The Power of Money: Wealth Effects in Contests*

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JEL codes: C72, D72.

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Abstract

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"Pecunia nervus belli."

1 Introduction

1.1 The general motivation

As popularized by Frank and Cook's (1995) best-selling book "The Winner-Take-All Society" many competitive situations in modern economies take the form of a contest. Examples include political lobbying, research and development, marketing, promotion, status-seeking, and litigation activities (Konrad 2009). In this paper, we are interested in the effect of wealth in contests. In particular, the motivation for our analysis includes general questions such as: Do rich people lobby more? Does poverty lead to more conflicts? Does low economic growth and wealth inequality increase redistributive politics?

The relationship between wealth and power has attracted attention for centuries (Marx 1867, Wright Mills 1956). The conventional wisdom suggests that the rich are more powerful than the poor.[1] Bartels (2005) concludes, for instance, that US senators are considerably more responsive to the opinions of their more affluent constituents (see also Gilens 2005). This idea has been extensively discussed in political science, as exemplified by Hacker and Pierson (2011)'s recent book "Winner-Take-All Politics: How Washington Made the Rich Richer–and Turned Its Back on the Middle Class". Nevertheless, in contrast, casual observation suggests that low wealth induces greater participation and effort in contest-type situations. People involved in highly predatory and competitive activities, such as thieves or athletes for instance, typically come from poorer segments of society. More corruption is also typically observed in poorer countries (Aidt 2009, Gundlach and Paldam 2009). Some groups (e.g., farmers), although often relatively poor,

[1] This is consistent with the beliefs of some prominent economists. For instance, Anne Krueger (1974), in her pioneering work on rent seeking argues that we can perceive the price system "as a mechanism rewarding the rich and well-connected". Likewise Jack Hirshleifer (1995) stresses that "the half of the population above the median wealth surely has greater political strength than the half below". Paul Krugman (2010) similarly observes that "the rich are different from you and me: they have more influence". Lastly, Daron Acemoglu (2012) declares that "the rise in inequality has created a class of very wealthy citizens who can use their wealth to gain more political power — partly to defend their wealth and partly to further their economic, political, and ideological agendas".

are well-known to be politically powerful. As a result, redistributive politics almost always goes from the rich to the poor. Poverty has also been found to be a robust factor in explaining violent crime and civil conflicts (Collier and Hoeffler 1998, Fajnzylberg, Lederman and Loayza 2002, Fearon and Laitin 2003, Blattman and Miguel 2010). Relatedly, it is often said that redistribution policies favour political stability and social peace.

1.2 The model, and the basic effects

Although the above observations concern disparate issues, they suggest that wealth may have fundamentally different, and perhaps opposing, effects in contests. This further suggests that economic theory may help us pin down and examine the strength of these effects. Yet, wealth effects have received little attention in the (otherwise vast) theoretical literature on contests (Tullock 1980, Garfinkel and Skaperdas 2007, Konrad 2009, Congleton, Hillman and Konrad 2010). Indeed, the "workhorse" model in this literature is based on a strategic game in which wealth plays essentially no role. In this game, each agent has the following payoff function:

$$U_i = w_i - x_i + \Pi_i r, \tag{1}$$

in which x_i is agent i's effort, r is the rent (i.e., the prize) for the contest winner and Π_i is the probability of winning the contest. Notice immediately then that individual wealth w_i enters separately in the payoff function (1), and thus has no effect on the agent's effort (hence, wealth is, without loss of generality, normalised to zero in the literature).

In this paper, we adapt minimally this basic contest model in order to examine the effects of wealth. Namely, we introduce in model (1) a utility function that displays the familiar property of decreasing marginal utility of wealth. Note that this introduction requires to specify whether the rent r and efforts x_i can be expressed in monetary terms, and thus are commensurable with wealth w_i. This specification is central. Indeed, it technically removes the separability of wealth with the rent and efforts in model (1). Moreover, it permits to pin down two basic effects that we believe should naturally arise in contests:

- First, wealth can reduce the marginal cost of effort. To illustrate, note that it is marginally less costly for a rich person than a poor person to offer a monetary payment to, e.g., a politician, in order to obtain some privilege. The rich can thus relatively more easily afford costly expenditures in a contest than the poor, other things being equal.

- Second, and in contrast to the first effect, wealth may decrease the marginal benefit of winning a contest. To illustrate, note that it is marginally more beneficial for the poor to obtain the monetary reward associated with victory in a contest. We may thus regard the poor as being relatively more motivated to exert effort in a contest than the rich, other things being equal.

Although these effects are simple and intuitive, their analysis is not trivial because of strategic considerations. If a change in wealth changes the level of effort of one player, the other player is expected to react to this change, which in turn affects the initial player and so on. There is thus a need to carefully examine the overall impact of wealth on the players' equilibrium efforts.

1.3 The organization of the paper

The paper is organized as follows. In the next Section, we define the general set-up of our models and derive some preliminary results. Using a simple single-crossing property of best response functions, we develop a comparative statics method to study wealth effects in a wide class of strategic games with two players. Our method permits to compare the levels of efforts of a poor and a rich within an equilibrium. It also permits to compare these levels across equilibria that differ only by a change in the wealth of one or both players. We characterize wealth effects both on individual efforts and on aggregate efforts, and also study the effect of a wealth transfer among the two players.

The following Sections then turn to the application of this method to the analysis of wealth effects in strategic contest games. As we said above, two wealth effects typically arise in contests: i) wealth decreases the marginal cost of effort, but also ii) decreases the marginal benefit of winning the contest. We thus introduce three strategic contest models depending on whether the first, second, or both wealth effects play a role, and further sharpen our characterizations of the wealth effects for utility functions that have a constant absolute risk averison (CARA), a constant relative risk aversion (CRRA) or are quadratic.

We consider in Section 3 a model in which only the first effect on marginal cost described above is active, the so-called "privilege contest" model. In this model, effort is monetary, but the rent —i.e., the privilege— is non-monetary and therefore its marginal value is independent of the level of wealth. We then consider in Section 4 a model in which only the second effect described above on the marginal benefit is active, the so-called "ability contest" model. In this alternative model, rent is monetary but effort —which determines ability— is non-monetary and so the marginal cost of effort is independent of wealth. We show that the effect of increasing wealth on agent effort is positive in the privilege contest model while it is negative in the ability contest model. We also examine the effect of wealth redistribution in both models, and find that this effect tends to decrease aggregate effort when the decisiveness of the contest (to be defined precisely in Section 2) is sufficiently low.

We then move to study in Section 5 a model in which both the rent and the efforts are monetary, the so-called "rent-seeking contest" model, corresponding to the rent-seeking model with risk aversion (Cornes and Hartley 2012). In this model, we show that under CARA, the two opposing wealth effects exactly offset each other so that wealth has no effect on the efforts of agents. Moreover, we show that wealth tends to increase effort under a specific condition on the utility. This condition appeared in the single-agent risk theory literature (Eeckhoudt, Gollier and Schlesinger 1996), and is such that more background risk increases risk aversion. We demonstrate that under this condition, a rich agent exerts relatively more effort than a poor agent, and that an isolated increase in the wealth of the rich agent always increases that agent's effort, but reduces the effort of the poor agent.

Finally, in Section 6 we discuss other possible wealth effects previously identified in the literature (Grossman 1991, Hirshleifer 1991, Skaperdas and Gan 1995, Che and Gale 1997). Section 7 concludes our analysis.

2 General set-up, and the comparative statics method

In our analysis, we study the effects of several types of wealth changes. We first present a general method and some preliminary results about the conditions that determine the sign of all these wealth effects in a general class of strategic models. This class includes the three contest models considered in the remainder of the paper.[2] Theorem 1 below provides a simple single crossing property that will turn out to be instrumental throughout the paper, while Theorem 2 derives a condition for signing the effect of a mean-preserving spread (MPS) in wealth. Section 2.2 discusses the assumptions on and properties of the contest success function (CSF).

2.1 Assumptions and some preliminary results

We consider a strategic game with two players, $i = a, b$, in which the only source of heterogeneity is wealth w_i. We assume without loss of generality that a is more wealthy than b: $w_a \geq w_b$ (with $w_a = w_b$ corresponding to the symmetric situation). Each player i chooses an effort level x_i from a convex feasible set. These effort levels determine the probability of winning a prize r. The CSF mapping the efforts into a winning probability is specified and discussed in the next sub-section. The nature of the prize, and the functions describing the utility over final wealth and the cost of effort are contest-dependent and we will specify them in Sections 3-5 below.

Since we use the comparative statics method, it is important to stress that we will focus on the part of the parameter space for which a unique pure strategy Nash equilibrium exists. The parameter space is made up of the two wealth levels, the CSF decisiveness parameter (to be discussed in Section 2.2), the size of the prize and the curvature parameters of the players' utility and cost functions. The theoretical literature on contest games

[2]Our strategic contest models belong to the class of "aggregative games" for which each individuals' payoffs only depend on their own effort and on the aggregate efforts of all players. It has been shown that aggregative games display special features that make their analysis simpler under some conditions (Bergstrom and Varian 1985, Corchon 1994, Acemoglu and Jensen 2013). Nevertheless, the following preliminary results are fairly general, and not restricted to aggregative games.

(Szidarovszky and Okuguchi 1997, Yamazaki 2009) has provided a set of sufficient conditions for a unique Nash equilibrium in pure strategies to exist. In section A.2 of the appendix, we cast these conditions in terms of our three contest models. They mainly consist of a tight upper bound on the decisiveness parameter of the CSF and we discuss how to relax these bounds in Section 2.2.

It is convenient to denote the best-response functions as $f(x_b, w_a)$ and $g(x_a, w_b)$ for players a and b, respectively, where x_i denotes the effort of player i. The effort levels (x_a, x_b) constitute a Nash equilibrium for the game with initial wealth (w_a, w_b) when

$$x_a = f(g(x_a, w_b), w_a), \tag{2}$$

$$x_b = g(f(x_b, w_a), w_b). \tag{3}$$

We write $x_a(w_a, w_b)$ and $x_b(w_a, w_b)$ as the unique pair of equilibrium effort levels for this game.

We now introduce the following single-crossing property.[3]

Theorem 1 *Suppose that* $x_a = x_b \Longrightarrow \frac{\partial x_a(w_a,w_b)}{\partial w_a} > (<) \frac{\partial x_b(w_a,w_b)}{\partial w_a}$. *Then* $w_a > w_b \Longrightarrow x_a(w_a, w_b) > (<) x_b(w_a, w_b)$.

This theorem implies that when $\frac{\partial x_a(w_a,w_b)}{\partial w_a}|_{w_a=w_b} > \frac{\partial x_b(w_a,w_b)}{\partial w_a}|_{w_a=w_b}$, player a exerts more effort than player b (when $w_a \geq w_b$). Thus, to compare within an equilibrium the relative effort of the rich and poor player, it is sufficient to examine at the symmetric equilibrium how each player comparatively reacts to an increase in the wealth of player a. This result is illustrated in Figure 1.

[3] All theorems are proven in the appendix.

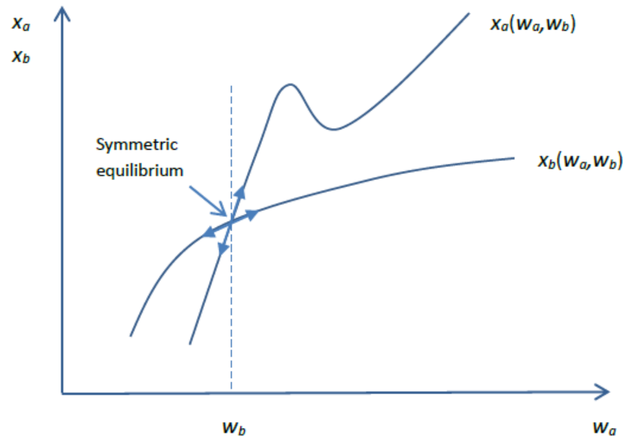

Figure 1. Single crossing of the best reponse functions.

We have characterized a property of the equilibrium in an asymmetric game. In addition, we assume in the following that the condition $1-f_1g_1>0$ is always satisfied. Note that this is the case if we assume that the equilibrium is locally stable, or $|f_1g_1|<1$ (see, e.g., Mas-Colell, Whinston and Green 1995, p. 414). We discuss this condition in detail in appendix A.3. for our three contest models.

Implicit differentiation of (2)-(3) gives the effects of isolated increases in wealth:

$$\frac{\partial x_a}{\partial w_a} = \frac{f_2}{1-f_1g_1}, \tag{4}$$
$$\frac{\partial x_b}{\partial w_a} = \frac{g_1f_2}{1-f_1g_1}, \tag{5}$$
$$\frac{\partial x_a}{\partial w_b} = \frac{f_1g_2}{1-f_1g_1}, \tag{6}$$
$$\frac{\partial x_b}{\partial w_b} = \frac{g_2}{1-f_1g_1}, \tag{7}$$

where the numerical subscripts with f and g denote partial derivatives and these functions are all evaluated at equilibrium. Thus, an increase in w_a increases player a's effort if and only if ("iff" hereafter) $f_2>0$ and increases player's b effort iff $g_1f_2>0$. The corresponding effects on aggregate effort are

$$\frac{\partial x_a}{\partial w_a}+\frac{\partial x_b}{\partial w_a}=\frac{f_2(1+g_1)}{1-f_1g_1}, \text{ and } \frac{\partial x_a}{\partial w_b}+\frac{\partial x_b}{\partial w_b}=\frac{g_2(1+f_1)}{1-f_1g_1}. \tag{8}$$

In a symmetric equilibrium (SE), $f_i = g_i$ ($i = 1, 2$). In that case, the change in individual effort following a common wealth increase is

$$\frac{\partial x_i}{\partial w_i}\Big|^{\mathrm{SE}}_{\mathrm{d}w_a=\mathrm{d}w_b} = \frac{f_2}{1-f_1}. \tag{9}$$

Finally, when wealth is redistributed from b to a, $\mathrm{d}w_a = -\mathrm{d}w_b$. Then

$$\frac{\mathrm{d}x_a}{\mathrm{d}w_a}\Big|_{\mathrm{d}w_a=-\mathrm{d}w_b} = \frac{f_2 - f_1 g_2}{1 - f_1 g_1}, \text{ and } \frac{\mathrm{d}x_b}{\mathrm{d}w_a}\Big|_{\mathrm{d}w_a=-\mathrm{d}w_b} = \frac{g_1 f_2 - g_2}{1 - f_1 g_1}.$$

In a symmetric equilibrium, a wealth transfer from b to a has no first-order effect on aggregate effort since

$$\frac{\mathrm{d}x_a}{\mathrm{d}w_a}\Big|^{\mathrm{SE}}_{\mathrm{d}w_a=-\mathrm{d}w_b} = -\frac{\mathrm{d}x_b}{\mathrm{d}w_a}\Big|^{\mathrm{SE}}_{\mathrm{d}w_a=-\mathrm{d}w_b} = \frac{f_2}{1+f_1}.$$

The second-order effect of such an MPS in wealth is given by the following theorem.

Theorem 2 *Consider a symmetric equilibrium. Let the stability condition $f_1^2 < 1$ be satisfied. The second-order effect of a MPS in wealth $dw_a = -dw_b$ on aggregate effort $x_a + x_b$ is given by*

$$\frac{(f_2)^2 f_{11} - 2(1+f_1) f_2 f_{12} + (1+f_1)^2 f_{22}}{(1+f_1)(1-f_1^2)}. \tag{10}$$

The numerator is a quadratic form in the Hessian of $f(\cdot)$.[4] *The denominator is positive under the stability condition.*

2.2 The contest success function

In standard strategic contest games, the contested rent, r, is indivisible in the sense that the winner takes all. Moreover, the players exert efforts, denoted x_i ($i = a, b$), to increase the probability of winning the rent (Nitzan 1994). For any player i, the probability of winning the contest, i.e., the CSF, is denoted $\Pi_i \equiv \Pi_i(x_a, x_b)$. Very often, we will denote the probability of a

[4] It can be written as $(-f_2, 1+f_1)\begin{pmatrix} f_{11} & f_{12} \\ f_{12} & f_{22}\end{pmatrix}\begin{pmatrix} -f_2 \\ 1+f_1 \end{pmatrix}$. Moreover, it can be easily checked that this form equals zero under the conditions identified in the theorem in Bergstrom and Varian (1985, p. 717). These conditions ensure that the distribution of agent characteristics has no effect on aggregate effort.

winning as $p(x_a, x_b)$ such that $\Pi_b = 1 - p(x_a, x_b)$, and the results will be given in terms of restrictions on (the derivatives of) $p(x_a, x_b)$. It is well known that the CSF plays a key role in strategic contest models, and this will also be the case in our analysis. We discuss here some of its key properties.

We assume throughout that the CSF has the power-logistic form (Tullock 1980):[5] i.e.,

$$p(x_a, x_b) = \frac{x_a^m}{x_a^m + x_b^m}, \tag{11}$$

with $x_i > 0$ $(i = a, b)$ and $m > 0$. If a exerts twice as much effort as b, the odds that he wins over b is $2^m : 1$. The parameter m is thus the contest-decisiveness parameter measuring how important relative effort $(\frac{x_i}{x_j})$ is compared to random factors for winning the contest (Hirshleifer 1991). If $m \to 0$, each player wins the contest with probability $\frac{1}{2}$ independently of the levels of effort. The larger m, the more pronounced the effects of relative differences in effort on the likelihood of winning.[6]

While this CSF is increasing and homogeneous of degree zero in its arguments, concavity is only guaranteed for arbitrary effort levels when $m \leq 1$. Because of the salient use of the "workhorse"-model (1) in the literature, the assumption that $m \leq 1$ is prevalent.[7] The reason for this is that it guarantees player i's payoff function to be concave in her strategy x_i, whatever the strategy of the opponent player. From an empirical stance, this assumption may

[5] This CSF is a special case of logistic class of functions, $p(x_a, x_b) = \frac{\Phi(x_a)}{\Phi(x_a)+\Phi(x_b)}$. Garfinkel and Skaperdas (2007) and Konrad (2009) discuss the axiomatic foundations and economic illustrations for this special, but common, class of CSFs. Moreover, Jia (2008) shows that this contest function can also be motivated as the probability of winning a rank-order tournament (as in Lazear and Rosen, 1981), when the noise terms are drawn from the inverse exponential distribution (see also Jia *et al.*, 2013). Under these conditions, the contests that we consider may therefore also be considered as rank-order tournaments.

[6] To illustrate, Hwang (2009) provides an idea about the order of magnitude of m in some contests. Using data from battles fought in 17th century Europe and during World War II, he obtains values of .704 (.120) and 3.420 (.678), respectively (standard errors in brackets).

[7] As an exception in the strategic contests literature, Perez-Castrillo and Verdier (1992) consider both the case decreasing returns of scale ($m \leq 1$) and of increasing returns of scale ($m > 1$). They show that the properties of equilibria differ significantly across these two cases.

seem unduly restrictive: exerting twice as much effort as your opponent does not bring your probability of winning above $\frac{2}{3}$. There are two ways to relax this assumption. The first is by restricting oneself to symmetric contests. As shown by Baye *et al.* (1994) in a symmetric Tullock contest, a symmetric Nash equilibrium in pure strategies exists as long as $m \leq 2$.[8] Since we are interested in the effects of changes in wealth and in wealth inequality, the symmetry assumption is too restrictive (though we shall maintain it when discussing the effects of introducing a mean-preserving spread in wealth). A second way out is to have sufficiently increasing marginal disutility of effort. In the privilege and rent-seeking contests to be discussed in Sections 4 and 6, resp., the effort is monetary and the convex disutility of effort directly stems from the concavity of the utility function over final wealth. In the ability contest model that we discuss in Section 5, effort is non-monetary with an increasing marginal cost. Hence we need not have to restrict m to be in the unit interval, and all our numerical examples are indeed for $m > 1$.

The properties of $p(\cdot)$ are given in Appendix A.1. Here, we draw attention to the important fact that

$$p_{12} = \frac{\partial^2 p}{\partial x_a \partial x_b} = \frac{m^2}{x_a x_b} \Pi_a \Pi_b (\Pi_a - \Pi_b), \tag{12}$$

meaning that the marginal productivity of one player's effort is enhanced by the other player's effort iff the former exerts more effort. Thus, the strategic models we will consider are neither games of strategic complements nor of strategic substitutes. In fact, as in Nti (1999) and Acemoglu and Jensen (2013) for instance, some interesting features arise in our contest models because the change in the effort of one player will either increase the effort of the other player (when this player wants to "keep up") or decrease this effort (because this other player "gives up").

We now turn to the three types of contest mentioned in the Introduction, starting with the privilege contest.

[8]For larger m, the first- and second-order conditions fail to characterise the best-response functions, the reason being that a single player can profitably deviate from the (symmetric) solution to these conditions by exerting zero effort.

3 The privilege contest model

In the privilege contest model, the rent is non-monetary. Our chief interpretation is that the benefit from winning the contest is only associated with a form of prestige (or "ego-utility"), without any financial counterpart. This model of contest may include, for instance, status-seeking activities, political campaign contributions or warfare for purely ideological motives.

Denoting the non-monetary benefit of the privilege as r, we model the preferences of player i $(= a, b)$ with wealth w_i and exerting effort x_i as

$$U_i = u(w_i - x_i) + \Pi_i r. \tag{13}$$

We assume that $u(\cdot)$ is concave, which ensures that the marginal willingness to pay for the privilege in terms of consumption, $\frac{\Pi_i}{u'_i}$, is decreasing (along the indifference curve) in consumption.[9] It also means that the marginal disutility of effort is increasing. Furthermore, we can express the dependency of this willingness to pay on wealth in terms of the coefficients of absolute risk aversion, $A_i \overset{\text{def}}{=} -\frac{u''(w_i - x_i)}{u'(w_i - x_i)}$, and absolute prudence, $P_i \overset{\text{def}}{=} -\frac{u'''(w_i - x_i)}{u''(w_i - x_i)}$:

$$\frac{\partial(-\frac{\mathrm{d}w_i}{\mathrm{d}r}|_{\mathrm{d}U_i=0})}{\partial w_i} = \frac{\Pi_i}{u'_i}A_i, \text{ and } \frac{\partial^2(-\frac{\mathrm{d}w_i}{\mathrm{d}r}|_{\mathrm{d}U_i=0})}{\partial w_i^2} = \frac{\Pi_i}{u'_i}A_i(2A_i - P_i). \tag{14}$$

In this model, the key property compared with the subsequent contest models is that the marginal benefit of exerting effort is independent of wealth.

Player a's best response $f(x_b, w_a)$ is defined by the necessary first- and second-order conditions

$$\begin{aligned} -u'(w_a - f(x_b, w_a)) + p_1(f(x_b, w_a), x_b)r &= 0, \\ u''(w_a - f(x_b, w_a)) + p_{11}(f(x_b, w_a), x_b)r &< 0. \end{aligned}$$

The latter condition can then also be written as

$$\frac{p_{11}}{p_1}x_a = m\frac{1 - (\frac{x_a}{x_b})^m}{1 + (\frac{x_a}{x_b})^m} - 1 < A_a x_a. \tag{15}$$

[9] As $-\frac{\mathrm{d}w_i}{\mathrm{d}r}|_{\mathrm{d}U_i=0} = \frac{\Pi_i}{u'_i}$, $\frac{\partial}{\partial r_i}(-\frac{\mathrm{d}w_i}{\mathrm{d}r}|_{\mathrm{d}U_i=0})|_{\mathrm{d}U_i=0} = \frac{\Pi_i^2}{u'_i}\frac{u''_i}{u'_i} < 0$.

Simple comparative statics show that

$$\begin{aligned} f_1 &= -\frac{p_{12}(x_a, x_b)r}{u''(w_a - x_a) + p_{11}(x_a, x_b)r} \text{ and} \\ f_2 &= \frac{u''(w_a - x_a)}{u''(w_a - x_a) + p_{11}(x_a, x_b)r} > 0, \end{aligned}$$

where the inequality follows from the concavity of u and the second-order condition. Therefore player a's best response increases when that player's wealth increases; i.e., effort is a normal good. The intuition is simple. When wealth increases, the marginal cost of exerting effort decreases (due to decreasing marginal utility) while the marginal benefit is unaffected. Likewise, player b's best response $g(x_a, w_b)$ satisfies the necessary first- and second-order conditions

$$\begin{aligned} -u'(w_b - g(x_a, w_b)) - p_2(x_a, g(x_a, w_b))r &= 0, \\ u''(w_b - g(x_a, w_b)) - p_{22}(x_a, g(x_a, w_b))r &< 0, \end{aligned}$$

where the last condition is equivalent to

$$\frac{p_{22}}{p_2} x_b = m \frac{1 - (\frac{x_b}{x_a})^m}{1 + (\frac{x_b}{x_a})^m} - 1 < A_b x_b. \tag{16}$$

Differentiating with respect to x_a and w_b, we obtain

$$\begin{aligned} g_1 &= \frac{-p_{12}(x_a, x_b)r}{-u''(w_b - x_b) + p_{22}(x_a, x_b)r}, \\ g_2 &= \frac{-u''(w_b - x_b)}{-u''(w_b - x_b) + p_{22}(x_a, x_b)r} > 0. \end{aligned}$$

At a symmetric equilibrium, $p_{12} = 0$ (cf. (12)) and therefore $f_1 = g_1 = 0$. Hence, at a symmetric equilibrium

$$\frac{\partial x_a}{\partial w_a} = f_2 > 0 \text{ and } \frac{\partial x_b}{\partial w_a} = 0,$$

and relying on Theorem 1 we can conclude that $x_a > x_b$ iff $w_a > w_b$. In view of (12), we can also conclude that $p_{12} > 0$. As a result, an isolated increase in the wealth of the poor player, b, increases both the equilibrium effort of that player (cf. (7) and $g_2 > 0$) as well as that of the rich player (cf. (6)

and $f_1, g_2 > 0$). Hence, total equilibrium efforts also increase. Alternatively, an increase in the wealth of the rich player, a, increases that player's own equilibrium effort (cf. (4) and $f_2 > 0$) but reduces that of the poor player, b (cf. (5) and $g_1 < 0 < f_2$). We know from (8) that this total effect depends on $(1 + g_1)$. Observe now that $1 + g_1 > 0$ iff

$$[p_{12}(x_a, x_b) - p_{22}(x_a, x_b)]\, r < -u''(w_b - x_b),$$

which, using the first-order condition for player b, may be written as

$$\pi(x_a, x_b) \stackrel{\text{def}}{=} \frac{p_{12}(x_a, x_b) - p_{22}(x_a, x_b)}{-p_2(x_a, x_b)} x_b < A_b x_b. \tag{17}$$

For the power-logistic CSF, $\pi(x_a, x_b) = m \frac{1-(\frac{x_b}{x_a})^m}{1+(\frac{x_b}{x_a})^m}(1 + \frac{x_b}{x_a}) - 1$.[10] Condition (17) compares two curvature measures. The *lhs*, $\pi(x_a, x_b)$, equals $\frac{\mathrm{d}\log \frac{\partial \Pi_b}{\partial x_b}}{\mathrm{d}\log x_b}|_{\mathrm{d}(x_a+x_b)=0}$, or the elasticity of b's marginal return to effort given that total effort remains constant. The *rhs* of (17) corresponds to $\frac{\mathrm{d}\log(-\frac{\partial u(w_b - x_b)}{\partial x_b})}{\mathrm{d}\log x_b}$, the elasticity of the marginal utility cost of effort. If the latter exceeds the former, b's reaction is temperate enough for aggregate effort to correlate with that of a.

We summarize this discussion as follows.

Proposition 1 *In the privilege contest model with unequal wealth, the rich player exerts more effort than the poor player. An isolated increase in the poor player's wealth always increases the equilibrium efforts of both players. An isolated increase in the rich player's wealth has a positive effect on his own effort but a negative effect on the effort of the poor player; under (17) it has a positive effect on total effort.*

Figure 2 illustrates these results. It is assumed that $u(w) = -w^{-1}$ (i.e., CRRA=2), $m = 2$, $r = 1$. The best-response functions of players a and

[10] Comparing (17) with (16) shows that when the SOC for b barely holds (i.e., close to zero), a sufficiently large p_{12} will result into $1 + g_1 < 0$: the increase in a's effort triggers such a strong negative reduction in b's effort that aggregate effort falls. A sufficiently large p_{12} can be provoked by both a high decisiviness parameter m and a sufficient degree of wealth inequality (small $\frac{x_b}{x_a}$, or large $\Pi_a - \Pi_b$).

b are drawn for $w_a = 10, 17$ and 20, and for $w_b = 10, 14$, and 20. First observe that for large efforts of the opponent, the best response for each player is zero effort. Hence the discontinuity in the best response functions (visible in the figure for w_a and w_b equal to 10). Note that these functions are first increasing and then decreasing, with a maximum at $x_a = x_b$. Point A represents a symmetric equilibrium with uniform low wealth ($w_a = w_b = 10$), while point D represents a symmetric equilibrium with uniform high wealth ($w_a = w_b = 20$). Point B represents an equilibrium with $w_a = 17 > w_b = 10$, and the move from B to C illustrates the effect of an increase in w_b from 10 to 14. Point E is the result of an increase in w_a from 17 to 20. Total efforts increase, despite the fact that the poor exerts less effort than in B.

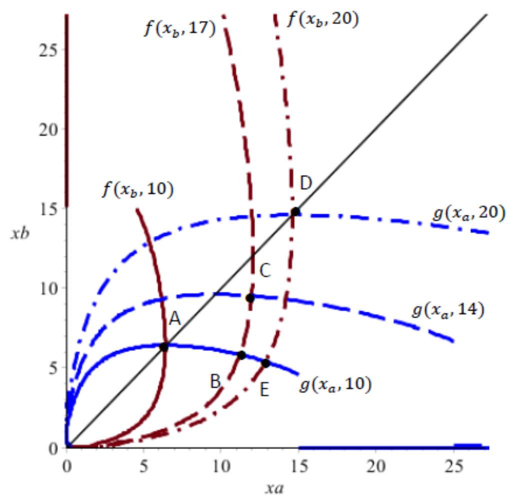

Figure 2. Equilibria in the privilege contest model for different wealth combinations.

What happens under uniform wealth growth? With unequal initial wealth, total effort will change with

$$(1 - f_1 g_1)(\mathrm{d}x_a + \mathrm{d}x_b) = [(1 + g_1)\, f_2 w_a + (1 + f_1)\, g_2 w_b]\, \mathrm{d}\log w, \tag{18}$$

where $\mathrm{dlog}\, w$ denotes the common growth rate in wealth. Thus, the same sufficient condition for total effort to increase when b gets richer, ensures that total effort is a normal good. In a symmetric game, $w_a = w_b$ and therefore $x_a = x_b$, so that (18) reduces to

$$(\mathrm{d}x_a + \mathrm{d}x_b) = 2 f_2 w \mathrm{d}\log w > 0. \tag{19}$$

This leads to the following result.

Proposition 2 *If (17) holds, a common increase in wealth increases total effort in the privilege contest model. With equal wealth, a common increase in wealth always increases the efforts of both players.*

We finally discuss the effects of wealth inequality. From Proposition 1, we observe that decreasing inequality in the sense of making the poor richer, or increasing inequality in the sense of making the rich richer, both increase total effort. Therefore, there is no systematic relationship between wealth inequality and effort in the privilege contest model. Now we study the effect of more wealth inequality when total wealth is constant. More precisely, we study the effect of a MPS in wealth. We can then invoke Theorem 2. In the Appendix, we prove the following result which involves the coefficients of absolute risk aversion A and prudence P defined at the symmetric equilibrium.

Theorem 3 *In the symmetric privilege contest model, the sign of the quadratic form (10) is positive iff*

$$2A(1-m^2) > P. \tag{20}$$

First, note that this inequality may also be written as $2A - P > 2Am^2$. Thus, if the marginal willingness to pay for rent is concave in final wealth (cf. (14)), a small MPS in wealth reduces total effort. When u is quadratic, $P = 0$, and the inequality reduces to $m < 1$. When u is CARA, $A = P$ and the inequality reduces to $m < 2^{-\frac{1}{2}} \simeq .707$. Thus the quadratic and CARA cases illustrate instances where the value of the decisiveness parameter of the CSF determines whether the effect of a MPS in wealth on total effort is positive or negative. If we multiply (20) by $(w-x)$, we may replace A and P by $-\frac{u''(w-x)}{u'(w-x)}(w-x)$ and $-\frac{u'''(w-x)}{u''(w-x)}(w-x)$, the coefficients of relative risk aversion and relative prudence, respectively. When $u(\cdot)$ has constant relative-risk aversion (CRRA) denoted by ρ, the inequality reduces to $\rho(\frac{1}{2}-m^2) > \frac{1}{2}$. We summarize these findings as follows.

Proposition 3 *In the symmetric privilege contest model, a small MPS in wealth increases total effort iff (20) is positive. Under CARA (resp. quadratic) preferences, this arises iff $m < \frac{1}{\sqrt{2}}$ (resp. $m < 1$). When u has CRRA ρ, this takes place iff $\rho(\frac{1}{2} - m^2) > \frac{1}{2}$. If the marginal willingness to pay for the rent is concave in final wealth, this never happens.*

These results indicate that a low decisiveness of the CSF is needed for a small MPS in wealth to increase aggregate effort. In the numerical example of Figure 2, which has CRRA= $m = 2$, aggregate effort in point D ($w_a = w_b = 20$) is 29.19. Redistributing one unit of wealth results in a lower aggregate effort (29.14).

4 The ability contest model

In the ability contest model, effort is non-monetary. Our principal interpretation is a situation in which players exert physical or mental efforts that increase their abilities, and thus put them in a better position to win a contest. Competitive sports, but also education filters, are examples of such contests.

In this model, player i's expected utility equals

$$U_i = \Pi_i u(w_i + r) + (1 - \Pi_i) u(w_i) - c(x_i),$$

with $c' > 0$ and $c'' \geq 0$. As before, we assume that $u(\cdot)$ is concave, which represents decreasing marginal utility of wealth (or risk aversion). The key property in this contest model is that the marginal cost of exerting effort is independent of wealth. As argued earlier, if this marginal cost is strictly increasing we can relax the assumption that $m \leq 1$ for a Nash equilibrium in pure strategies to exist.

The best response of player a, $f(x_b, w_a)$, is defined by the necessary first- and second-order conditions

$$\begin{aligned} p_1(f(x_b, w_a), x_b)\Delta u_a - c'(f(x_b, w_a)) &= 0, \\ p_{11}(f(x_b, w_a), x_b)\Delta u_a - c''(f(x_b, w_a)) &< 0, \end{aligned}$$

where $\Delta u_i \stackrel{\text{def}}{=} u(w_i + r) - u(w_i) > 0$ ($i = a, b$), and similar definitions for $\Delta u_i'$ and $\Delta u_i''$. Eliminating Δu_a from the second-order condition, this may also be written as

$$\frac{p_{11}}{p_1} x_a < \frac{c''(x_a)}{c'(x_a)} x_a. \tag{21}$$

Simple comparative statics show that

$$f_1 = -\frac{p_{12}\Delta u_a}{p_{11}\Delta u_a - c''(x_a)} \text{ and } f_2 = -\frac{p_1 \Delta u_a'}{p_{11}\Delta u_a - c''(x_a)} < 0,$$

where the inequality follows from the concavity of $u(\cdot)$ and the second-order condition. Player a's best response is now an inferior good. The intuition is simple. An increase in wealth decreases the marginal benefit of effort, but has no effect on the marginal cost. Similarly, player b's best response $g(x_a, w_b)$ satisfies the necessary first- and second-order conditions

$$\begin{aligned} -p_2(x_a, g(x_a, w_b))\Delta u_b - c'(g(x_a, w_b)) &= 0, \\ -p_{22}(x_a, g(x_a, w_b))\Delta u_b - c''(g(x_a, w_b)) &< 0, \end{aligned}$$

and differentiating with respect to x_a and w_b yields

$$g_1 = \frac{-p_{21}\Delta u_b}{p_{22}\Delta u_b + c''(x_b)}, \text{ and } g_2 = \frac{-p_2 \Delta u_b'}{p_{22}\Delta u_b + c''(x_b)} < 0.$$

Again, at a symmetric equilibrium, $p_{12} = 0$ and therefore $f_1 = g_1 = 0$. Hence, at a symmetric equilibrium (cf. (4) and (5))

$$\frac{\partial x_a}{\partial w_a} = f_2 < 0 = \frac{\partial x_b}{\partial w_a},$$

and Theorem 1 allows us to conclude that $x_a < x_b$ iff $w_a > w_b$. Unlike the privilege contest model, the rich player now exerts less effort than the poor player. At such an asymmetric equilibrium, $p_{12} < 0$.

An increase in player a's wealth reduces that player's equilibrium effort (cf. (4) and $f_2 < 0$). And because $p_{12} < 0$, the equilibrium effort of the poorer player, b, will also fall (cf. (5) and $f_2 < 0 < g_1$): that is, the poor player's effort is a strategic complement to that of the rich player. Total equilibrium effort then unambiguously declines ($\frac{f_2(1+g_1)}{1-f_1 g_1} < 0$).

Conversely, an isolated increase in the wealth of the poor player, b, reduces that player's own equilibrium effort, (cf (7) and $g_2 < 0$), but increases the equilibrium effort of the rich player (cf. (6) and $f_1, g_2 < 0$). Without further restrictions, the sign of the effect on total equilibrium effort, $\frac{g_2(1+f_1)}{1-f_1 g_1}$, is then ambiguous. Using the first-order condition for a, we show that a necessary and sufficient condition for $1 + f_1$ to be positive, i.e., for aggregate effort to fall, is

$$\frac{p_{11} - p_{12}}{p_1} x_a < \frac{c''(x_a)}{c'(x_a)} x_a. \tag{22}$$

Notice that $\frac{p_{11}-p_{12}}{p_1}x_a = \pi(x_b, x_a) < 0$ (where $\pi(\cdot)$ was defined in (17)) and we therefore obtain a similar sufficient condition as for the privilege contest model.[11] The *lhs* of (22) can be interpreted as $\frac{\mathrm{d}\log p_1}{\mathrm{d}\log x_a}|_{\mathrm{d}(x_a+x_b)=0}$, the elasticity of a's marginal return to effort given that total effort remains constant. The *rhs* corresponds to the elasticity of a's marginal cost of effort. If the latter elasticity exceeds the former, player a's reaction is temperate enough for aggregate effort to correlate with that of b. Observe that for $m < 1$, $\pi < 0$, (22) is always satisfied and aggregate effort falls when agent b alone gets wealthier. This leads to the following result.

Proposition 4 *In the ability contest model with unequal wealth, the rich player exerts less effort than the poor player. An isolated increase in the rich player's wealth always reduces the equilibrium effort of both players. An isolated increase in the poor player's wealth reduces his own effort but increases the effort of the rich player; aggregate effort falls iff (22) holds.*

Figure 3 depicts the results obtained in this section. For this example $u(w) = \log(w)$ (i.e., CRRA=1), $c(x) = x^2$, $m = 2$, and $r = 4$. The best-response functions of players a and b are drawn for $w_a = 6, 12$ and 16, and for $w_b = 6, 8$, and 16.[12] It should be remembered that an increase in wealth decreases the best-response functions in the ability contest model. Thus, point A represents a symmetric equilibrium with low wealth ($w_a = w_b = 6$), and the move from A to D illustrates the effect of a common increase in wealth from 6 to 16. Similarly, point B represents an equilibrium with $w_a = 12 > w_b = 6$, and the move from B to E illustrates the effect of an increase in w_a, resulting in a downward adjustment in both effort levels. The move from B to C on the other hand, represents an increase in w_b from 6 to 8, resulting in opposing adjustments in the effort levels of the two players. Still, aggregate effort correlates with the effort of b because condition (22) is met.

[11] If the second-order condition for a barely holds, a reduction in x_b triggers a huge increase in x_a, resulting in a larger total effort. Both a large m or a high degree of wealth inequality ($\frac{x_a}{x_b} \ll 1$, i.e. $\Pi_b - \Pi_a \gg 0$) will contribute to this.

[12] For the ability contest, a sufficient condition for the set of first and second order conditions to identify the Nash equilibrium is that $p(x_a^*, x_b^*)\Delta u_a - c(x_a^*) \geq 0$. With a power-logistic CSF and a power cost of effort function, this amounts to $\frac{m}{\gamma}\frac{(\Delta u_b/\Delta u_a)^{m/\gamma}}{1+(\Delta u_b/\Delta u_a)^{m/\gamma}} > 1$, which is satisfied for the chosen parameters.

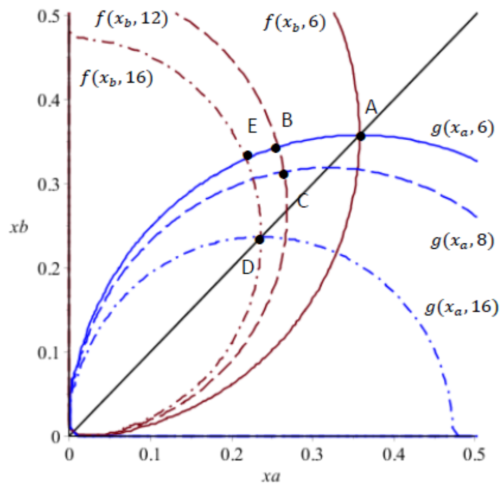

Figure 3. Equilibria in the ability contest model for different wealth combinations.

With initially unequal wealth, general wealth growth affects total effort by (18), with both terms on the *rhs* negative if (22) holds; total effort is an inferior good. In a symmetric contest, the effect is given by (19) and therefore negative (as $f_2 < 0$). The intuition is once again that an increase in wealth lowers the marginal benefit of effort, resulting in lower effort to win the rent.

Proposition 5 *If (22) holds, a common increase in wealth decreases total effort in the ability contest model. With equal wealth, a common increase in wealth always decreases the efforts of both players.*

We now discuss the effects of wealth inequality. As in the privilege contest, we first observe that there is no systematic relationship between wealth inequality and effort in the ability contest model. Indeed, decreasing inequality in the sense of making the poor richer, or increasing inequality in the sense of making the rich richer, both decrease total effort. We then examine the effect of a small MPS in wealth. In the appendix, we prove the following theorem.

Theorem 4 *Consider the symmetric ability contest model with a convex power cost of effort, i.e.,* $c(x) = x^\gamma$ $(\gamma \geq 1)$*. The sign of the quadratic form (10) is positive iff*

$$\left(\frac{\Delta u''/\Delta u'}{\Delta u'/\Delta u}\gamma^2 - (\gamma-1)\gamma\right) - 2m^2 > 0. \tag{23}$$

With CARA preferences, $A = -\frac{\Delta u''}{\Delta u'} = -\frac{\Delta u'}{\Delta u}$, and the large round bracket term simplifies to γ. With quadratic preferences, $\frac{\Delta u''}{\Delta u'} = 0$, and the large round bracket term becomes $-(\gamma - 1)\gamma \leq 0$. Under CRRA, it can be shown that the term $\frac{\Delta u''/\Delta u'}{\Delta u'/\Delta u}$ has the Taylor expansion

$$\frac{1+\rho}{\rho}(1 + \frac{1}{12}\left(\frac{r}{w}\right)^2) + O(\left(\frac{r}{w}\right)^3).$$

We summarize these results in the following proposition.

Proposition 6 *In the symmetric ability contest model with $c(x) = x^\gamma$ ($\gamma \geq 1$), a small MPS in wealth increases total efforts iff (23) holds. This is never the case with quadratic preferences. Under CARA, this happens iff $m < \sqrt{\frac{\gamma}{2}}$. When u has CRRA equal to ρ, this happens if $m < \sqrt{\frac{1}{24}\frac{\rho+1}{\rho}\left(12 + (\frac{r}{w})^2\right)\gamma^2 - \frac{1}{2}(\gamma - 1)\gamma}$.*

Recall that m measures the decisiveness of the contest. These results suggest that with a sufficiently low contest decisiveness, aggregate effort rises following the introduction of a small wealth inequality. In the numerical example of Figure 3, $m = 2$. When $w_a = w_b = 6$, aggregate effort is .7147 and the final root expression in Proposition 6 takes the value 1.77. Thus the inequality is not satisfied and redistributing one unit of wealth from a to b leads to a lower aggregate effort (.7133). We finally turn to the rent-seeking contest model.

5 The rent-seeking contest model

In the rent-seeking contest model, both rent and effort are monetary. This model can then accommodate many contest-type situations including lobbying, marketing, and litigation activities where both the rent and the effort have a direct monetary counterpart.[13] In this model, player i's expected utility equals

$$U_i = \Pi_i u(w_i + r - x_i) + (1 - \Pi_i)u(w_i - x_i). \tag{24}$$

[13] We observe that the economics literature on contests has traditionally (and often implicitly) assumed that both the rent and the effort are monetary. For instance, an important focus in this literature has concerned the rate of rent dissipation, i.e., $\frac{\Sigma_i x_i}{r}$, which assumes that the rent and the efforts are expressed in the same units, typically a monetary unit.

The concavity of $u(\cdot)$ is usually interpreted as risk aversion (Cornes and Hartley 2012), and we retain this interpretation in what follows.

We proceed as before and first characterize the best responses. For player a, $f(x_b, w_a)$, is now defined by

$$\begin{aligned} p_1(f(x_b,w_a),x_b)\Delta u_a - Eu_a' &= 0, \\ p_{11}(f(x_b,w_a),x_b)\Delta u_a - 2p_1(f(x_b,w_a),x_b)\Delta u_a' + Eu_a'' &< 0, \end{aligned}$$

where Eu_i' and Eu_i'' denote expected marginal utility and its second-order equivalent $(i = a, b)$. Simple computations show that

$$\begin{aligned} f_1 &= -\frac{p_{12}\Delta u_a - p_2\Delta u_a'}{p_{11}\Delta u_a - 2p_1\Delta u_a' + Eu_a''}, \text{ and} \\ f_2 &= -\frac{p_1\Delta u_a' - Eu_a''}{p_{11}\Delta u_a - 2p_1\Delta u_a' + Eu_a''}. \end{aligned} \tag{25}$$

Unlike the privilege and ability contest models, an increase in wealth has an ambiguous effect on the best-response function. The reason is that additional wealth reduces both the marginal benefit of winning the rent *and* the (expected) marginal cost of effort.

Similarly, player b's best response $g(x_a, w_b)$ satisfies the necessary first- and second-order conditions

$$\begin{aligned} -p_2(x_a, g(x_a,w_b))\Delta u_b - Eu_b' &= 0, \\ -p_{22}(x_a, g(x_a,w_b))\Delta u_b + 2p_2(x_a, g(x_a,w_b))\Delta u_b' + Eu_b'' &< 0. \end{aligned}$$

Differentiating with respect to x_a and w_b, we obtain

$$\begin{aligned} g_1 &= -\frac{-p_{21}\Delta u_b + p_1\Delta u_b'}{-p_{22}\Delta u_b + 2p_2\Delta u_b' + Eu_b''}, \text{ and} \\ g_2 &= -\frac{-p_2\Delta u_b' - Eu_b''}{-p_{22}\Delta u_b + 2p_2\Delta u_b' + Eu_b''}. \end{aligned}$$

At a symmetric equilibrium, $p_{12} = 0$, and therefore, $f_1, g_1 < 0$. Hence, at a symmetric equilibrium

$$\frac{\partial x_a}{\partial w_a} = \frac{f_2}{1 - f_1 g_1} \text{ and } \frac{\partial x_b}{\partial w_a} = \frac{g_1 f_2}{1 - f_1 g_1},$$

and we may claim that $\frac{\partial x_a}{\partial w_a}|^{\text{SE}} \gtrless 0 \gtrless \frac{\partial x_b}{\partial w_a}|^{\text{SE}}$ iff $f_2 \gtrless 0$.

Note that the sign of f_2 is given by the sign of its numerator, which upon using the first-order condition for a can be written as

$$Eu_a' \left(\frac{\Delta u_a'}{\Delta u_a} - \frac{Eu_a''}{Eu_a'} \right). \tag{26}$$

Let us now define two lotteries: a uniformly distributed lottery $\widetilde{z} =_d \text{U}(w_a - x_a, w_a - x_a + r)$ and a binary lottery $\widetilde{y} =_d (w_a - x_a + r, \frac{1}{2}; w_a - x_a, \frac{1}{2})$, so that the term in round brackets can be written as[14]

$$\frac{-Eu_a''(\widetilde{y})}{Eu_a'(\widetilde{y})} - \frac{-Eu_a''(\widetilde{z})}{Eu_a'(\widetilde{z})}.$$

Given that the binary lottery $(\widetilde{y})$ is a MPS of the uniform lottery $(\widetilde{z})$, the sign of f_2 is positive (resp. negative) if the MPS of a background risk increases (resp. decreases) the coefficient of absolute risk aversion. Let us introduce the following definition.

Definition 1 *Let Ω be the class of utility functions so that a MPS of a background risk increases absolute risk aversion.*

It sounds intuitive that additional background risk induces greater risk aversion, i.e., $u \in \Omega$. Eeckhoudt, Gollier and Schlesinger (1996, Prop. 3) show, however, that the conditions on u so that extra background risk makes an agent more risk averse are complex, involving restrictions on higher-order attitudes towards risk, such as the degree of temperance of u, i.e., $-u''''/u'''$. A necessary condition for $u \in \Omega$ is that risk aversion increases when a zero-mean background risk is introduced. Gollier and Pratt (1996) called this condition "risk vulnerability" and it is a stronger condition than the familiar DARA (decreasing absolute risk aversion).

For a small rent, a second-order Taylor approximation of the term in the round bracket in (26) helps us understand why DARA is necessary in our problem for wealth to increase effort. Let $t(r) \overset{\text{def}}{=} \Delta u_a' E u_a' - \Delta u_a E u_a''$. Then $t(0) = 0$, $t'(0) = 0$, and $t''(0)$ has the sign of $P_a - A_a$. Therefore, DARA

[14] $E_U u'(\widetilde{z}) = \int_{w_a - x_a}^{w_a - x_a + r} u_a'(\widetilde{z}) \frac{1}{r} \text{d}\widetilde{z} = \frac{1}{r} \Delta u_a$ and $E_U u''(\widetilde{z}) = \int_{w_a - x_a}^{w_a - x_a + r} u_a''(\widetilde{z}) \frac{1}{r} \text{d}\widetilde{z} = \frac{1}{r} \Delta u_a'$.

ensures that $t''(0) \geq 0$.[15] The intuition for this result may be given as follows. Investing in a contest is very much like gambling, where one spends money to increase the probability of winning the monetary prize. For the same reason that gambling activities should be reduced under increased risk aversion, efforts in a contest should also be reduced with increasing risk aversion (Treich 2010). By a similar reasoning, an increase in wealth —which reduces risk aversion under DARA— should increase effort in a contest.

We now use these results to compare the efforts of the rich and the poor within an equilibrium. If $u \in \Omega$, then $\frac{\partial x_a}{\partial w_a} > 0 > \frac{\partial x_b}{\partial w_a}$ at a symmetric equilibrium as $g_1 < 0$. Hence, Theorem 1 allows us to conclude that for $u \in \Omega$, in an asymmetric rent-seeking game $w_a > w_b$ implies $x_a > x_b$, and therefore $p_{12} > 0$.

As a result, $u \in \Omega$ ensures that an isolated increase in a's wealth will raise that player's equilibrium effort level. The equilibrium reaction of the poorer agent, b, is negative. As before, aggregate effort will increase iff $1 + g_1 > 0$. For the rent-seeking contest model, this condition is equivalent to

$$\frac{p_{21} - p_{22}}{-p_2} + \frac{2p_2 - p_1}{-p_2}\frac{\Delta u_b'}{\Delta u_b} + \frac{Eu_b''}{Eu_b'} < 0$$

$$\Updownarrow \; \frac{p_1}{p_2} = \frac{x_b}{x_a}$$

$$\pi(x_a, x_b) + \left(1 + \frac{x_b}{x_a}\right)\left(-\frac{\Delta u_b'}{\Delta u_b}\right) x_b < \left[\left(-\frac{Eu_b''}{Eu_b'}\right) - \left(-\frac{\Delta u_b'}{\Delta u_b}\right)\right] x_b. \quad (27)$$

We know that the *rhs* is positive if $u \in \Omega$. But as the second *lhs* term is positive, $\pi(x_a, x_b) < 0$ is no longer sufficient for $1 + g_1 > 0$.

If the poor person becomes wealthier, that player's effort changes with $\frac{g_2}{1-f_1 g_1}$, which is positive if $u \in \Omega$ (the reasoning is the same as for f_2). The rich agent's equilibrium effort changes with $\frac{f_1 g_2}{1-f_1 g_1}$. From (25), it transpires that $f_1 > 0$ iff $\frac{p_{12}}{-p_2} > -\frac{\Delta u_a'}{\Delta u_a}$. Since the sign of p_{12} depends on that of $x_a - x_b$, a necessary condition for a to increase effort is that a is sufficiently richer than b. As b's wealth approaches that of a, the latter will begin to reduce effort despite the fact that b is increasing effort. The two effort levels then

[15] $\frac{\partial}{\partial w}(-\frac{u''}{u'}) = A_a(A_a - P_a)$. Hence DARA$\Longleftrightarrow P_a - A_a > 0$.

turn into strategic substitutes. Thus, in the rent-seeking contest model, the nature of the strategic interaction depends on wealth levels. This possibility of strategic substitutability also blurs the effect of w_b on aggregate effort. Indeed, a similar argument as above shows that $1 + f_1 > 0$ iff

$$\frac{p_{11} - p_{12}}{p_1} - \frac{2p_1 - p_2}{p_1}\frac{\Delta u_a'}{\Delta u_a} + \frac{Eu_a''}{Eu_a'} < 0$$
$$\Updownarrow$$
$$\pi(x_b, x_a) + \left(1 + \frac{x_a}{x_b}\right)\left(-\frac{\Delta u_a'}{\Delta u_a}\right)x_a < \left[\left(-\frac{Eu_a''}{Eu_a'}\right) - \left(-\frac{\Delta u_a'}{\Delta u_a}\right)\right]x_a. \qquad (28)$$

Given $\frac{x_a}{x_b} > 1$, the first *lhs* term is negative (since $p_{12} > 0 > p_{11}$). The *rhs* is positive if $u \in \Omega$. Once again, the positive second *lhs* term blurs the inequality. We summarize these results as follows.

Proposition 7 *Suppose that $u \in \Omega$. In a rent-seeking contest model with unequal wealth, the rich player exerts more effort than the poor player. An isolated increase in the wealth of the rich player increases that player's effort, but reduces the poor player's effort. An isolated increase in the wealth of the poor player increases that player's effort. With "sufficient wealth inequality", an isolated increase in the wealth of the poor player also increases the effort of the rich player.*

The CARA utility function satisfies the conditions for Ω "just" (since background risk has no effect on absolute risk aversion under CARA). Hence, it provides a boundary case where $f_2 = 0$ and $g_2 = 0$, which is easily checked as both $\frac{-\Delta u_i'}{\Delta u_i}$ and $\frac{-Eu_i''}{Eu_i'}$ equal the coefficient of absolute risk aversion. The quadratic utility function provides another case where $f_2 = 0$ and $g_2 = 0$.[16] In both cases, aggregate effort is unaffected by an isolated increase in wealth.

With a common increase in wealth, aggregate efforts change with $2\frac{f_2}{1-f_1}$. Hence, $u \in \Omega$ ensures that uniform growth in wealth will increase the representative agent's effort. We summarize this discussion as follows.

Proposition 8 *If u is CARA or quadratic, an isolated and therefore a common increase in wealth leaves equilibrium efforts unaffected in the rent-seeking*

[16] Observe that $E\widetilde{z} = E\widetilde{y} = w_a - x_a + \frac{1}{2}r$. If $u(w) = w - \frac{\beta}{2}w^2$, then $\frac{-Eu_a''(\widetilde{y})}{Eu_a'(\widetilde{y})} = \frac{-Eu_a''(\widetilde{z})}{Eu_a'(\widetilde{z})} = \frac{\beta}{1-\beta(w_a - x_a + \frac{1}{2}r)}$.

contest model. In a symmetric rent-seeking contest model, a common increase in wealth increases equilibrium efforts under $u \in \Omega$.

Figure 4 illustrates the different wealth effects occurring under $u \in \Omega$. It is assumed that $u(w) = \log(w)$ (i.e., CRRA=1), $m = 1.5$, $r = 10$. The best-response functions of players a and b are drawn for $w_a = 10, 15$ and 18, and for $w_b = 10, 14$ and 18. Again, the best-response functions display a discontinuity but without upsetting the existence of a Nash equilibrium in pure strategies for our range of parameters. Point A is a symmetric equilibrium where $w_a = w_b = 10$. A common increase in wealth with 8 units moves the equilibrium to D. Point B is an asymmetric equilibrium with $w_a = 15 > w_b = 10$. The move from B to E is then because of an increase in w_a from 15 to 18: x_a increases, but x_b falls. Conversely, the move from B to C is because of an increase in w_b from 10 to 14. While raising x_b, this leads to a fall in x_a, illustrating the above mentioned ambiguity when wealth, and thus effort, are sufficiently close.

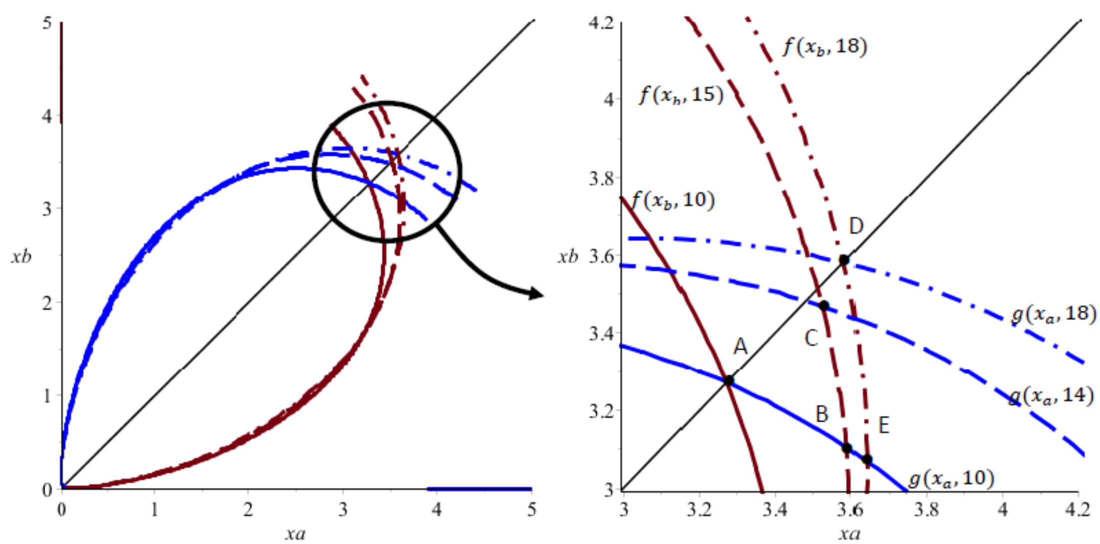

Figure 4. Equilibria in the rent-seeking model for different wealth combinations.

We finally discuss the effect of wealth inequality on aggregate effort in the rent-seeking contest model. For the reason discussed earlier, there is no effect of wealth distribution across players under CARA or quadratic utility. The following theorem is proven in the appendix.

Theorem 5 *Consider the rent-seeking contest model. The sign of the quadratic form (10) is positive iff*

$$p_{11}^2 p_1 T_1 + 4p_{11} p_1^2 T_2 + (3p_{112} - p_{111})\, p_1^2 T_3 + 4p_1^3 T_4 < 0 \tag{29}$$

where

$$\begin{aligned}
T_1 &= \frac{Eu''}{Eu'}\left(\frac{Eu'''}{Eu''} - 2\frac{\Delta u'}{\Delta u}\right) - \frac{\Delta u'}{\Delta u}\left(\frac{\Delta u''}{\Delta u'} - 2\frac{\Delta u'}{\Delta u}\right), \\
T_2 &= \frac{\Delta u'}{\Delta u}\left[\frac{\Delta u'}{\Delta u}\left(\frac{Eu''}{Eu'} - \frac{\Delta u'}{\Delta u}\right) - \frac{Eu''}{Eu'}\left(\frac{Eu'''}{Eu''} - \frac{\Delta u''}{\Delta u'}\right)\right], \\
T_3 &= \left(\frac{Eu''}{Eu'} - \frac{\Delta u'}{\Delta u}\right)^2, \\
T_4 &= \left(\frac{\Delta u'}{\Delta u}\right)^2\left[\frac{\Delta u''}{\Delta u'}\left(\frac{\Delta u'}{\Delta u} - \frac{Eu''}{Eu'}\right) + \frac{Eu''}{Eu'}\left(\frac{Eu'''}{Eu''} - \frac{\Delta u''}{\Delta u'}\right)\right].
\end{aligned}$$

With CARA preferences, all ratios in T_1, T_2, T_3, and T_4 coincide with $-A$, and therefore the four terms equal zero. The same is true for quadratic preferences, $u(y) = y - \frac{\beta}{2}y^2$.[17] In the appendix, we prove the following theorem.

Theorem 6 *With CRRA preferences and small $\frac{r}{w}$, the inequality (29) is violated.*

We can therefore summarize our findings as follows.

Proposition 9 *Consider the rent-seeking contest model. Under CARA and quadratic preferences, a MPS in wealth does not affect aggregate effort. Under CRRA preferences, when the stake of the contest, $\frac{r}{w}$, is small, a small MPS in wealth reduces aggregate effort.*

In the numerical example of Figure 4, which has CRRA= 1, aggregate effort when $w_a = w_b = 18$ is 7.159. Redistributing one unit of wealth leads to an aggregate effort of 7.155. Thus even with a $\frac{r}{w} = .55$, the prediction of the proposition holds.

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[17] In that case, $\Delta u = \Delta y(1 - \beta Ey)$, $\Delta u' = -\beta \Delta y$, $\Delta u'' = 0$, $Eu' = 1 - \beta Ey$, $Eu'' = -\beta, Eu''' = 0$, where $\Delta y = r$ and $Ey = w - x + \frac{1}{2}r$. Then $\frac{\Delta u'}{\Delta u} = \frac{Eu''}{Eu'} = -\frac{\beta}{1-\beta Ey}$ and $\frac{\Delta u''}{\Delta u'} = \frac{Eu'''}{Eu''} = 0$. Once again, all four terms vanish.

None of the preferences considered in Proposition 9 (i.e., quadratic, CARA, and CRRA) result in larger aggregate efforts. At the same time, these three types of preferences share a non-negative third derivative of $u(\cdot)$ ("prudence"). This suggests that a negative third derivative ("imprudence") may be a necessary condition for a MPS in wealth to raise aggregate effort. This conjecture is supported by the following example.

Example 1 *Suppose that* $u(y) = y - \frac{\beta}{2}y^2 + \frac{\gamma}{3}y^3$ *with* $\beta = \frac{1}{15}$ *and* $\gamma \leq \frac{2}{1000}$*, such that* $u''(y) < 0$ *for all* $y < 15$*. Then for a symmetric rent-seeking contest model with* $w = 10, r = 1, m = 1.5$*, and for* $\gamma \in [-.002, 0]$*, a small MPS in wealth results in higher aggregate efforts, as shown in Table 1.*

Table 1. Results for cubic preferences.[a]

γ	x^*	SOC	$Eu(x^*) - Eu(0)$	qf
-.0020	.3760	-.2114	.0023	.0052
-.0015	.3755	-.3583	.0099	.0006
-.0010	.3753	-.5055	.0176	.0001
-.0005	.3751	-.6527	.0252	.00002
0	.375	-.8	.0328	0
.0005	.3749	-.9473	.0404	-.000004
.0010	.3749	-1.0947	.0480	-.000003
.0015	.3748	-1.2420	.0557	-.000001

[a]The columns respectively provide the values of the "prudence coefficient" γ, the equilibrium effort (x^*), the value of the second-order condition (SOC), the gain in expected utility when playing x^* rather than 0 against x^*, and the value of the lhs of (29) (qf).

6 Other wealth effects

A few studies have discussed the effect of wealth in strategic models of contests. These effects differ significantly from those considered in the present analysis. In this section, we present a short summary of these other wealth effects previously studied in the literature.

But first let us briefly discuss the issue of redistributive politics (as mentioned in the Introduction). Redistributive politics may be interpreted as a contest where r is a transfer from the loser to the winner. One may think that this transfer could introduce a new wealth effect in our different models. That is not the case as our three contest models can accommodate this interpretation, given a basic change in notation.[18]

Che and Gale (1997) examine the effect of budget constraints in a basic contest model as in (1). They show that each agent's equilibrium effort is a weakly-increasing function of the agent's budget constraint, and that the presence of budget constraints lowers aggregate effort. Therefore, if one naturally assumes that a wealthier agent is less budget-constrained, wealth has a positive effect on effort.[19] The effect of a budget constraint can be interpreted as an extreme utility curvature of u at zero. However, in Che and Gale (1998), as the utility function is otherwise linear, there are no wealth effects when wealth changes occur within the bounds of unconstrained efforts, i.e., using our notation, when $x_i < w_i$. Moreover, Che and Gale's (1998) model does not capture the effect that wealthier agents may have a lower marginal benefit of obtaining rent.

Hirshleifer (1991) studies the so-called "paradox of power". In its weak form, this paradox states that the final distribution of wealth will have less dispersion than the initial distribution of wealth. In its strong form, it states that there should be equal final distribution of wealth. Hirshleifer (1991) considers a two-player contest model in which the payoff of agent a is, using our notation, written as follows:

$$U_a = p(x_a, x_b)[(w_a - x_a)^{1/s} + (w_b - x_b)^{1/s}]^s,$$

where the quantity within the bracket is interpreted as the aggregate production in the economy with $s \geq 1$ a "complementarity index" parameter in the production functions of the two agents (and where U_b is defined analogously). It is easy to see that when $s = 1$ then the model is symmetric; efforts are thus equal, implying that the strong paradox of power holds, i.e., $w_a/w_b = U_a/U_b$. Hirshleifer (1991) then presents numerical examples for various values of s

[18] The change in notation can be defined as follows. In the privilege contest model, let $U_i = u(w_i - x_i) + \Pi_i r + (1 - \Pi_i)(-r) = u(w_i - x_i) + \Pi_i r_0$ with $r_0 = 2r$ and a renormalisation. In the ability contest model, let $U_i = \Pi_i[u(w_i + r) - u(w_i - r)] - c(x_i) = \Pi_i[u(w_0 + r_0) - u(w_0)] - c(x_i)$ with $w_0 = w - r$. In the rent-seeking contest model, let $U_i = \Pi_i u(w + r - x_i) + (1 - \Pi_i)u(w - r - x_i) = \Pi_i u(w_0 + r_0 - x_i) + (1 - \Pi_i)u(w_0 - x_i)$.

[19] Note, however, that in an all-pay auction, the introduction of budget constraints may surprisingly increase effort (Che and Gale 1998).

and the decisiveness parameter m for which the paradox of power in its weak form does, or does not, hold. The key difference to our model is that in Hirshleifer's (1991) model, the rent, i.e., aggregate production, is endogenous and increases with wealth. As a result, the rich player always exerts (weakly) more effort than the poor player as the marginal utility of exerting effort is (weakly) lower. In fact, the ability contest model we introduced displays a much stronger form of the paradox of power compared to Hirshleifer's (1991) model. Indeed, in the ability contest model, the poor player always exerts strictly greater effort, in absolute terms, than the rich player.

A related model is the "winner take all with limited liability" model introduced in Skaperdas and Gan (1995). Using our notation again, the agent's payoff in this model essentially writes as follows

$$U_i = \Pi_i u(w_i - x_i).$$

An interpretation of this model is that the loser "dies" and obtains utility $u(0) = 0$. Although Skaperdas and Gan (1995) are uninterested in the effect of wealth (but study that of risk aversion), it is easy to understand that wealth has a positive effect in this model. The intuition is that the two effects we have identified go in the same direction in Skaperdas and Gan's (1995) model. Wealth decreases the marginal cost of effort (as usual), but wealth also increases the marginal benefit of effort. Indeed, this last effect simply means that it "pays off" more to be alive when wealthier. It is also possible to show that a rich player always exerts more effort than the poor in the two-player version of Skaperdas and Gan's (1995) model.

Finally, we discuss Grossman's (1991) model of insurrections.[20] Grossman (1991) considers a general equilibrium model in which agents choose how to devote their time between production, soldiering (for the government) and insurrection. This implies that income (generated by production) and conflict expenditures are endogenously determined in equilibrium. Grossman (1991) is especially interested in how the equilibrium depends on the exogenous

[20] This model has been influential in the conflict literature and has been used as a benchmark to understand the relationship between wealth and conflict (Azam 2006, Chassang and Padro-i-Miquel 2009, Besley and Persson 2012). However, this model is significantly different from the standard Tullock (1980) contest model and the few extensions studied in our analysis. We thus merely present some key insights of the Grossman (1991)'s model, and attempt to relate them to the results we obtain.

CSF properties. Typically, the more favourable to a successful insurrection is the parameter in the CSF, the larger is the fraction of time devoted to insurrection as opposed to production. As a result, there is an equilibrium associating low wealth and high conflict expenditures. A key insight from this model is that participation in soldiering increases with the opportunity cost of fighting. In a contest model, this effect could be somehow captured by allowing the cost of effort, $c(x_i)$ using our notation, to depend directly on wealth w_i. This dependence could reflect that the marginal cost of conscription is higher in rich countries. This effect may then counteract the other possible positive effect of wealth on conflict. To see this, consider the following payoff function

$$U_i = \Pi_i f(w_i) - c(x_i, w_i).$$

Then wealth has a positive effect, both on the marginal benefit of effort, through f when $f' > 0$ (as in Hirshleifer 1991 and Skaperdas and Gan 1995), and also on the marginal cost of effort, through c when $\partial c / \partial w_i > 0$. However, it is unclear which effect prevails. For instance, taking f and c linear in w_i, then the two effects cancel each other out. This point has been observed, and extensively discussed, in Fearon (2007).

7 Discussion and conclusion

The archetype of a contest is warfare. Following the old Latin saying (often attributed to Cicero) "pecunia nervus belli", or "money is the sinews of war", wealth is expected to play an important role in warfare. But, is this theoretically true? Our answer to this question is based on a theoretical analysis of strategic models of contests. These models capture economic situations with elements of warfare. The simplest conclusion we can offer from our analysis is that the relationship beween contestants' efforts and wealth is strongly "contest-dependent". Hence, our analysis does not support without qualifications general claims that the rich lobby more, nor that low economic growth and inequality increase conflict.

The more precise answer is that wealth effects critically depend on the nature of the rent and of the type of efforts exerted in a contest. They depend in particular on whether the rent and/or efforts are commensurable with wealth. We have especially stressed the importance of the property of decreasing marginal utility of wealth. This property plays a fundamental role in our analysis through two basic effects. First, wealth decreases the marginal cost of monetary effort. Second, wealth decreases the marginal benefit of winning a monetary rent. The first effect tends to increase efforts, as we have shown in the "privilege contest" model. The second effect tends to decrease efforts, as we have shown in the "ability contest" model. Therefore, the disparate effects of wealth in contests that we colloquially discussed in the introduction may well reflect these two fundamentally opposing forces that our models identify.

However, these basic effects go in an opposite direction when both the rent and efforts are monetary. Therefore the total effect of wealth is complex and potentially limited, as we have shown in our "rent-seeking contest" model. In the special, but common, case of a CARA utility function, the two effects exactly offset each other, and wealth therefore has no impact on effort. Moreover, we have shown that wealth increases effort in the rent-seeking contest model under the assumption on the utility function that more background risk increases risk aversion in a single decision making setting. This assumption involves higher-order derivatives of the utility function, and is stronger than DARA. All of these results are summarized in Table 2 below.

Table 2. Summary of wealth effects in the privilege, ability, and rent-seeking contest models under $w_a \geq w_b$.[a]

Contest models:		Privilege	Ability	Rent-seeking
Rich vs poor	$x_a - x_b$	$+$	$-$	$+$
Isolated increase in w_a	$\frac{\partial x_a}{\partial w_a}$	$+$	$-$	$+$
	$\frac{\partial x_b}{\partial w_a}$	$-$	$-$	$-$
	$\frac{\partial (x_a+x_b)}{\partial w_a}$	$+$ iff (17)	$-$	$+$ iff (27)
Isolated increase in w_b	$\frac{\partial x_a}{\partial w_b}$	$+$	$+$	?
	$\frac{\partial x_b}{\partial w_b}$	$+$	$-$	$+$
	$\frac{\partial (x_a+x_b)}{\partial w_b}$	$+$	$-$ iff (22)	$+$ iff (28)
Common increase	$\frac{\partial x_i}{\partial w_i}\Big\vert_{\mathrm{d}w_a=\mathrm{d}w_b}^{w_a=w_b}$	$+$	$-$	$+$
	$\frac{\partial (x_a+x_b)}{\partial w_i}\Big\vert_{\mathrm{d}w_a=\mathrm{d}w_b}^{w_a=w_b}$	$+$	$-$	$+$
"Small" MPS in wealth	$\frac{\partial x_i}{\partial w_i}\Big\vert_{\mathrm{d}w_a=-\mathrm{d}w_b}^{w_a=w_b}$	$+$ iff (20)	$+$ iff (23)	$+$ iff (29)

[a]Symbols $+$ and $-$ indicate the sign of the effects mathematically described in the second column. Symbol ? indicates that this sign is indeterminate. In the rent-seeking contest

model, we assume $u \in \Omega$ (cf. Definition 1).

Table 2 also illustrates the subtle effects of isolated changes in wealth compared to other possible parametric changes. Nti (1999) for instance examines in an asymmetric contest the effect of changes in the valuation of the rent, and concludes that "competition becomes keener when the gap between the contestants is not too large" (Nti 1999, p. 425). Based on this insight, one would expect that increasing the wealth of the rich should decrease aggregate efforts, while increasing the wealth of the poor should increase aggregate efforts. Table 2 illustrates a much more complex pattern. Indeed, the total effect of an isolated increase in wealth of either the poor or the rich may be always positive, or always negative. Moreover this effect critically depends on the type of contest (and the curvature of the utility function of wealth).

The last observation relates to our analyis of the effects of wealth inequality. These effects are the most complex. To illustrate, take the privilege contest model. In this model, an isolated increase in wealth always increases aggregate effort. Therefore, a decrease in inequality (through an increase in the wealth of the poor) or an increase in inequality (through an increase in the wealth of the rich) both increase aggregate effort. Observe then that wealth redistribution, i.e., transferring money from the rich to the poor, combines the first effect and the opposite of the second effect (for a fixed total wealth). It thus essentially involves two opposing effects, and it is difficult to sign the effect of such a MPS in wealth without further restricting the functional forms. In fact, we have shown in our three contest models that the effect of a MPS in wealth depends on the property of the CSF, as well as on the sign of the higher-order derivatives of the utility functions. Interestingly, these results stand in sharp contrast with the "neutrality result" concerning the effect of wealth redistribution in the celebrated private provision of public goods model (Bergstrom, Blume and Varian 1986). An implication of our analysis is that the consequences of a wealth redistribution policy in terms of political stability and social peace are by no means obvious.

To conclude, let us add that there exist some natural extensions to our results. To start with, one may wish to consider other CSFs, an arbitrary number of players, other dimensions of heterogeneity (e.g., on the cost or value of rent), and to assume that the rent itself depends on wealth. One may also want to explore welfare effects. This could be interesting in that an increase in the wealth of one or more players need not have a positive effect on overall welfare despite increasing utility. This is because of the strategic effects that may serve to increase overall effort. However, a general study of welfare effects in contest models must also explicitly discuss to which extent efforts are socially (un)productive. Finally, it could also be interesting to explore the dynamic effects: wealth affects conflict, which in turn affects wealth, and so on. Such a dynamic analysis would permit a better understanding of the somewhat elusive relationship between power and money.

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A Appendix

A.1 Power-logistic contest success functions

In this appendix, we display the properties of the power-logistic CSF used to derive our results. The CSF for player a is given by

$$\Pi_a(x_a, x_b) = p(x_a, x_b) = \frac{x_a^m}{x_a^m + x_b^m},$$

where $m > 0$. Then $\Pi_b(x_a, x_b) = 1 - p(x_a, x_b) = \frac{x_b^m}{x_a^m + x_b^m}$. The derivatives of $p(x_a, x_b)$ are (where $\overset{\text{SE}}{=}$ denotes evaluation at $x_a = x_b = x$)

$$\begin{aligned}
p_1 &= \frac{m}{x_a}\Pi_a\Pi_b \overset{\text{SE}}{=} \frac{1}{4}\frac{m}{x} > 0 \text{ and } p_2 = -\frac{m}{x_b}\Pi_a\Pi_b \overset{\text{SE}}{=} -\frac{1}{4}\frac{m}{x} < 0;\\
p_{12} &= \frac{m^2}{x_a x_b}\Pi_a\Pi_b(\Pi_a - \Pi_b) > 0 \text{ iff } x_a > x_b, \text{ and } \overset{\text{SE}}{=} 0;\\
p_{11} &= \frac{m}{x_a^2}\Pi_a\Pi_b[-1 + m(\Pi_b - \Pi_a)] \overset{\text{SE}}{=} -\frac{1}{4}\frac{m}{x^2};\\
p_{22} &= \frac{m}{x_b^2}\Pi_a\Pi_b[1 - m(\Pi_a - \Pi_b)] \overset{\text{SE}}{=} \frac{1}{4}\frac{m}{x^2};\\
p_{111} &= 2\frac{m}{x_a^3}\Pi_a\Pi_b - \frac{m^2}{x_a^3}\Pi_a\Pi_b^2 + \frac{m^2}{x_a^3}\Pi_a^2\Pi_b - \frac{m^2}{x_a^3}\Pi_a\Pi_b(\Pi_b - \Pi_a)\\
&\quad + \frac{m^2}{x_a^3}\Pi_a\Pi_b[-1 + m(\Pi_a - \Pi_b)](\Pi_b - \Pi_a) - 2\frac{m^3}{x_a^3}\Pi_a^2\Pi_b^2\\
&\overset{\text{SE}}{=} \frac{1}{2}\frac{m}{x^3} - \frac{1}{8}\frac{m^3}{x^3};\\
p_{122} &= \frac{m}{x_a}p_{22}(\Pi_b - \Pi_a) - 2\frac{m^3}{x_a x_b^2}\Pi_a^2\Pi_b^2 \overset{\text{SE}}{=} -\frac{1}{8}\frac{m^3}{x^3};\\
p_{112} &= \frac{m}{x_a}\left(p_{12} - p_2\frac{1}{x_b}\right)(\Pi_b - \Pi_a) + 2\frac{m^3}{x_a^2 x_b}\Pi_a^2\Pi_b^2 \overset{\text{SE}}{=} \frac{1}{8}\frac{m^3}{x^3}.
\end{aligned}$$

For future reference, we also note that

$$3p_{112} - p_{111} \overset{\text{SE}}{=} \frac{1}{2}\frac{m}{x^3}(m^2 - 1).$$

A.2 Existence and uniqueness in (asymmetric) contests

This section builds on the literature on (asymmetric) contests to identify conditions ensuring the existence of a unique pure Nash equilibrium. Remember that the payoff function in the privilege, ability, and rent-seeking contest models can be written respectively:

$$\begin{aligned} U_i &= u(w_i - x_i) + \Pi_i r. \\ U_i &= \Pi_i u(w_i + r) + (1 - \Pi_i) u(w_i) - c(x_i), \\ U_i &= \Pi_i u(w_i + r - x_i) + (1 - \Pi_i) u(w_i - x_i). \end{aligned}$$

Observe first that a sufficient condition for these payoff functions to be concave in x_i is that Π_i is concave in x_i, which under (11) implies $m \leq 1$. This assumption is enough to ensure that best response functions are unique and continuous in each contest model. We now turn to the result of existence and uniqueness, and show that a sufficient condition under our assumptions is also $m \leq 1$.

Proposition 10 *There exists a unique equilibrium:*
i) in the privilege contest model, if $\frac{-u''(w-x)}{u'(w-x)} > \frac{m-1}{x}$;
ii) in the ability contest model, if $\frac{c''(x)}{c'(x)} > \frac{m-1}{x}$;
iii) in the rent-seeking contest model, if $u(\cdot)$ *has non-increasing absolute risk aversion and* $m < 1$.

Proof of Proposition 10

We follow the proof of Szidarovszky and Okuguchi (1997). They show that there always exists a unique equilibrium when the form of the payoff function for each player i can be written as follows:

$$U_i = \frac{y_i}{\sum_j y_j} - g_i(y_i) \text{ with } g_i' > 0 \text{ and } g_i'' > 0.$$

In the privilege contest model, we obtain this form of the payoff function under the following change in variable $g_i(y_i) = -u(w_i - y_i^{\frac{1}{m}})/r$. Then it is immediate that $g_i' > 0$, and that $g_i'' > 0$ iff $\frac{-u''(w-x)}{u'(w-x)} > \frac{m-1}{x}$.

In the ability contest model, we obtain the above form of the payoff function under the following change in variable $g_i(y_i) = c(y_i^{\frac{1}{m}}/(u(w_i + r) - u(w_i)))$. Then it is immediate that $g_i' > 0$, and that $g_i'' > 0$ iff $\frac{c''(x)}{c'(x)} > \frac{m-1}{x}$.

Therefore, under $u'' < 0$ and $c'' > 0$, conditions i) and ii) hold as soon as $m < 1$.

Finally, Yamazaki (2009) proves that there always exists a unique equilibrium in the rent-seeking contest model with CSF $\frac{\Phi(x_a)}{\Phi(x_a)+\Phi(x_b)}$ under non-increasing absolute risk aversion and $\Phi(\cdot)$ concave, which for the power-logistic function corresponds to $m < 1$.

A.3 The condition $1 - f_1 g_1 > 0$ and the stability condition

Throughout our analysis, we have assumed that the condition $1 - f_1 g_1 > 0$ is satisfied. We first show that this is always the case in the privilege and ability contest models, and then resort to a stability condition which ensures that is the case as well in the rent-seeking contest model.

In the privilege contest model, we have $g_1 = \frac{p_{12}(x_a,x_b)r}{u''(w_b - x_b) - p_{22}(x_a,x_b)r}$ and $f_1 = \frac{p_{12}(x_a,x_b)r}{-u''(w_a - x_a) - p_{11}(x_a,x_b)r}$ so that f_1 and g_1 have opposite signs. This implies $f_1 g_1 < 0$ and the condition is satisfied.

Similarly, in the ability contest model, we have that $g_1 = \frac{-p_{12}(x_a,x_b)}{p_{22}(x_a,x_b)}$ and $f_1 = \frac{-p_{12}(x_a,x_b)}{p_{11}(x_a,x_b)}$, which also have opposite signs so that $f_1 g_1 < 0$ and the condition is also satisfied

In the rent-seeking contest model, f_1 and g_1 need not have opposite signs, and the condition $1 - f_1 g_1 > 0$ is not necessarily verified. We thus impose a stability condition, i.e., $|f_1 g_1| < 1$, which ensures that the condition $1 - f_1 g_1 > 0$ is indeed satisfied. See Nti (1997) for a discussion of a related stability condition and of similar assumptions made in the literature on strategic contest models.

A.4 Proof of Theorems 1 and 2

Proof of Theorem 1

We need to prove that if (i) $x_a = x_b \Longrightarrow \frac{\partial x_a(w_a,w_b)}{\partial w_a} > \frac{\partial x_b(w_a,w_b)}{\partial w_a}$, then (ii) $w_a > w_b \Longrightarrow x_a(w_a,w_b) > x_b(w_a,w_b)$. Since $x_a(w_b,w_b) = x_b(w_b,w_b)$ (i.e., the unique equilibrium is the symmetric equilibrium), it follows from (i) that $\frac{\partial x_a(w_a,w_b)}{\partial w_a}|_{w_a=w_b} > \frac{\partial x_b(w_a,w_b)}{\partial w_a}|_{w_a=w_b}$. If for some $w_a > w_b$, we have $x_a(w_a,w_b) \leq x_b(w_a,w_b)$ this implies, due to the continuity of best responses, that there exists $w_c \in (w_a, w_b]$, such that $x_a(w_c,w_b) = x_b(w_c,w_b)$ and $\frac{\partial x_a(w_a,w_b)}{\partial w_a}|_{w_a=w_c} \leq$

$\frac{\partial x_b(w_a,w_b)}{\partial w_a}|_{w_a=w_c}$, contradicting (i). Hence, we must have (ii) $x_a(w_a, w_b) > x_b(w_a, w_b)$. The case with reverse inequalities can be proved in an analogous fashion. This proves Theorem 1.

Proof of Theorem 2

For a symmetric equilibrium,

$$x_a = f(f(x_a, w_b), w_a) \text{ and } x_b = f(f(x_b, w_a), w_b).$$

These expressions may be solved for the reduced form expressions for equilibrium effort:

$$x_a = F(w_a, w_b) \text{ and } x_b = F(w_b, w_a).$$

Theorem 2 is now proven with the help of two lemmas.

Lemma 1 *A small redistribution in wealth $dw_a = -dw_b = t$ increases aggregate effort $x_a + x_b$ iff $F_{11} - 2F_{12} + F_{22}$ evaluated at (w, w) is positive.*

Proof of Lemma 1

Starting from an equal wealth distribution (w, w), the new effort level for player a following a transfer t from b to a is then

$$x_a(w+t, w-t) = F(w+t, w-t) \simeq F(w, w) + (F_1 - F_2)t + \frac{1}{2}(F_{11} - 2F_{12} + F_{22})t^2,$$

where F_i means the partial w.r.t. the ith argument and all derivatives are evaluated at (w, w). Likewise, the new effort level for player b is approximately

$$x_b(w+t, w-t) = F(w-t, w+t) \simeq F(w, w) - (F_1 - F_2)t + \frac{1}{2}(F_{11} - 2F_{12} + F_{22})t^2.$$

Hence, aggregate equilibrium efforts are equal to

$$x_a(w + t, w - t) + x_b(w + t, w - t) \simeq 2F(w, w) + (F_{11} - 2F_{12} + F_{22})t^2.$$

Lemma 2 *At a symmetric equilibrium,*

$$F_{11} - 2F_{12} + F_{22} = \frac{(f_2)^2 f_{11} - 2(1 + f_1)f_2 f_{12} + (1 + f_1)^2 f_{22}}{(1 + f_1)(1 - f_1^2)},$$

where f_i (f_{ij}) denotes the first- (second-)order partial w.r.t. arguments i (ij).

Proof of Lemma 2

Let $f(x_b, w_a)$ be the best-response function for agent a, and $g(x_a, w_b)$ be the best-response function for agent b. Then

$$x_a = f(g(x_a, w_b), w_a).$$

Implicit differentiation then gives

$$\begin{aligned}
\mathrm{d}x_a &= f_1 g_1 \mathrm{d}x_a + f_2 \mathrm{d}w_a + f_1 g_2 \mathrm{d}w_b \\
F_1 = \frac{\partial x_a}{\partial w_a} &= \frac{f_2(g(x_a, w_b), w_a)}{1 - f_1(g(x_a, w_b), w_a) g_1(x_a, w_b)} \\
F_2 = \frac{\partial x_a}{\partial w_b} &= \frac{f_1(g(x_a, w_b), w_a) g_2(x_a, w_b)}{1 - f_1(g(x_a, w_b), w_a) g_1(x_a, w_b)}.
\end{aligned}$$

Differentiating one more time gives

$$\begin{aligned}
F_{11} = & \frac{1}{1 - f_1 g_1} \left\{ f_{21} g_1 \frac{\partial x_a}{\partial w_a} + f_{22} \right. \\
& \left. + \frac{f_2}{1 - f_1 g_1} \left[\left(f_{11} g_1 \frac{\partial x_a}{\partial w_a} + f_{12} \right) g_1 + f_1 g_{11} \frac{\partial x_a}{\partial w_a} \right] \right\} \\
= & \frac{1}{1 - f_1 g_1} \left\{ f_{21} g_1 \frac{f_2}{1 - f_1 g_1} + f_{22} \right. \\
& \left. + \frac{f_2}{1 - f_1 g_1} \left[\left(f_{11} g_1 \frac{f_2}{1 - f_1 g_1} + f_{12} \right) g_1 + f_1 g_{11} \frac{f_2}{1 - f_1 g_1} \right] \right\}
\end{aligned}$$

$$\begin{aligned}
F_{12} = & \frac{1}{1 - f_1 g_1} \left\{ f_{21} \left(g_1 \frac{\partial x_a}{\partial w_b} + g_2 \right) \right. \\
& \left. + \frac{f_2}{1 - f_1 g_1} \left[f_{11} \left(g_1 \frac{\partial x_a}{\partial w_b} + g_2 \right) g_1 + f_1 \left(g_{11} \frac{\partial x_a}{\partial w_b} + g_{12} \right) \right] \right\} \\
= & \frac{1}{1 - f_1 g_1} \left\{ f_{21} \left(g_1 \frac{f_1 g_2}{1 - f_1 g_1} + g_2 \right) \right. \\
& \left. + \frac{f_2}{1 - f_1 g_1} \left[f_{11} \left(g_1 \frac{f_1 g_2}{1 - f_1 g_1} + g_2 \right) g_1 + f_1 \left(g_{11} \frac{f_1 g_2}{1 - f_1 g_1} + g_{12} \right) \right] \right\}
\end{aligned}$$

$$
\begin{aligned}
F_{22} &= \frac{1}{1-f_1g_1}\left\{f_{11}\left(g_1\frac{\partial x_a}{\partial w_b}+g_2\right)g_2+f_1\left(g_{21}\frac{\partial x_a}{\partial w_b}+g_{22}\right)\right.\\
&\left.+\frac{f_1g_2}{1-f_1g_1}\left[f_{11}\left(g_1\frac{\partial x_a}{\partial w_b}+g_2\right)g_1+f_1\left(g_{11}\frac{\partial x_a}{\partial w_b}+g_{12}\right)\right]\right\}\\
&= \frac{1}{1-f_1g_1}\left\{f_{11}\left(g_1\frac{f_1g_2}{1-f_1g_1}+g_2\right)g_2+f_1\left(g_{21}\frac{f_1g_2}{1-f_1g_1}+g_{22}\right)\right.\\
&\left.+\frac{f_1g_2}{1-f_1g_1}\left[f_{11}\left(g_1\frac{f_1g_2}{1-f_1g_1}+g_2\right)g_1+f_1\left(g_{11}\frac{f_1g_2}{1-f_1g_1}+g_{12}\right)\right]\right\}.
\end{aligned}
$$

At a symmetric equilibrium, $f_{12}=f_{21}=g_{21}=g_{12},\ \ f_1=g_1,\ f_2=g_2$, and $f_{22}=g_{22}$. Then, using the above expressions, it can be shown that

$$
F_{11}-2F_{12}+F_{22}=\frac{(f_2)^2f_{11}-2(1+f_1)f_2f_{12}+(1+f_1)^2f_{22}}{(1+f_1)(1-f_1^2)}.
$$

The numerator is a quadratic form in the Hessian of the best-response function $f(x_2,w_1)$:

$$
\begin{bmatrix} -f_2 & 1+f_1 \end{bmatrix}\begin{bmatrix} f_{11} & f_{12}\\ f_{12} & f_{22}\end{bmatrix}\begin{bmatrix} -f_2\\ 1+f_1\end{bmatrix}.
$$

The denominator will be positive under the stability assumption: $|f_1g_1|=|f_1^2|=f_1^2<1\Longrightarrow|f_1|<1$.

The proof of Theorem 2 then follows immediately from Lemmas 1 and 2.

A.5 Proofs of Theorems 3, 4, and 5

For all three models, we can say that the first- and second-order conditions for agent a are given by

$$
\begin{aligned}
h(x_a,w_a,x_b) &= 0,\\
h_1(x_a,w_a,x_b) &< 0.
\end{aligned}
$$

Hence, the optimal responses to $\mathrm{d}w_a$ and $\mathrm{d}x_b$ are given by

$$
\frac{\partial x_a}{\partial w_a}=-\frac{h_2}{h_1}\text{ and }\frac{\partial x_a}{\partial x_b}=-\frac{h_3}{h_1}.
$$

The second-order responses are then given by

$$\begin{aligned} \frac{\partial^2 x_a}{\partial w_a^2} &= \frac{\partial(-\frac{h_2}{h_1})}{\partial w_a} + \frac{\partial(-\frac{h_2}{h_1})}{\partial x_a}\frac{\partial x_a}{\partial w_a} \\ &= \left(-\frac{1}{h_1}\right)\left[h_{22} - 2\frac{h_2}{h_1}h_{12} + \left(\frac{h_2}{h_1}\right)^2 h_{11}\right] \qquad (30) \\ \frac{\partial^2 x_a}{\partial w_a \partial x_b} &= \frac{\partial(-\frac{h_2}{h_1})}{\partial x_b} + \frac{\partial(-\frac{h_2}{h_1})}{\partial x_a}\frac{\partial x_a}{\partial x_b} \\ &= \left(-\frac{1}{h_1}\right)\left[h_{23} - \frac{h_2}{h_1}h_{13} - \frac{h_3}{h_1}h_{21} + \frac{h_2}{h_1}\frac{h_3}{h_1}h_{11}\right] \qquad (31) \\ \frac{\partial^2 x_a}{\partial x_b^2} &= \frac{\partial(-\frac{h_2}{h_1})}{\partial x_b} + \frac{\partial(-\frac{h_2}{h_1})}{\partial x_a}\frac{\partial x_a}{\partial x_b} \\ &= \left(-\frac{1}{h_1}\right)\left[h_{33} - 2\frac{h_3}{h_1}h_{13} + \left(\frac{h_3}{h_1}\right)^2 h_{11}\right]. \qquad (32) \end{aligned}$$

Proof of Theorem 3

For the privilege contest model, we have the following h-functions:

$$\begin{aligned} h &= -u'(w_a - x_a) + p_1 r = 0 \\ h_1 &= u''(w_a - x_a) + p_{11} r < 0 \\ h_2 &= -u_a'', h_3 = p_{12} r \overset{\text{SE}}{=} 0, \\ h_{11} &= -p_{111} r - u_a''', h_{12} = u_a''', h_{13} = p_{112} r, \\ h_{22} &= -u_a''', h_{23} = 0, h_{33} = p_{122} r. \end{aligned}$$

With the help of (**??**)-(32), the partials of $f(x_b, w_a)$ can then be computed

$$\begin{aligned} f_{11} &= \left(-\frac{r}{h_1^3}\right)\left(-u_a''' p_{11}^2 r + (u_a'')^2 p_{111}\right), \\ f_{22} &= \left(-\frac{r}{h_1^3}\right) p_{122} h_1^2, \\ f_{12} &= \left(-\frac{r}{h_1^3}\right) u_a'' p_{112}\left(u_a'' + p_{11} r\right). \end{aligned}$$

Given $h_3 = 0$, we obtain that $f_1 = 0$ and $1 + f_1 = 1$. Applying Theorem 2 then obtains that the sign of the quadratic form (10) is given by the sign of

$$A\left(p_{111} - 3p_{112}\right) - P\frac{p_{11}^2}{p_1},$$

where $A \overset{\text{def}}{=} -\frac{u''(w-x)}{u'(w-x)}$ and $P \overset{\text{def}}{=} -\frac{u'''(w-x)}{u''(w-x)}$. Making use of the expressions for the p-derivatives gives

$$A\frac{1}{2}\frac{m}{x_3}\left(1-m^2\right) - P\frac{1}{4}\frac{m}{x_3}.$$

This proves Theorem 3.

Proof of Theorem 4

For the ability contest model, we obtain the following expressions for the h function

$$\begin{aligned}
h &= \frac{\partial p}{\partial x_a}\Delta u_a\left[u(w_a+r)-u(w_a)\right] - c_a' = 0 \\
h_1 &= \frac{\partial^2 p}{\partial x_a^2}\Delta u_a - c_a'' < 0 \\
h_2 &= \frac{\partial p}{\partial x_a}\Delta u_a', h_3 = \frac{\partial^2 p}{\partial x_a \partial x_b}\Delta u_a \overset{\text{SE}}{=} 0 \\
h_{11} &= \frac{\partial^3 p}{\partial x_a^3}\Delta u_a - c_a''', h_{12} = \frac{\partial^2 p}{\partial x_a^2}\Delta u_a' \\
h_{13} &= \frac{\partial^3 p}{\partial x_a^2 \partial x_b}\Delta u_a, h_{22} = \frac{\partial p}{\partial x_a}\Delta u_a'' \\
h_{23} &= \frac{\partial^2 p}{\partial x_a \partial x_b}\Delta u_a' \overset{\text{SE}}{=} 0, h_{33} = \frac{\partial^3 p}{\partial x_a \partial x_b^2}\Delta u_a,
\end{aligned}$$

where $\Delta u_a \overset{\text{def}}{=} [u(w_a+r)-u(w_a)]$. Making use of (**??**)-(32), we obtain the following curvatures for the best-response function:

$$\begin{aligned}
f_{22} &= \left(-\frac{1}{h_1^3}\right)\left(p_1 \Delta u_a'' h_1^2 - 2p_1 p_{11}(\Delta u_a')^2 h_1 + p_1^2(\Delta u_a')^2(p_{111}\Delta u_a - c''')\right), \\
f_{11} &= \left(-\frac{1}{h_1^3}\right) p_{122}\Delta u_a h_1^2, \\
f_{12} &= \left(-\frac{1}{h_1^3}\right)\left(-p_{112}p_1 \Delta u_a' \Delta u_a h_1\right).
\end{aligned}$$

Given $h_3 = 0$, $f_1 = 0$ and $1 + f_1 = 1$, application of Theorem 1 gives that the quadratic form (10) is given by

$$\left(-\frac{1}{h_1^3}\right)\left\{-3(\Delta u')^2\Delta u p_1^2 p_{112} + [\Delta u''\Delta u - 2(\Delta u')^2]\Delta u p_1 (p_{11})^2 \right.$$
$$+(\Delta u')^2\Delta u p_1^2 p_{111} - 2[\Delta u''\Delta u - (\Delta u')^2]p_1 p_{11} c'' + \Delta u'' p_1 c''^2$$
$$\left. -(\Delta u')^2 p_1^2 c'''\right\}.$$

Since $h_1 < 0$ (SOC), the sign of the quadratic form is the sign of the term in curly brackets. Making use of the derivatives of the power-logistic CSF in the symmetric equilibrium, the power specification for $c(x)$, $c(x) = x^\gamma$, and the FOC evaluated at the symmetric equilibrium, $\frac{m}{4x}\Delta u = c'$, this term can be written as (up to a positive constant)

$$\left(-2m^2 + \frac{\Delta u''/\Delta u'}{\Delta u'/\Delta u}\right) + (\gamma - 1)\left(\frac{\Delta u''/\Delta u'}{\Delta u'/\Delta u}(1+\gamma) - \gamma\right).$$

Then the quadratic form is therefore positive iff

$$\frac{\Delta u''/\Delta u'}{\Delta u'/\Delta u}\gamma^2 - (\gamma - 1)\gamma > 2m^2.$$

This proves Theorem 4.

Proof of Theorem 5

For the rent-seeking contest model, the h-functions are given by

$$\begin{aligned}
h &= p_1\Delta u_a - Eu_a' = 0,\\
h_1 &= p_{11}\Delta u_a - 2p_1\Delta u_a' + Eu_a'' < 0\\
h_2 &= p_1\Delta u_a' - Eu_a'', h_3 = p_{12}\Delta u_a - p_2\Delta u_a' \overset{\text{SE}}{=} p_1\Delta u_a'\\
h_{11} &= p_{111}\Delta u_a - 3p_{11}\Delta u_a' + 3p_1\Delta u_a'' - Eu_a'''\\
h_{12} &= p_{11}\Delta u_a' - 2p_1\Delta u_a'' + Eu_a'''\\
h_{13} &= p_{112}\Delta u_a - 2p_{12}\Delta u_a' + p_2\Delta u_a'' \overset{\text{SE}}{=} p_{112}\Delta u_a - p_1\Delta u_a''\\
h_{22} &= p_1\Delta u_a'' - Eu_a''', h_{23} = p_{12}\Delta u_a' - p_2\Delta u_a''\\
h_{33} &= p_{122}\Delta u_a - p_{22}\Delta u_a' \overset{\text{SE}}{=} -p_{112}\Delta u_a + p_{11}\Delta u_a',
\end{aligned}$$

where $\Delta u_a \overset{\text{def}}{=} u(w_a + r - x_a) - u(w_a - x_a)$. With the help of these derivatives and expressions (30)-(32), the curvatures f_{11}, f_{12}, and f_{22} for a's best-response function are computed. Using Theorem 2, and simple, but tedious, factorization, it can be shown (the Maple files are available from the authors upon request) that the sign of the quadratic form $F_{11} - 2F_{12} + F_{22}$ can be written as

$$\frac{1}{G}\left[p_{11}^2 p_1 T_1 + 4p_{11}p_1^2 T_2 + \left(3p_{112} - p_{111}\right) p_1^2 T_3 + 4p_1^3 T_4\right], \tag{33}$$

where

$$\begin{aligned}
T_1 &= \frac{Eu''}{Eu'}\left(\frac{Eu'''}{Eu''} - 2\frac{\Delta u'}{\Delta u}\right) - \frac{\Delta u'}{\Delta u}\left(\frac{\Delta u''}{\Delta u'} - 2\frac{\Delta u'}{\Delta u}\right),\\
T_2 &= \frac{\Delta u'}{\Delta u}\left[\frac{\Delta u'}{\Delta u}\left(\frac{Eu''}{Eu'} - \frac{\Delta u'}{\Delta u}\right) - \frac{Eu''}{Eu'}\left(\frac{Eu'''}{Eu''} - \frac{\Delta u''}{\Delta u'}\right)\right],\\
T_3 &= \left(\frac{Eu''}{Eu'} - \frac{\Delta u'}{\Delta u}\right)^2,\\
T_4 &= \left(\frac{\Delta u'}{\Delta u}\right)^2\left[\frac{\Delta u''}{\Delta u'}\left(\frac{\Delta u'}{\Delta u} - \frac{Eu''}{Eu'}\right) + \frac{Eu''}{Eu'}\left(\frac{Eu'''}{Eu''} - \frac{\Delta u''}{\Delta u'}\right)\right], \text{ and}\\
G &= \left(p_{11} - p_1\frac{\Delta u'}{\Delta u} + p_1\frac{Eu''}{Eu'}\right)\left(p_{11} - 3p_1\frac{\Delta u'}{\Delta u} + p_1\frac{Eu''}{Eu'}\right)^2.
\end{aligned}$$

Note that G is negative given the term in the first round brackets can be written as $\frac{h_1}{\Delta u} + \frac{\partial p_a}{\partial x_a}\frac{\Delta u'}{\Delta u}$ and both h_1 and $\Delta u'$ are negative because of the second-order condition and risk aversion, respectively. This proves Theorem 5.

A.6 Proof of Theorem 6

The first-order condition for x is given by $h(x_a, w_a, x_b) = 0$, where

$$\begin{aligned}
h(x_a, w_a, x_b) = {} & \frac{\partial p_a(x_a, x_b)}{\partial x_a}[u(w_a + r - x_a) - u(w_a + r - x_a)]\\
& -[p(x_a, x_b)u(w_a + r - x_a) + (1 - p(x_a, x_b))u(w_a - x_a)].
\end{aligned}$$

At a symmetric equilibrium ($x = x_a = x_b$), $\frac{\partial p_a(x,x)}{\partial x_a} = \frac{1}{4}\frac{m}{x}$ and $p_a(x, x) = \frac{1}{2}$. Using for $u(y)$ the CRRA form, $u(y) = \frac{y^{1-\rho}}{1-\rho}$, and taking a Taylor expansion of degree 2 around $r = 0$, results in

$$h(x, w, x) \simeq (w-x)^{-\rho}\left[-1 + \left(\frac{1}{4}\frac{m}{x} + \frac{1}{2}\frac{\rho}{w - x}\right) r - \left(\frac{1}{8}\frac{m}{x}\frac{\rho}{w - x} + \frac{1}{4}\frac{\rho(1 + \rho)}{(w - x)^2}\right) r^2\right].$$

Equating the *rhs* to zero and solving for x gives three roots, with the real solution being $\frac{1}{4}mr + O(r^3)$ (in fact $\frac{1}{4}mr$ is the solution to the case of quadratic preferences). Replacing x by $\frac{1}{4}mr$, the obtained expressions for T_1, T_2, T_3, and T_4 are then Taylor-approximated around $r = 0$:

$$
\begin{aligned}
T_1 &= \frac{1}{2}\frac{\rho(1+\rho)}{w^4}r^2 + O(r^3), \\
T_2 &= \frac{1}{3}\frac{\rho^2(1+\rho)}{w^5}r^2 + O(r^3), \\
T_3 &= O(r^3), \text{ and} \\
T_4 &= \frac{2}{3}\frac{\rho^3(1+\rho)}{w^6}r^2 + O(r^3).
\end{aligned}
$$

Next, the coefficients with T_1, T_2, and T_4 are computed using the earlier derived expressions for the probability function and its derivatives, and evaluating them at $x = \frac{1}{4}mr$. Finally, the numerator of (33) is computed. Up to a negative proportionality factor, it is equivalent to

$$1 - \frac{2}{3}\left(\rho m \frac{r}{w}\right) + \frac{1}{2}\left(\rho m \frac{r}{w}\right)^2.$$

The expression has no real roots and is always positive. Hence, for CRRA preferences and a rent that is small w.r.t. the initial wealth w, a small MPS in wealth reduces aggregate effort. This proves Theorem 6.