_{-i} > 0$, the first-order conditions for the maximization of (4) with respect to x_i are

$$\frac{\partial U_i}{\partial x_i} = E\left[\left(\frac{\phi'(x_i)b_{-i}}{\left(\phi(x_i)+b_{-i}\right)^2}\widetilde{v} - 1\right)u'\left(\frac{\phi(x_i)}{\phi(x_i)+b_{-i}}\widetilde{v} - x_i\right)\right] = 0, \quad i \in \{1, 2, ..., n\}. \tag{5}$$

The second-order conditions are written as

$$\frac{\partial^2 U_i}{\partial x_i^2} = E\left[\frac{\phi''(x_i)\left(\phi(x_i)+b_{-i}\right)b_{-i} - 2(\phi'(x_i))^2 b_{-i}}{\left(\phi(x_i)+b_{-i}\right)^3}\widetilde{v}u'\left(\frac{\phi(x_i)}{\phi(x_i)+b_{-i}}\widetilde{v} - x_i\right)\right]$$

$$+E\left[\left(\frac{\phi'(x_i)b_{-i}}{\left(\phi(x_i)+b_{-i}\right)^2}\widetilde{v} - 1\right)^2 u''\left(\frac{\phi(x_i)}{\phi(x_i)+b_{-i}}\widetilde{v} - x_i\right)\right] < 0, \quad i \in \{1, 2, ..., n\}.$$

Note that under (A.1), (A.2) and (A.3), the second-order conditions are satisfied such that U_i is strictly concave in $x_i \geq 0$ for any given $b_{-i} > 0$. Therefore, the existence of a pure-strategy equilibrium is equivalent to the

[5] More generally, if $x_{-i} = 0$, the expected payoff of agent i approaches $E[u(\widetilde{v})]$ as $x_i \to 0$. Thus, agent i's best response does not exist for $x_{-i} = 0$.

case where the system of the first-order conditions (5) has a solution $x^* = (x_1^*, x_2^*, ..., x_n^*)$.

We start by a technical condition under which the system of first-order conditions (5) can only have symmetric solutions.[6]

Proposition 1 *Let* $H(x,v) = \ln[u'(v\phi(x) - x)]$*. If* $\frac{\partial^2 H(x,v)}{\partial x \partial v} < 0$*, the risky contest cannot have asymmetric equilibria.*

Note that the above condition is fulfilled, for example, for u CARA. Indeed, if $u(x) = -\exp(-\lambda x)$, then $H(x,v)$=ln$[\lambda - \lambda(v\phi(x) - x)]$ and

$$\frac{\partial^2 H(x,v)}{\partial x \partial v} = -\lambda\phi'(x) < 0.$$

The above condition is also satisfied under u CRRA and ϕ strictly concave. Indeed, if $u(w) = \frac{w^{1-\gamma}}{1-\gamma}$ and $\phi'' < 0$, we have

$$\frac{\partial^2 H(x,v)}{\partial x \partial v} = \gamma \frac{x\phi'(x) - \phi(x)}{(x - v\phi(x))^2} < 0.$$

From now on, we focus on symmetric equilibria only. We first show that a symmetric equilibrium always exists.

Proposition 2 *The risky contest has a symmetric equilibrium.*

Proof: Let $f(x)$ denote any of the first-order conditions in (5) evaluated at the symmetric solution $x_1 = x_2 = ... = x_n = x > 0$. Omitting the i-subscript, $f(x)$ can be written as follows:

$$f(x) = E\left[(\Delta(x)\widetilde{v} - 1)\, u'\left(\frac{\widetilde{v}}{n} - x\right)\right] = 0, \tag{6}$$

where $\Delta(x) = \frac{n-1}{n^2}\frac{\phi'(x)}{\phi(x)}$. Note that $\lim_{x \longrightarrow 0} f(x) = +\infty$ since $\lim_{x \longrightarrow 0} \Delta(x) = +\infty$. Note also that $\frac{\phi'(x)}{\phi(x)} \leq \frac{1}{x}$ under our assumptions on ϕ. Accordingly, $\Delta(x)$ can be made small enough, and thus $f(x)$ negative, for x large enough. This implies that $f(x) = 0$ has a solution $x^* > 0$ that, given the strict concavity of the expected payoff function, is an interior symmetric equilibrium of the game.■

We then prove the uniqueness of the symmetric equilibrium under common assumptions on the utility function.

[6] The proof of this Proposition is presented in the Appendix.

Proposition 3 *The symmetric equilibrium of the risky contest is unique for u CARA or u DARA.*

Proof: The symmetric equilibrium is unique if the following single crossing property is satisfied: $f(x) = 0 \Longrightarrow f'(x) < 0$ where f is defined as in (6). We have

$$f'(x) = E\left[(\Delta'(x)\widetilde{v})u'\left(\frac{\widetilde{v}}{n} - x\right)\right] - E\left[(\Delta(x)\widetilde{v} - 1)u''\left(\frac{\widetilde{v}}{n} - x\right)\right].$$

Note that

$$E\left[(\Delta'(x)\widetilde{v})u'\left(\frac{\widetilde{v}}{n} - x\right)\right] < 0$$

since $\Delta'(x) < 0$ under ϕ concave. Thus, it is sufficient to show that

$$E\left[(\Delta(x)\widetilde{v} - 1)\, u'\left(\frac{\widetilde{v}}{n} - x\right)\right] = 0 \Longrightarrow -E\left[(\Delta(x)\widetilde{v} - 1)u''\left(\frac{\widetilde{v}}{n} - x\right)\right] \leq 0.$$

Note that

$$-E\left[(\Delta(x)\widetilde{v} - 1)u''\left(\frac{\widetilde{v}}{n} - x\right)\right] = E\left[(\Delta(x)\widetilde{v} - 1)\, u'\left(\frac{\widetilde{v}}{n} - x\right) A\left(\frac{\widetilde{v}}{n} - x\right)\right],$$

where $A(.) = -\frac{u''(.)}{u'(.)}$ is the degree of absolute risk aversion. Under CARA, $A(.)$ is a constant so that

$$E\left[(\Delta(x)\widetilde{v} - 1)\, u'\left(\frac{\widetilde{v}}{n} - x\right) A\left(\frac{\widetilde{v}}{n} - x\right)\right] = 0.$$

Thus, we have

$$E\left[(\Delta(x)\widetilde{v} - 1)u''\left(\frac{\widetilde{v}}{n} - x\right)\right] = 0$$

under CARA, and the sufficient condition for uniqueness is satisfied.

Under DARA, $A(\frac{v}{n} - x)$ is decreasing in v. Now consider two cases. When $v \geq \frac{1}{\Delta(x)}$, then

$$(\Delta(x)v - 1)\, u'\left(\frac{v}{n} - x\right) A\left(\frac{v}{n} - x\right) \leq (\Delta(x)v - 1)\, u'\left(\frac{v}{n} - x\right) A\left(\frac{1}{n\Delta(x)} - x\right).$$

When $v \leq \frac{1}{\Delta(x)}$, the previous inequality is also verified. Therefore we always have

$$\begin{aligned} & E\left[\left(\Delta(x)\widetilde{v}-1\right)u'\left(\frac{\widetilde{v}}{n}-x\right)A\left(\frac{\widetilde{v}}{n}-x\right)\right] \\ & \leq E\left[\left(\Delta(x)\widetilde{v}-1\right)u'\left(\frac{\widetilde{v}}{n}-x\right)A\left(\frac{1}{n\Delta(x)}-x\right)\right]=0. \end{aligned}$$

Thus, we have

$$-E\left[(\Delta(x)\widetilde{v}-1)u''\left(\frac{\widetilde{v}}{n}-x\right)\right]\leq 0$$

under DARA.■

From the above analysis, we can conclude that for the common CARA or CRRA utility functions the risky contest has a symmetric equilibrium, and that it is the unique equilibrium in the class of all pure-strategy equilibria of the game.

4 The effects of risk and risk aversion

In this section, we examine the effects of risk aversion and of the riskiness of the rent on equilibrium effort levels in the risky contest.

4.1 Comparison with risk neutrality and certainty

From the above conditions, the equilibrium of the risky contest x^* can be characterized by the following implicit solution:

$$\frac{\phi\left(x^*\right)}{\phi'\left(x^*\right)}=\frac{n-1}{n^2}\frac{E\left[\widetilde{v}u'\left(\frac{\widetilde{v}}{n}-x^*\right)\right]}{E\left[u'\left(\frac{\widetilde{v}}{n}-x^*\right)\right]}. \tag{7}$$

Under risk neutrality, the symmetric equilibrium, denoted x^{RN}, is thus characterized by

$$\frac{\phi\left(x^{RN}\right)}{\phi'\left(x^{RN}\right)}=\frac{n-1}{n^2}E[\widetilde{v}].$$

Note that under (A.1), the function $\frac{\phi(x)}{\phi'(x)}$ is equal to 0 at $x=0$; moreover, this function is strictly increasing in x, so that it can cross $\frac{n-1}{n^2}E[\widetilde{v}]$ only once. As a result, the symmetric equilibrium x^{RN} is unique under risk neutrality. We now have the following Proposition.

Proposition 4 *In the risky contest, the symmetric equilibrium effort under risk aversion is lower than the symmetric equilibrium effort under risk neutrality, that is,* $x^* < x^{RN}$*.*

Proof: Using the definition of the covariance and after simplification, we can rewrite (7) as follows:

$$\frac{\phi(x^*)}{\phi'(x^*)} = \frac{n-1}{n^2} E\left[\widetilde{v}\right] + \frac{COV\left[\widetilde{v}, u'\left(\frac{\widetilde{v}}{n} - x^*\right)\right]}{E\left[u'\left(\frac{\widetilde{v}}{n} - x^*\right)\right]}. \quad (8)$$

Hence $x^* < x^{RN}$ if and only if the covariance term is negative. This is true under risk aversion, since $u'(\frac{v}{n} - x)$ decreases when v increases.■

Thus, we find that risk introduces an additional covariance term into the characterization of the equilibrium condition. This term can be interpreted as a risk premium associated with rent-seeking effort, and is negative if and only if the agent is risk-averse.

We now consider the certainty case. Note that if the rent is risk-free and of value v, the symmetric equilibrium under certainty, denoted x^C, is characterized by

$$\frac{\phi(x^C)}{\phi'(x^C)} = \frac{n-1}{n^2} v.$$

Hence, if the value of the risk-free rent is greater than or equal to the expected value of the risky rent, that is, $E[\widetilde{v}] \leq v$, we obviously have $x^{RN} \leq x^C$. Using the previous result, we can directly compare the equilibrium under a risk-free rent with that under a risky rent.

Proposition 5 *If* $E[\widetilde{v}] \leq v$*, then the symmetric equilibrium effort in the risky contest is lower than the symmetric equilibrium effort under certainty, that is,* $x^* < x^C$*.*

Note that the previous results do not require the assumption of a unique equilibrium under risk aversion. Since the equilibrium under risk neutrality or under certainty is unique, the result indeed shows that any equilibrium under risk aversion (whether unique or not) is below the unique equilibrium under risk neutrality or under certainty.

4.2 The effect of more risk aversion

We now examine the effect of more risk aversion on equilibrium efforts. To do so, we compare the equilibria between two contests differing only in terms of the common degree of risk aversion of the agents participating in each contest. We obtain the following Proposition.

Proposition 6 *Assume that x^* (resp. $\widehat{x}$) is the unique symmetric equilibrium of a risky contest with a utility function u (resp. $\widehat{u}$). Assume also that $\widehat{u}$ is more risk-averse than u. Then, we have $\widehat{x} \leq x^*$.*

Proof: Consider the implicit solution (7). Note that the right-hand side is strictly positive at $x^* = 0$ and therefore crosses the left-hand side from above. Thus, we only have to show that the term $E[\widetilde{v}u'(\frac{\widetilde{v}}{n}-x)]/E[u'(\frac{\widetilde{v}}{n}-x)]$ decreases with risk aversion. Let $\widehat{u}(.) \equiv T(u(.))$. As is usual in the comparative statics of risk aversion, we examine the effect of an increasing and concave transformation $T(.)$ (Pratt 1964). We thus want to show that

$$\frac{E\left[\widetilde{v}u'\left(\frac{\widetilde{v}}{n}-x\right)T'\left(u\left(\frac{\widetilde{v}}{n}-x\right)\right)\right]}{E\left[u'\left(\frac{\widetilde{v}}{n}-x\right)T'\left(u\left(\frac{\widetilde{v}}{n}-x\right)\right)\right]} \leq \frac{E\left[\widetilde{v}u'\left(\frac{\widetilde{v}}{n}-x\right)\right]}{E\left[u'\left(\frac{\widetilde{v}}{n}-x\right)\right]}.$$

We now introduce the following probability density function:

$$m(v) = \frac{d(v)u'\left(\frac{v}{n}-x\right)}{E\left[u'\left(\frac{\widetilde{v}}{n}-x\right)\right]},$$

where $d(v)$ is the probability density function of the random variable $\widetilde{v}$. Then the previous inequality can be simply rewritten

$$\frac{\widehat{E}\left[\widetilde{v}T'\left(u\left(\frac{\widetilde{v}}{n}-x\right)\right)\right]}{\widehat{E}\left[T'\left(u\left(\frac{\widetilde{v}}{n}-x\right)\right)\right]} \leq \widehat{E}\left[\widetilde{v}\right],$$

where $\widehat{E}$ is the expectation operator taken with respect to the probability density function $m(v)$. Observe finally that this last inequality holds if and only if $COV[\widetilde{v}, T'(u(\frac{\widetilde{v}}{n}-x))] \leq 0$, namely if and only if $T'(u(\frac{v}{n}-x))$ is decreasing in v. This is always true under T concave.■

This last Proposition is thus a generalization of Proposition 4 to the common notion of increased risk aversion. The result seems intuitive. In our game, an increase in an agent's effort, the other agents' efforts remaining fixed, increases her share of the risky rent. Yet, from portfolio theory, we know that an increase in risk aversion reduces the share invested in net risky assets (Pratt 1964). Following this insight from portfolio theory, more risk aversion should thus naturally decrease the effort in our game. Note, however, that the negative effect of risk aversion holds at the equilibrium of the game and thus does not rely on the assumption that the other agents' effort is kept fixed.

4.3 The effect of more risk

We have just shown that more risk aversion induces lower efforts at equilibrium. We next examine the effect of more risk on the rent. It turns out that this case needs an additional condition to sign the comparative statics analysis.

Proposition 7 *Assume that x^* (resp. x^{**}) is the unique symmetric equilibrium of a risky contest with rent $\widetilde{v}$ (resp. $\widetilde{v}'$). Assume also that $\widetilde{v}'$ is more risky than $\widetilde{v}$. Then, we have $x^{**} \leq x^*$ if and only if*

$$\left(\frac{v}{n} - \frac{n}{n-1}\frac{\phi(x)}{\phi'(x)}\right)\left(-\frac{u'''(\frac{v}{n}-x)}{u''(\frac{v}{n}-x)}\right) \leq 2.$$

Proof: We have to show that $f(x) = E[(\Delta(x)\widetilde{v} - 1)u'(\frac{\widetilde{v}}{n} - x)]$ decreases when $\widetilde{v}$ becomes more risky. As is usual in the comparative statics of increasing risk (Rothschild and Stiglitz 1970), this is equivalent to showing that

$$g(v) = (\Delta(x)v - 1)u'(\frac{v}{n} - x)$$

is concave in v. We have

$$g'(v) = \Delta(x)u'(\frac{v}{n} - x) + \frac{(\Delta(x)v-1)}{n}u''(\frac{v}{n} - x)$$

and

$$g''(v) = 2\frac{\Delta(x)}{n}u''(\frac{v}{n} - x) + \frac{(\Delta(x)v-1)}{n^2}u'''(\frac{v}{n} - x).$$

Therefore, $g(v)$ is concave if and only if

$$\left(\frac{\Delta(x)v-1}{\Delta(x)n}\right)\left(-\frac{u'''(\frac{v}{n}-x)}{u''(\frac{v}{n}-x)}\right) \leq 2$$

which, by using $\Delta(x) = \frac{n-1}{n^2}\frac{\phi'(x)}{\phi(x)}$, yields the result.■

The condition in the above Proposition places no restriction on the probability distribution of the rent but combines restrictions on the functional forms of u and ϕ. Note that this condition holds for all ϕ when $u''' = 0$.[7] However, this last restriction on the utility function is very strong. The weaker restriction $u''' \geq 0$, coined "prudence" (Kimball 1990),[8] is more common in the literature. Using the above result, we can now derive a simple sufficient condition for the comparative statics analysis of risk that only relies on a property of the utility function.

Corollary *Assume* $u''' \geq 0$. *If* $\frac{-wu'''(w)}{u''(w)} \leq 2$ *for all* w, *then* $x^{**} \leq x^*$.

Proof: From the concavity of ϕ, we know that $\frac{\phi(x)}{\phi'(x)} \geq x$. This implies that

$$\frac{v}{n} - \frac{n}{n-1}\frac{\phi(x)}{\phi'(x)} \leq \frac{v}{n} - x,$$

and therefore

$$\left(\frac{v}{n} - \frac{n}{n-1}\frac{\phi(x)}{\phi'(x)}\right)\left(-\frac{u'''\left(\frac{v}{n}-x\right)}{u''\left(\frac{v}{n}-x\right)}\right) \leq \left(\frac{v}{n} - x\right)\left(-\frac{u'''(\frac{v}{n}-x)}{u''(\frac{v}{n}-x)}\right) \leq 2$$

under $u''' \geq 0$. The result in Proposition 7 concludes the proof.■

Given a CRRA utility function (which has $u''' \geq 0$), that is, $u(w) = (1-\gamma)^{-1}w^{1-\gamma}$ with $\gamma > 0$, the condition $\frac{-wu'''(w)}{u''(w)} \leq 2$ for all w simplifies to $\gamma < 1$. It may thus appear unexpected that the condition assumes that agents are not "too" risk-averse to ensure that they decrease their efforts when the rent becomes more risky. However, this result is not very surprising given previous results in the literature. Indeed, referring again to portfolio theory, an increase in risk has been shown to not necessarily decrease risk taking (Rothschild and Stiglitz 1971). In fact, Hadar and Seo (1990) obtained a similar condition on the utility function, that is, $\frac{-wu'''(w)}{u''(w)} \leq 2$, to sign the comparative statics of more risk in the standard portfolio model.

[7] This is simply because the left hand side of the inequality condition in Proposition 7 is equal to 0 under $u''' = 0$. Equivalently, it is easy to see from (7) that, when the utility function is quadratic (implying $u''' = 0$), the right hand side always decreases with the variance of the risk.

[8] Sometimes this condition is also called downside risk aversion (Menezes et al. 1980). Note that DARA implies prudence.

Moreover, note that the condition above means that "relative prudence" is positive and lower than 2. Therefore, the result identifies in fact a condition on relative prudence, and not one on relative risk aversion. Intuitively, there are two effects (see the computation of $g''(v)$ above). On the one hand, risk aversion decreases efforts, as shown in Proposition 4. This is a "risk aversion effect", simply because an increase in risk decreases risk taking. But there is also a "prudence effect", in the sense that risk also increases the marginal utility of effort. Indeed, when the rent is more risky, each agent has an additional incentive to exert effort under prudence. The intuition is that it is relatively more beneficial for a prudent agent to secure an increasing portion of a valuable rent under more risky conditions. Our result above thus reflects the tension between these two effects, a risk aversion effect and a prudence effect.

5 Conclusion

This paper has examined a contest with a shared risky rent. This model can be interpreted as a general model of "markets for influence" (Menezes and Quiggin 2010), with applications to marketing, advertising or lobbying. Our analysis delivers a simple message: risk-averse agents exert less efforts when they become (more) risk-averse or when the rent becomes (more) risky. Importantly, our model differs from the standard probabilistic contest game usually considered in the literature. Nevertheless, the main insight does not differ much. Indeed, it has been shown that risk aversion and risk tend to decrease effort in the standard contest model (Treich 2010, Cornes and Hartley 2012).

We must add, however, that the effect of risk in our model depends on a condition on the third derivative of the agents' utility function. Thus, there is an additional force beyond risk aversion, coined the "prudence effect", which leads to increased efforts under risky conditions (see Section 4.3). Moreover, we have examined the case with only concave impact functions (see A.1), thus introducing a common but severe constraint on the contest technology. Under convex impact functions, the message above must be qualified, or even reversed. More precisely, risk-averse agents can be shown to exert greater efforts under special conditions when the rent becomes more risky.[9] Finally,

[9] This result is available upon request to the corresponding author.

we should mention that an important limitation of our analysis is that we have considered only a symmetric game.

6 Appendix

Proof of Proposition 1. The proof follows from the proof of Proposition 3.1 in Corchón (2007). Rearranging terms, (5) can be written as follows:

$$\frac{\phi'(x_i)b_{-i}}{(\phi(x_i)+b_{-i})^2} = \frac{E\left[u'\left(\frac{\phi(x_i)}{\phi(x_i)+b_{-i}}\widetilde{v}-x_i\right)\right]}{E\left[\widetilde{v}u'\left(\frac{\phi(x_i)}{\phi(x_i)+b_{-i}}\widetilde{v}-x_i\right)\right]}.$$

Let $x_m = \min_{i\in I} x_i$ and $x_M = \max_{i\in I} x_i$. If the solution is not symmetric, then $x_m < x_M$. Since $\phi'(x_m)b_{-m} > \phi'(x_M)b_{-M}$ and $\phi(x_m)+b_{-m} = \phi(x_M)+b_{-M}$, we have

$$\frac{\phi'(x_m)b_{-m}}{(\phi(x_m)+b_{-m})^2} > \frac{\phi'(x_M)b_{-M}}{(\phi(x_M)+b_{-M})^2},$$

which in turn implies

$$\frac{E\left[\widetilde{v}u'\left(\frac{\phi(x_m)}{\phi(x_m)+b_{-m}}\widetilde{v}-x_m\right)\right]}{E\left[u'\left(\frac{\phi(x_m)}{\phi(x_m)+b_{-m}}\widetilde{v}-x_m\right)\right]} < \frac{E\left[\widetilde{v}u'\left(\frac{\phi(x_M)}{\phi(x_M)+b_{-M}}\widetilde{v}-x_M\right)\right]}{E\left[u'\left(\frac{\phi(x_M)}{\phi(x_M)+b_{-M}}\widetilde{v}-x_M\right)\right]}.$$

But this last inequality is not possible if the function $\Psi(x) = \frac{E\left[\widetilde{v}u'\left(\frac{\phi(x)}{B}\widetilde{v}-x\right)\right]}{E\left[u'\left(\frac{\phi(x)}{B}\widetilde{v}-x\right)\right]}$ is decreasing in x for any $B > 0$. We have

$$\Psi'(x) = \frac{E\left[(\frac{\phi'(x)}{B}\widetilde{v}-1)\widetilde{v}u''(.)\right]E\left[u'\left(\frac{\phi(x)}{B}\widetilde{v}-x\right)\right] - E\left[(\frac{\phi'(x)}{B}\widetilde{v}-1)u''(.)\right]E\left[\widetilde{v}u'\left(\frac{\phi(x)}{B}\widetilde{v}-x\right)\right]}{E^2\left[u'\left(\frac{\phi(x)}{B}\widetilde{v}-x\right)\right]}$$

$$= \frac{E\left[(\frac{\phi'(x)}{B}\widetilde{v}-1)\frac{u''}{u'}(.)\widetilde{v}u'\right]}{E\left[u'\left(\frac{\phi(x)}{B}\widetilde{v}-x\right)\right]} - \frac{E\left[(\frac{\phi'(x)}{B}\widetilde{v}-1)\frac{u''}{u'}(.)u'\right]}{E\left[u'\left(\frac{\phi(x)}{B}\widetilde{v}-x\right)\right]} \times \frac{E\left[\widetilde{v}u'\left(\frac{\phi(x)}{B}\widetilde{v}-x\right)\right]}{E\left[u'\left(\frac{\phi(x)}{B}\widetilde{v}-x\right)\right]}$$

Similarly to the proof of Proposition 6, we now introduce the following probability density function

$$m(v) = \frac{d(v)u'\left(\frac{\phi(x)}{B}\widetilde{v}-x\right)}{E\left[u'\left(\frac{\phi(x)}{B}\widetilde{v}-x\right)\right]},$$

where $d(v)$ is the probability density function of the random variable $\widetilde{v}$. With this new probability density function, we can rewrite the equality above as

$$\Psi'(x) = COV\left[\widetilde{v}, \left(\frac{\phi'(x)}{B}\widetilde{v} - 1\right)\frac{u''\left(\frac{\phi(x)}{B}\widetilde{v} - x\right)}{u'\left(\frac{\phi(x)}{B}\widetilde{v} - x\right)}\right]$$
$$= COV\left[\widetilde{v}, \frac{d}{dx}\ln\left[u'\left(\frac{\widetilde{v}}{B}\phi(x) - x\right)\right]\right].$$

Without loss of generality, we can normalize B to 1. Denoting $H(x,v) = \ln\left[u'(v\phi(x) - x)\right]$, the covariance is thus negative if and only if $\frac{\partial^2 H(x,v)}{\partial x \partial v} < 0$, which concludes the proof.■

7 References

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