Overbidding in First-Price Auctions: Risk Aversion vs. Probability Weighting Function

Olivier Armantier* Nicolas Treich†

January 2008

Abstract

There is a long standing debate about whether or not risk aversion under an expected utility framework may be considered the main source of overbidding in a first-price independent private values auction. As an alternative, we adopt a non-expected utility framework, and identify an interpretable property on the probability weighting function (PWF) which always induces overbidding. This property, called the star-shaped property, is weaker than the notion of “strong risk aversion” relevant in Rank-dependent expected utility models.

Keywords: Probability Weighting Function, Auctions, Overbidding, Non-expected Utility, Risk Aversion.

JEL Classification: C70, C92, D44, D81.

*Federal Reserve Bank of New York, Université de Montréal, CIRANO, and CIREQ.
†Toulouse School of Economics, (LERNA-INRA), Aile J.-J. Laffont, 21 all. de Brienne, 31042 Toulouse, France. Email: ntreich@toulouse.inra.fr.
1 Introduction

It has been repeatedly observed in experiments that subjects participating in first-price independent private values auctions tend to bid above the risk neutral Bayesian Nash equilibrium. This tendency to “overbid” has often been rationalized by risk aversion within the expected utility framework (see, e.g., Cox, Smith and Walker, 1985, 1988). Indeed, standard economic theory predicts that the equilibrium bidding strategy in first-price independent private values auctions is higher when the utility function is concave rather than linear (Milgrom and Weber, 1982). However, this rationale based on risk aversion to explain overbidding in experiments has led a number of criticisms.1 In particular, risk aversion is often viewed with skepticism to organize behavior in experiments due to the low financial incentives typically involved in laboratory experiments (Rabin, 2000).

Hence, there is still no consensus in the literature about the causes of overbidding in first-price independent private values auctions. In particular, Goeree, Holt and Palfrey (2002) adopt a non-expected utility approach and show that a convex probability weighting function (hereafter PWF) “fits the data just as well as the risk aversion model” (Goeree, Holt and Palfrey, 2002, p. 265).2 Likewise, Armantier and Treich (2007) estimate a model accounting for a non-linear PWF, and find that the concavity of the utility function play a lesser role than previously believed in explaining overbidding. Finally, numerous individual decision-making experiments have suggested that subjects weight non-linearly probabilities when making risky choices (Camerer, 1995; Dawes, 1998). It appears therefore natural to expect that agents may use a non-linear PWF in games where probabilities are involved, as it is typically the case in an auction.

This raises the question of the theoretical effect of a PWF in a standard first-price independent private values auction. The objective of the paper is to address this question. To do so, we consider one of the leading

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1 Among those criticisms, risk aversion cannot explain the observed persistent overbidding in second-price or third-price auctions. See the December 1992 issue of the American Economic Review for a sample of the debates around the causes of overbidding, and Kagel (1995) for an excellent literature survey.

2 In fact, Cox, Smith and Walker (1985) were the first to study first-price auctions allowing for a non-linearity in the probabilities. They also concluded that “two theories are observationally equivalent on the basis of all experimental tests to date” (Cox, Smith and Walker, 1985, p. 162).
non-expected utility model involving a PWF, the Rank-dependent expected utility model. This model has been able to organize data in a large number of single decision-maker experiments. However, the application of the Rank-dependent expected utility model to strategic situations has been scarce.\footnote{A possible explanation may be that non-expected utility models may induce unappealing properties, like time-inconsistent decisions in dynamic games. Likewise, non-linear probabilities may affect the behavior toward randomization, which may be problematic for the existence of mixed-strategy equilibria (Crawford, 1990). Some stronger conditions may thus be required to ensure the existence of equilibria under the Rank-dependent expected utility model (Ritzberger, 1996; Chen and Neilon, 1999).} In fact, the present paper constitutes, to the best of our knowledge, the first attempt at explaining a puzzle in a game with the Rank-dependent expected utility model. More specifically, we identify a simple property on the PWF, the so-called “star-shaped” PWF, that always induces overbidding in first-price independent private values auction. As we shall see, this property is weaker than the convexity of the PWF, but stronger than a mere underweighting of probabilities. Interestingly, the “star-shaped” property is relevant to characterize the notion of risk aversion in Rank-dependent expected utility models.

2 The Model

2.1 Risk Preferences

We use the Rank-dependent expected utility model (Quiggin, 1982), which is one of the best-known non-expected utility model involving a non-linear PWF.\footnote{There exist several versions of this model (Chew, 1983; Yaari, 1987; Viscusi, 1989; Luce, 1991; Luce and Fishburn, 1991; Gul, 1991; Tversky and Kahneman, 1992; Wakker and Tversky, 1993; Schmidt and Zank, 2001; Safra and Segal, 2001).} Assume that possible outcomes expressed in terms of final wealth $w_k$ are indexed by $k = 1, \ldots, m$, and are ranked such that $w_1$ is the worst and $w_m$ is the best. Let $p_k$ denote the probability of occurrence of $w_k$, with $\sum_{k=1}^{m} p_k = 1$. Under the Rank-dependent expected utility model, individuals’ preferences are characterized by

$$\sum_{k=1}^{m} \left\{ \Phi(\sum_{k'=k}^{m} p_{k'}) - \Phi(\sum_{k'=k+1}^{m} p_{k'}) \right\} u(w_k)$$

\footnote{A possible explanation may be that non-expected utility models may induce unappealing properties, like time-inconsistent decisions in dynamic games. Likewise, non-linear probabilities may affect the behavior toward randomization, which may be problematic for the existence of mixed-strategy equilibria (Crawford, 1990). Some stronger conditions may thus be required to ensure the existence of equilibria under the Rank-dependent expected utility model (Ritzberger, 1996; Chen and Neilon, 1999).}
where \( \Phi \) is a PWF. We assume that \( \Phi \) and \( u \) are strictly increasing and differentiable functions. Also, we assume that \( \Phi(0) = 0 \) and \( \Phi(1) = 1 \).\(^5\)

In this model, one can distinguish the effects of a concave utility function \( u \) - corresponding to the standard notion of risk aversion in the expected utility model - from that of a non-linear PWF \( \Phi \). When \( u(w) = w \), the model essentially reduces to the dual theory of Yaari (1987). When \( \Phi(p) = p \), we are back to the standard expected utility model. Notice also that the Rank-dependent expected utility model reduces to

\[
\Phi(p)u(w_2) + (1 - \Phi(p))u(w_1)
\]

in the case of a binary lottery, where \( p \) is the probability of the best outcome.

The Rank-dependent expected utility model has proved powerful to organize single individual decision-making data. In particular, it has been shown to explain experimental puzzles such as the Allais paradox and the common ratio effect (Camerer, 1995, Starmer, 2000). The most common PWF \( \Phi \) consistent with choice data is such that small probabilities are inflated and large probabilities are deflated (Kahneman and Tversky, 1979), giving rise to the common inverse S-shape PWF (Edwards, 1961; Tversky and Fox, 1995; Prelec, 1998; Gonzalez and Wu, 1999; Abdellaoui, 2000).

Some implications of this model for the theory of choice have been analyzed. In particular, it is known that this model enables one to distinguish between the notions of weak and strong risk aversion. An individual is weakly risk averse if and only if he always declines a random variable in favor of its expected value. An individual is strongly risk averse if and only if he always dislikes any mean-preserving spread of risk in the sense of Rothschild and Stiglitz (1970). Under expected utility, weak and strong risk aversion are equivalent to the concavity of \( u \). Under Rank-dependent expected utility, strong risk aversion is equivalent to \( u \) concave and \( \Phi \) convex (Chew, Karni and Safra, 1987), and weak risk aversion is equivalent to \( u \) concave and \( \Phi(p) \leq p \) for every \( p \in [0, 1] \) (Cohen, 1995).

### 2.2 The Auction Game

We consider a standard first-price independent private values auction model. \( N \) agents with identical risk preferences (i.e. identical \( u \) and \( \Phi \), as well as

\(^5\)An alternative model is \( \sum_{k=1}^{m} \Phi(p_k)u(w_k) \). Although more intuitive, it is well-known that this model does not satisfy the first-order stochastic dominance property, unlike the Rank-dependent expected utility model (see, e.g., Starmer, 2000).
identical initial wealth \( w \) participate in an auction in which they each submit a sealed bid for an indivisible object. Agent \( i = 1, ..., N \) has a private-value \( v_i \) for the object. This private-value is drawn independently from a distribution with cumulative \( F(\cdot) \), density \( f(\cdot) \) and support \([\underline{v}, \overline{v}]\). The highest bidder gets the object. His payoff is equal to his own valuation of the object minus his bid, \( v_i - b_i \). The other bidders receive no payoffs.

We denote \( p(b_i, B) \equiv P[b_i > B(v_j), \forall j \neq i] \) the probability that bidder \( i \) wins the auction when she selects a bid \( b_i \), while each of her opponents uses the bid function \( B \). A strategy \( B^\ast \) is then a symmetric Bayesian Nash equilibrium of this game if it satisfies the following optimization and fixed point problems,

\[
B^\ast(v_i) = \text{ArgMax}_{v \leq b_i \leq v_i} \Phi(p(b_i, B^\ast))u(w + v_i - b_i) + (1 - \Phi(p(b_i, B^\ast)))u(w) \\
\forall v_i \in [\underline{v}, \overline{v}] \text{ and } \forall i = 1, ..., N. \tag{1}
\]

In words, \( B^\ast(v_i) \) is bidder \( i \)'s best-reply when the other bidders' select the equilibrium strategy \( B^\ast(\cdot) \), given that all bidders have the same preferences. The only difference compared to the standard first-price independent private values auction model is that the PWF \( \Phi \) may be non-linear.

If we restrict our attention to monotonic strategies, then the maximization in (1) is equivalent to maximizing over \( b_i \)

\[
\Phi(F(B^{\ast-1}(b_i))^{N-1})U(v_i - b_i)
\]

where \( B^{\ast-1}(\cdot) \) stands for the inverse of \( B^\ast(\cdot) \), and \( U \) is an indirect utility function defined by \( U(x) = u(w + x) - u(w) \). Notice that \( U \) is concave if and only if \( u \) concave.

Differentiating with respect to \( b_i \) yields

\[
\Phi'(F(B^{\ast-1}(b_i))^{N-1})(N - 1)F(B^{\ast-1}(b_i))^{N-2} \frac{f(B^{\ast-1}(b_i))}{B^\ast(B^{\ast-1}(b_i))}U'(v_i - b_i) - \Phi(F(B^{\ast-1}(b_i))^{N-1})U'(v_i - b_i).
\]

Setting this expression equal to zero gives the first order differential equation

\[
B^\ast'(v_i) = (N - 1)f(v_i) \frac{\Phi(F(v_i)^{N-1})F(v_i)^{N-2} U(v_i - B^\ast(v_i))}{\Phi(F(v_i)^{N-1})} U'(v_i - B^\ast(v_i)) \forall v_i \in [\underline{v}, \overline{v}]. \tag{2}
\]

Together with the boundary condition \( B^\ast(\underline{v}) = \underline{v} \), this differential equation characterizes a Bayesian symmetric Nash equilibrium bidding behavior in an auction where bidders have a PWF \( \Phi(\cdot) \).
3 The Effect of a Non-linear PWF

3.1 The Star-shaped PWF and the Main Result

We introduce a specific class of PWF that will permit to sign the effect of a non-linear PWF on the equilibrium bidding strategy compared to a linear PWF. We coin this class the star-shaped PWF.

Definition Let a PWF $\Phi(p)$ with $\Phi(0) = 0$ and $\Phi(1) = 1$; then $\Phi(p)$ is star-shaped if $\Phi(p)/p$ is non-decreasing in $p$.

The term star-shaped is taken from Chateauneuf, Cohen and Meilijson (2004).6 As explained in Chateauneuf, Cohen and Meilijson (2004), a star-shaped PWF is useful to capture risk aversion toward one of their four specific types of increase in risk, that is, a right-monotone increase in risk.7 In short, this type of increase in risk corresponds to a risk spread which is limited to the domain of gains. To have an intuition for this, consider a lottery which gives $x/p$ with probability $p$, or 0 otherwise. The expectation of this lottery is $x$. Notice that a decrease in $p$ corresponds to a specific mean-preserving spread in the domain of gains. Denote $C(p)$ the risk premium toward this lottery, defined by $u(w + x - C(p)) = \Phi(p)u(w + x/p) + (1 - \Phi(p))u(w)$; for a $u$ linear we thus have $C(p) = x(1 - \Phi(p)/p)$. Hence, assuming a linear utility function, the risk premium increases in this particular type of mean-preserving spread if and only $\Phi$ is star-shaped.

We have plotted on Figure 1 a star-shaped PWF. A star-shaped PWF implies that the chord to the PWF drawn from 0 to $p$ must lay above $\Phi(p)$ for every $p$. This property is equivalent to assuming that the slope of the chord is lower than the slope of the tangent to $\Phi(p)$ at $p$, namely

$$\Phi(p)/p \leq \Phi'(p)$$

for every $p \in [0, 1]$. (3)

Figure 1 makes it clear that this last inequality is satisfied at $p_1$ and $p_2$.

It is immediate that any convex PWF is star-shaped. Hence the notion of strong risk aversion as defined Chew, Karni and Safra (1987) implies that, together with $u$ concave, the PWF is star-shaped in the Rank-dependent

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6A function $\Phi$ is star-shaped at $\mu$ if $(\Phi(x) - \Phi(\mu))/(x - \mu)$ is non-decreasing in $x$. The exact term would thus be that $\Phi$ is star-shaped at 0. See also Landsberger and Meilijson (1990)’s paper where the characterization is applied to utility functions.

7The four types are respectively i) a mean-preserving increase in risk, ii) a monotone increase in risk, iii) a left-monotone increase in risk and iv) a right-monotone increase in risk.
expected utility model. Furthermore, observe that $\Phi$ star-shaped implies underweighting everywhere, i.e. $\Phi(p) \leq p$. Hence, assuming that $u$ is concave, a star-shaped PWF implies weak risk aversion.

We are now in a position to show that a star-shaped PWF is a sufficient condition on the PWF to increase the equilibrium bidding strategy compared to a linear PWF, that is compared to the equilibrium bidding strategy of expected utility maximizers. Let us compare $B^*(v)$, the symmetric equilibrium bidding strategy under a PWF $\Phi$ as given by the general condition (2), to the corresponding equilibrium condition in the expected utility case ($\Phi(p) = p$) denoted $B^*_0(x)$. Assume that $\Phi$ is star-shaped, or equivalently that $\Phi_0(p) \geq \Phi(p)/p$ for every $p \in [0, 1]$. Then, we get

$$B^*_0(v) - B^*_0(v) = (N - 1)f(v)F(v)^{N-2}\left[\frac{\Phi'(F(v)^{N-1})}{\Phi(F(v)^{N-1})} U(v - B^*(v))\right]$$

$$\geq (N - 1)f(v)\left[U(v - B^*(v)) - U(v - B^*_0(v))\right]$$

by assumption and since $U, U'_0, \Phi$ and $\Phi'$ are positive.

From the last inequality, we have that, for any $v$, $B^*(v) = B^*_0(v)$ implies $B^*(v) \geq B^*_0(v)$. We thus have a single crossing property. This property means that the function $B^*(.)$ can only cross the function $B^*_0(.)$ from below. Since $B^*(v) = B^*_0(v) = v$, the function $B^*(v)$ will always be larger than $B^*_0(v)$ for any $v$ such that $v \geq \underline{v}$. Therefore, individuals increase their bids when the PWF $\Phi$ is star-shaped compared to a linear PWF (i.e., compared to the expected utility model). Also, it is easy to generalize the Milgrom and Weber (1982)’s result to show that a concave $u$ always leads to increase the equilibrium bid function compared to a linear $u$, even in the presence of a non-linear PWF.\(^9\)

We now derive a closed-form solution of the equilibrium bidding strategy that will illustrate the effect of a non-linearity both in the PWF and in

\(^8\)Suppose it is not the case. Then there exists $p_0$ such that $\Phi(p_0) > p_0$. As a result, $\Phi(p_0)/p_0 > 1 = \Phi(1)/1$, which contradicts $\Phi$ star-shaped.

\(^9\)The proof is similar to the one above, based on the single-crossing property and on the fact that $U(x)/U'(x)$ is always larger than $x$ under $U$ concave and $U(0) = 0$. 

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the utility function. To do so, we consider the special case of power utility function \( u(w) = w^{1-r}/(1 - r) \) where \( r \) may be interpreted as the constant relative risk aversion parameter in the traditional expected utility framework. We also assume a zero initial wealth so that we have \( U(x) = x^{1-r}/(1 - r) \) together with \( r \in [0, 1] \). Note that most empirical estimations of first-price auction models rely on a similar specification of the utility function. Moreover, we assume \( \Phi(p) = p^\beta \), with \( \beta \geq 1 \). Therefore, \( \Phi \) is star-shaped since it is convex. We finally assume a uniform distribution \( F(v) = v \) over the support \([0, 1]\). Under these assumptions, we derive a closed-form solution to the differential equation (2), which in this case simply reduces to

\[
B(v) = \beta v (1 + \beta - r)^{-1}
\]

for the two-bidder case.

We can draw some observations from this closed-form solution. First, even if we have introduced a convex PWF the bidding strategy remains a linear function of the private value, as in the expected utility model using a power utility function (see, e.g., Cox, Smith and Walker, 1988). Second, compared to a linear PWF (i.e. \( \beta = 1 \)), a star-shaped PWF (i.e. \( \beta > 1 \)) yields higher equilibrium bid, consistent with our main result. Third, the equilibrium bid can increase indifferently with either \( \beta \) the curvature of the PWF, or \( r \) the curvature of the utility function; this simple first-price auction model therefore illustrates the identification problem between the curvature of the PWF and that of the utility function.

### 3.2 Intuition in a Single Decision-Maker Environment

Our main result relies on a trade-off faced by each participant in an auction. To understand this trade-off intuitively, we simplify the problem by assuming that an agent considers the strategies of his opponents as fixed.\(^{10}\) Consider an agent who maximizes over \( b \) the objective function

\[
\Phi(p(b))U(v - b),
\]

\(^{10}\)See, e.g., Gradstein, Nitzan and Slutsky (1992) for examples in which simplifying the environment to a single decision-maker context provides intuitive insights into the equilibrium behavior in a game.
where \( p(b) \) is the probability of getting a prize of value \( v \), and \( \Phi(.) \) is a PWF.\(^{11}\) Assume that this objective function is concave in \( b \). Notice that if \( \Phi(p) = p \) the solution is simply given by \( b^* \) solving

\[
p'(b^*)U(v - b^*) - p(b^*)U'(v - b^*) = 0. \tag{6}
\]

This condition simply reflects the trade-off between the marginal benefit of increasing the probability of winning \( p'(b^*)U(v - b^*) \) and the marginal cost of reducing the value of the payoff \( p(b^*)U'(v - b^*) \).

Our objective is to compare the maximizer of (5) to \( b^* \). This maximizer will be higher if and only if the slope of the tangent of \( \Phi(p(b))U(v - b) \) at \( b^* \) is positive. This is equivalent to

\[
\Phi'(p(b^*))p'(b^*)U(v - b^*) - \Phi(p(b^*))U'(v - b^*) \geq 0. \tag{7}
\]

Using condition (6) we have \( p'(b^*) = \frac{p(b^*)U'(v - b^*)}{U(v - b^*)} \), so that the inequality (7) simply reduces to \( \Phi'(p(b^*))p(b^*) \geq \Phi(p(b^*)) \). Since \( b^* \) can take any value (as it depends on the parameters of the model) this last inequality is thus equivalent to (3). Hence, a star-shaped PWF leads to increase the optimal value of \( b \) compared to \( b^* \) in this simple decision-making environment.

Let us now interpret this result. Observe that the result derives from the comparison of (6) to (7). This comparison shows the tension between two effects: i) an increase in the probability of winning and ii) a decrease in the payoff contingent on winning. How does the shape of the PWF affect each of these two effects? First, there is the effect of the change in the probability of winning. Directly comparing the first terms on the left hand side of (6) and (7) shows that this effect is controlled by \( \Phi'(p) \) compared to 1. This means that when \( \Phi'(p) \) is larger than 1, increasing \( b \) by one unit is perceived as relatively more profitable at the margin, so that the agent has an incentive to increase \( b \). Second, there is the effect related to the decrease in the amount of the payoff. Comparing the second terms on the left hand side of (6) and (7) shows that this second effect is controlled by \( \Phi(p) \) compared to \( p \). When the probability is underweighted \( \Phi(p) \leq p \), this effect leads to reduce the perceived cost associated with the reduced payoff, so that this effect gives the agent an incentive to increase \( b \) as well.

\(^{11}\)Notice that this model resembles a self-protection model in which an agent selects an effort to increase the probability of a good outcome, the bad outcome being normalized to zero (which could be interpreted as the utility of death assuming no bequest motive).
Hence the two conditions i) $\Phi'(p) \geq 1$ and ii) $\Phi(p) \leq p$ give an incentive to increase $b$. However, there does not exist any continuous PWF with $\Phi(0) = 0$ and $\Phi(1) = 1$ together with $\Phi(p) \leq p$ such that $\Phi'(p) \geq 1$ hold for every $p \in [0, 1]$, unless $\Phi(p) = p$. Hence, there is no hope of finding a non-linear PWF for which both effects go in the same direction for every $p$. Yet, our result shows that the aggregate effect, that simply condenses the two effects mentioned above, critically depends on condition (3), namely on whether the PWF $\Phi$ is star-shaped.

4 Estimates of Risk Aversion in First-Price Auctions

In this section, we briefly discuss some issues related to the estimation of risk aversion in first-price independent private values auction. Using experimental and field data, several attempts have been made at estimating the bidders risk aversion within the expected utility framework. Most of these studies rely on the closed-form solution (4). The constant relative risk aversion parameter $r$ is then estimated under the constraint $\beta = 1$. The resulting estimates of $r$ tend to lie around 0.6 (see e.g. Harrison 1990, Cox and Oaxaca 1996, Chen and Plott 1998, Campo, Perrigne and Vuong 2001, as well as Pezanis-Christou and Romeu 2003). This value for $r$ may be considered low compared to values obtained in the risky choice literature (e.g., in finance or insurance), usually larger than 1, and often in the vicinity of 3 or 4 (see, e.g., Barsky, Juster, Kimball and Shapiro, 1997).

This comparison to other risk aversion estimates is, however, somewhat misleading. Indeed, auction models are typically estimated under the assumption that wealth $w$ is equal to 0. Yet, we may expect constant relative risk aversion estimates to increase with wealth. To understand why, it is useful to look at the equilibrium condition (2). Observe indeed that the utility function plays a role only through the index $U(x)/U'(x)$ where $x = v_i - B(v_i)$. Notice that this index is equal to $(u(w + x) - u(w))/u'(w + x)$; this index may thus be affected by a change in risk aversion, or by a change in wealth. For instance, assuming a power utility function with $r = 0.6$ and a zero wealth $w = 0$ this index is equal to $2.5x$. Yet, if $w = 10$, then $r$ must be equal to

\footnote{Surprisingly, this assumption has rarely been discussed in the first-price auctions literature. In particular, notice that it constrains $r$ to be lower than one.}
17 for the index to remains equal to 2.5x at x = 1 (and if w = 100, r must be approximately equal to 164!). In other words, much higher (and arguably implausible) levels of risk aversion are required for risk aversion to have the same effect on bidding behavior if wealth becomes positive. This simple exercise therefore raises serious doubts about the ability of risk aversion in an expected utility framework to fully explain the overbidding puzzle.

The above remark is in line with the common criticism that the expected utility framework may provide inaccurate predictions of behavior in lottery choice experiments. Specifically, it is in line with the idea that risk aversion can hardly organize behavior in experiments due to the low incentives involved (Rabin, 2000). As an alternative, Goeree, Holt and Palfrey (2002) in their section 7 consider a Rank-dependent expected utility model. However, they assume a linear utility function (as in Yaari, 1987), that is, they assume $r = 0$. Further assuming a power form for the PWF, they obtain an estimated value for $\beta$ in the range of 2. Notice that this value is not surprising since the pair $(r, \beta) = (0, 2)$ yields an equilibrium bidding strategy in (4) roughly equivalent to the pair $(r, \beta) = (0.6, 1)$, namely roughly equivalent to a constant relative risk aversion parameter equal to 0.6 consistent with previous empirical studies assuming expected utility.

To the best of our knowledge, Armantier and Treich (2007) is the only study in which $r$ and $\beta$ are jointly estimated. To circumvent the identification problem between the two parameters, Armantier and Treich (2007) conduct an experiment in which they both elicit probabilistic beliefs and the bidding strategy. They assume a linear utility function (as in Yaari, 1987), that is, they assume $r = 0$. Further assuming a power form for the PWF, they obtain an estimated value for $\beta$ in the range of 2. Notice that this value is not surprising since the pair $(r, \beta) = (0, 2)$ yields an equilibrium bidding strategy in (4) roughly equivalent to the pair $(r, \beta) = (0.6, 1)$, namely roughly equivalent to a constant relative risk aversion parameter equal to 0.6 consistent with previous empirical studies assuming expected utility.

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13 Armantier and Treich (2007)’s experimental outcomes indicate that subjects underestimate their probability of winning the auction, and overbid. Nevertheless, when provided with feedback on the precision of their predictions, subjects learn to make better predictions, and to reduce their tendency to overbid.

14 Note that the PWF may be non-linear for two reasons, i) because subjects misperceive the probabilities of winning the auction, and ii) because of the decision weights attached by subjects when making risky choices. Armantier and Treich (2007) only estimate the first type of non-linearities. As a result, they cannot distinguish between risk aversion and decision weights. Hence, Armantier and Treich (2007) may still overestimate the risk aversion parameter $r$, as they do not account for decision weights.
5 Conclusion

We find that the overbidding commonly observed in first-price independent private values auctions could be fully rationalized by a non-linear PWF. An alternative interpretation of this result is that explaining overbidding in the presence of a non-linear PWF may require a lower level of risk aversion than previously assumed in the literature within the expected utility framework. This last interpretation is relevant for two reasons: first, it has been suggested that the levels of risk aversion estimated to explain overbidding in experimental auctions may be considered unreasonably high. Second, it is consistent with the conclusions of the recent experimental studies of Goeree, Holt and Palfrey (2002) and Armantier and Treich (2007).

We finally emphasize, however, that our main result need not be inconsistent with the general idea that “risk aversion” is the main determinant of overbidding in first-price independent private values auctions. Indeed, the particular class of PWF justifying overbidding identified in our paper is consistent with broader notions of risk aversion that have been proposed in Rank-dependent expected utility models (Chew, Karni and Safra, 1987; Chateauneuf, Cohen and Meilijson, 2004).
References


Figure 1
A Star-Shaped Probability Weighting Function