7. Experimental Evidence

Outline
- Tests of Expected Utility
- Probability triangle
- Preference reversal
- The debate about incentives
- Are subjects risk-averse?
- Field experiments
Experimental Economics

- Hundreds of experiments on risky decisions in psychology and economics

- Will select a small sample of them. Mostly on the tests of foundations, i.e., tests of EU and risk-aversion.

- Focus on the experimental results. (Not so much on experimental design)
Test of Expected Utility

• Early experiments, especially Allais (Etca, 1953)

• Generalized by Kahneman and Tversky (Etca, 1979)

⇒ Huge impact on economics

Early Experiments

Four main effects:
1. Certainty effect (or common ratio effect)
2. Allais paradox (or common consequence effect)
3. Isolation effect
4. Reflection effect
Certainty Effect

X1 = (4000, 80%; 0, 20%)
X2 = (3000, 100%)
X3 = (4000, 20%; 0, 80%)
X4 = (3000, 25%; 0, 75%)

Most subjects prefer X2 to X1 and X3 to X4
A Variant

\[ X_1' = (6000, 45\%; 0, 55\%) \]
\[ X_2' = (3000, 90\%; 0; 10\%) \]
\[ X_3' = (6000, 0.1\%; 0, 99.9\%) \]
\[ X_4' = (3000, 0.2\%; 0, 99.8\%) \]

Most subjects prefer \( X_2' \) to \( X_1' \) and \( X_3' \) to \( X_4' \)
Allais Paradox

$X_1^* = (2500, 33\%; 2400, 66\%; 0, 1\%)$

$X_2^* = (2400, 100\%)$

$X_3^* = (2500, 33\%; 0, 67\%)$

$X_4^* = (2400, 34\%; 0, 66\%)$

Most subjects prefer $X_2^*$ to $X_1^*$ and $X_3^*$ to $X_4^*$
Isolation Effect

$X_1 = (4000, 80\%; 0, 20\%)$

$X_2 = (3000, 100\%)$

$X_3 = (4000, 20\%; 0, 80\%)$

$X_4 = (3000, 25\%; 0, 75\%)$

Two-stage gamble:

$X_5 = (X_1, 25\%; 0, 75\%)$

$X_6 = (X_2, 25\%; 0, 75\%)$

Most subjects prefer $X_3$ to $X_4$ and $X_6$ to $X_5$
Two-stage gambles

Gamble 1

Gamble 2

Which gamble you prefer?
Two-stage gambles

Gambles are identical

People are not indifferent between both gambles. Usually prefer gamble 1.

Choices affected by the probability of winning in the « gain wheel » compared to probability of loosing in the « loss wheel ».
Reflection Effect

X1= (4000, 80%; 0, 20%)
X2= (3000, 100%)

Y1= (-4000, 80%; 0, 20%)
Y2= (-3000, 100%)

Most subjects prefer X2 to X1 and Y1 to Y2
Would need a « specific » utility function to accommodate for this
Three-outcome Lotteries

• One of the difficulty is to « organize » these anomalies/effects

• Observe that these experiments involve lotteries with at most three outcomes

• Can be represented using a triangle of probabilities - Machina (1982, Etca)
Any lottery with the same 3 outcomes
Only the vector \((p_1, 1-p_1-p_3, p_3)\)
matters

Any lottery can thus be represented
on the following triangle
Risk Preferences

\[ K = p_1 u(x_1) + (1 - p_1 - p_3) u(x_2) + p_3 u(x_3) \]

Indifference curves over \((p_1, p_3)\)
Slope: \[ \frac{[u(x_2) - u(x_1)]}{[u(x_3) - u(x_2)]} \]

Without loss of generality, \(x_1 < x_2 < x_3\)
Machina Triangle

Upwards movements increase $p_3$ (at the expense of $p_2$).
Leftwards movements reduce $p_1$ (to the benefit of $p_2$).
Risk Preferences

Increasing Preferences

Risk Aversion
Risk Neutrality

$p_3$

$p_1$
Certainty Effect

Increasing Preferences

\[ X_1 = (4000, 80\%; 0, 20\%) \]
\[ X_2 = (3000, 100\%) \]
\[ X_3 = (4000, 20\%; 0, 80\%) \]
\[ X_4 = (3000, 25\%; 0, 75\%) \]
Increasing preferences; but preferences fan out within the triangle: Machina’s hypothesis

\[ X_1 = (4000, 80\%; 0, 20\%) \]
\[ X_2 = (3000, 100\%) \]
\[ X_3 = (4000, 20\%; 0, 80\%) \]
\[ X_4 = (3000, 25\%; 0, 75\%) \]
Allais Paradox

\[ X_1^* = (2500, 33\%; 2400, 66\%; 0, 1\%) \]
\[ X_2^* = (2400, 100\%) \]
\[ X_3^* = (2500, 33\%; 0, 67\%) \]
\[ X_4^* = (2400, 34\%; 0, 66\%) \]
Test of Fanning Out

• Machina’s hypothesis: more risk aversion in the region of more preferred lotteries

• This hypothesis holds at the extremes of the triangle but does not hold in general (Conlisk, 1989, AER)

• Raises the question of the appropriate alternative theory, see chapter 9
Incentives

• Most early experiments: no monetary incentives (Incentives: subjects are paid in real money) – Common in psychology

• What about the experimental tests of EU under appropriate monetary incentives?

• With incentives, and when lotteries are in the interior of the triangle, violations of EU are rare! (Camerer, 1989, JRU, 1995)

• Later we will study the effect of incentives on risk aversion assuming EU
Preference Elicitation

Preferences should be invariant to the way the question is asked (framing)

Kahneman and Tversky (1979, Etca):

You have been given 1000. You are now asked to choose between 50% of a gain of 1000 or a sure gain of 500. (84% of subjects choose the sure gain)

You have been given 2000. You are now asked to choose between 50% of a loss of 1000 or a sure loss of 500. (69% of subjects choose the gamble)
Preference Reversal

Let X1 and X2 two lotteries and C(X1) and C(X2) their certainty equivalent

In EU (and most) models, X1 prefered to X2 is equivalent to C(X1)>C(X2)

Lichtenstein and Slovic (1971, JExpP) show that this equivalence fails in experiments => preference reversal

But, again, this phenomenon may virtually disappear with monetary incentives (Grether and Plott, 1979, AER)
Are people risk-averse?

• Next step. Are subjects risk-averse in experiments with monetary incentives?

• But raise another question: How much should we pay subjects to induce appropriate incentives?

• Traditionally too small incentives in experiments (a few dollars) => Individuals should be nearly risk-neutral. Rabin’s critique (Rabin, Etca, 2000)
Rabin’s Calibration Theorem

• Under EU: « Suppose that, from any initial wealth, a person turns down gambles where she loses $100 or gains $110, each with 50% probability. Then she will turn down 50-50 bets of losing $1000 or gaining any sum of money ». 

• Risk-aversion needed to explain low-stakes situations implies an absurd amount of risk-aversion in high-stakes lotteries
Rabin’s critique

• Intuition: turn down gamble from any initial wealth is equivalent to CARA utility function: $u(x)=-\exp(-\lambda x)$. Scaling up payoffs by 100 under CARA is equivalent to $u(x)=-\exp(-\lambda 100x)$ which is equivalent to having 100 times as much risk aversion.

• But Rabin’s critique implicitly assumes CARA. He « somewhat » extends to DARA though.
Rabin’s critique

• Suggests that strong incentives are important. But difficult to induce strong incentives (costly!).

• Kahneman and Tversky (1979)’s approach: « By default, the method of hypothetical choices emerges as the simplest procedure »

• Rely on the assumptions that i) people know how they would behave with high stakes and ii) do not disguise their true preferences
Risk-aversion and incentives

Holt and Laury (AER, 2002):

Table 1. The Ten Paired Lottery-Choice Decisions with Low Payoffs

<table>
<thead>
<tr>
<th>Option A</th>
<th>Option B</th>
<th>Expected Payoff Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/10 of $2.00, 9/10 of $1.60</td>
<td>1/10 of $3.85, 9/10 of $0.10</td>
<td>$1.17</td>
</tr>
<tr>
<td>2/10 of $2.00, 8/10 of $1.60</td>
<td>2/10 of $3.85, 8/10 of $0.10</td>
<td>$0.83</td>
</tr>
<tr>
<td>3/10 of $2.00, 7/10 of $1.60</td>
<td>3/10 of $3.85, 7/10 of $0.10</td>
<td>$0.50</td>
</tr>
<tr>
<td>4/10 of $2.00, 6/10 of $1.60</td>
<td>4/10 of $3.85, 6/10 of $0.10</td>
<td>$0.16</td>
</tr>
<tr>
<td>5/10 of $2.00, 5/10 of $1.60</td>
<td>5/10 of $3.85, 5/10 of $0.10</td>
<td>-$0.18</td>
</tr>
<tr>
<td>6/10 of $2.00, 4/10 of $1.60</td>
<td>6/10 of $3.85, 4/10 of $0.10</td>
<td>-$0.51</td>
</tr>
<tr>
<td>7/10 of $2.00, 3/10 of $1.60</td>
<td>7/10 of $3.85, 3/10 of $0.10</td>
<td>-$0.85</td>
</tr>
<tr>
<td>8/10 of $2.00, 2/10 of $1.60</td>
<td>8/10 of $3.85, 2/10 of $0.10</td>
<td>-$1.18</td>
</tr>
<tr>
<td>9/10 of $2.00, 1/10 of $1.60</td>
<td>9/10 of $3.85, 1/10 of $0.10</td>
<td>-$1.52</td>
</tr>
<tr>
<td>10/10 of $2.00, 0/10 of $1.60</td>
<td>10/10 of $3.85, 0/10 of $0.10</td>
<td>-$1.85</td>
</tr>
</tbody>
</table>
Figure 1. Proportion of Safe Choices in Each Decision: Data Averages and Predictions. Key: Data Averages for Low Real Payoffs (Solid Line with Dots), 20x, 50x, and 90x Hypothetical Payoffs (Thin Lines) Risk Neutral Prediction (Dashed Line).
Incentives

Strong incentives effects!

Figure 2. Proportion of Safe Choices in Each Decision: Data Averages and Predictions. Averages for Low Real Payoffs (Solid Line with Dots), 20x Real (Squares), 50x Real (Diamonds), 90x Real Payoffs (Triangles), and Risk Neutral Prediction (Dashed Line).
Field Experiments

• Binswanger (1980, AJAE) show that most farmers in rural India exhibit significant risk aversion

• Kachelmeier and Shehata (1992, AER) in their study in China report significant increase in risk aversion as incentives increase
Field Experiment

- List (2003, QJE, 2004, Etca) - Sportscard game

- Inexperienced subjects display inconsistencies like those identified by Kahneman and Tversky

- But show that experience and learning significantly eliminates these inconsistencies
Animal Experiments

• Presumably, one might induce salient rewards with animals: food.

• Rats may be taught about the returns (food) of a specific lever. Then, they face a « safe lever » vs. a « random lever »

• Rats display inconsistencies, like the certainty effect Battalio et al. (1985, AER), Kagel et al. (1990) – Same with monkeys (Chen et al., JPE, 2004)

• Further research needed to examine the persistence of these inconsistencies among animals
Eliminate Risk-Aversion

• Some experimenters do not want to elicit risk-aversion, but instead do want to «eliminate» it (e.g., to discriminate between two hypotheses, one being risk aversion)
• Assuming EU, it is possible in experiments to eliminate, in principle, risk-aversion.
• Idea: pay subjects with « probability points »
Eliminate Risk-Aversion

• Example: 10% of $100 is not twice better than 10% of $50. But under EU, it is twice better than 5% of $100. And it is the same as ½ of 15% of $100 and ½ of 5% of $100.

• Under EU, the objective function is linear in probability points. If subjects are paid with probability points, they behave like risk-neutral agents.

• Called the lottery-ticket procedure. (Roth and Malouf, PR, 1979)

• But the lottery-ticket procedure does not perform well experimentally (Millner and Pratt, 1989, PC)
8. Uncertainty and Ambiguity

Outline

- Risk perceptions
- Probability elicitation, scoring rule
- Mistakes, heuristics and biases
- Hindsight bias, overconfidence
- Bounded rationality
- Ellsberg’s paradox
- A theory of ambiguity
Introductory Remark

Here: no more objective probabilities (contrary to previous chapters)

=> Calls for a distinction:
- Judgments: How do people perceive uncertainty?
- Choices: How do they behave in face of uncertainty?
Mortality Risk Perceptions

![Graph showing the relationship between estimated and actual frequency of death](image)
Probability Perceptions

The Inverted S-shape Pattern. Most common shape in probability judgments.
Data on Subjective Probabilities?

- The assumption of rational perception (and in turn the “rational expectation” paradigm) is challenged by psychologists.
- An implication: Choice data are consistent with many combinations of choice/perceptions models (identification problem); see Manski (Etca, 2004).
- By imposing rational perceptions economists may distort estimates of individual preferences.
- Using data on probability perceptions (e.g. surveys, opinion polls) may thus help address this problem.
- Problem: how reliable are these data?
Elicitation of Probabilities

- How to know people’s estimates of probabilities?
- Psychology of judgment: people are not paid according to their estimates
- But incentives usually may matter
- Incentives must induce a truthful revelation of subjective probabilities

=> Scoring rules (used in statistics and economics)
Scoring Rule

• Suppose a subject truly believes that the probability of an event (e.g., Obama wins the election) is $q$

• Objective: obtain truthful subjects’ estimates of the probability she assigns to this event

• Suppose the subject announces $p$, then let $S(p)$ the monetary amount the subject is paid if the event occurs, and $SN(p)$ if the event does not occur
Scoring Rule

- Under risk neutrality, the subject thus maximizes

\[ \max_p qS(p) + (1-q)SN(p) \]

\[ \text{FOC: } qS'(p) + (1-q)SN'(p) = 0 \]

\[ \text{SOC: } qS''(p) + (1-q)SN''(p) < 0 \]

- Let consider a quadratic scoring rule:

\[ S(p) = 2p - p^2 \text{ and } SN(p) = 1 - p^2 \]

- FOC: \( q(1-p) - (1-q)p = 0 \) \implies \( p^* = q \) (truthful revelation)
Quadratic Scoring Rule

- Statisticians have shown that other scoring rules are incentive compatible (Savage, 1971, JASA), e.g. the logarithmic scoring rule (Godd, 1952, JRSS)

- Selten (1998, EE) provides an axiomatic foundation for the quadratic scoring rule (based on four axioms: symmetry, elongation invariance, incentive compatibility and neutrality)
Efficiency of Scoring Rules

• Allow beliefs to be elicited even without mentioning the word “probability”, or defining it

• Yet, the efficiency of a scoring rule in experiments is not clear (Camerer, 1995). It is often too complex (Hogarth, 1987). It may alter the behavior of the subject during the course of an experiment (Croson, 2000, JEBO).

• Predictions are too extreme (Palfrey and Wang, 2007), and erratic (Nyarko and Schotter, 2002, Etca)

• Theoretically, it is not robust to the introduction of risk-aversion
Mistakes

• Milton Friedman: people make mistakes. Just not systematic.

• Tversky and Kahneman (1974, Science): many systematic mistakes (e.g., overestimate small risks, and underestimate large risks, see above)

• Confronted with uncertainty, all individuals use similar heuristics, that are useful, but that lead to systematic biases
Two-Card Game

- Two cards
- Each has the two faces colored
- One card has red/red
- The other card has red/green
- You observe red
- What is probability that the other side is red?
Small Sample

• Hospital A : 45 babies/day
• Hospital B : 15 babies/day
• Nb of days for which >60% of babies are male: larger for A or B?

• Answers: 21% A, 53% the same
• The law of large numbers does not belong to the repertory of intuitions
Law of Small Numbers

• Coin: Tail (T) or Head (H) ; proba = 1/2
• Results: TTTT
• Probability that H comes out next flip?
• Many subjects say >1/2
• “gambler’s fallacy”
• Misperception of correlations
Repeated Sequences

• Gneezy (1976, AP)
• A financial asset follows a random walk
• +$1 or -$1 with p and 1- p
• Assume p=0.7
• Stop if the asset reaches either 0 or 10
• Probability to reach 10?
  • If starting value $1, proba=0.57; if $5 proba=0.9857
• It has been shown that exponential growth is considerably underestimated (Wagenaar and Timmers, 1979, AP)
Compound Events

• Suppose you were able to build a ballistic missile defense that depended on the successful performance of 500 independent parts or subsystems and suppose that each part or subsystem were 99% reliable. What are the chances that a system would work in a first attempt?

• Let a group composed of N people. What is the value of N necessary to make it more that 50% the chances of 2 people having the same birthday?
Compound Events

• Suppose you were able to build a ballistic missile defense that depended on the successful performance of 500 independent parts or subsystems and suppose that each part or subsystem were 99% reliable. What are the chances that a system would work in a first attempt? (p<1%)

• Let a group composed of N people. What is the value of N necessary to make it more that 50% the chances of 2 people having the same birthday? (N=23)
Sampling and Bayes’ Rule

• You can bet on three gambles:
  - 1) Drawing a red ball from a bag containing 50% red balls
  - 2) Drawing a red ball for seven successive times, with replacement, from a bag containing 90% red balls
  - 3) Drawing a red ball at least once in seven successive tries with replacement, from a bag containing 10% red balls

Usually people prefer 2 (p=0.48) to 1 (p=0.5) and 1 to 3 (p=0.52)
Sampling and Bayes’ Rule

• Imagine an urn filled with balls, of which 2/3 are of one color and 1/3 of another. One individual has drawn 5 balls from the urn, and found that 4 were red and 1 was white. Another individual has drawn 20 balls and found that 12 were red and 8 were white. Which of the two individuals should be more confident that the urn contains 2/3 red balls and 1/3 white balls?

• Correct posterior odds are resp. 8 to 1 and 16 to 1
• “Conservative estimates”: usually estimates are far less extreme than the correct values
Heuristics

• Need to explain and organize patterns of probability (mis-)perceptions

• Tversky and Kahneman (1974, Science): People use heuristics when they have to assess probabilities

• Three main heuristics: “Representativeness”, “Availability” and “Anchoring”
Representativeness

• Heuristic: Probabilities are evaluated by the degree something resembles to something else

• Typical induced bias: Detailed scenarios (who seem more realistic) are judged more likely; But more details in fact lessen likelihood

• Implications: May include preconception of chance, misconception of regression to the mean
Availability

• Heuristic: Probabilities evaluation depends on the ease in which instances or occurrences can be brought to mind

• Typical induced bias: More salient (e.g., sensational) events are judged more likely

• Implications: May include overestimation of small probabilities, illusory correlation
Anchoring

• Heuristic: People make probability estimates by starting from an initial value that is adjusted insufficiently

• Typical induced bias: Estimates end up around a “natural” anchor

• Implications: May include conjunctive vs disjunctive events, anchoring to the mean, insufficient adjustment
Hindsight Bias

• Limited memory
• Hindsight bias or “I knew it all along”
• Systemic bias in how people recall their own past judgments of probabilities
• People believe that an event was inevitable; Determinism
• After an event occurs, people overestimate their own past probabilities predictions that this event would occur (Fischhoff, 1975, JExp.Psych.)
Overconfidence

• Calibration: a judge is calibrated if, over the long run, for all propositions assigned a given probability, the probability that it is true equals the probability assigned.

• Example. “Absinthe is a a) precious stone, b) a liqueur, c) a Caribbean island”. The assessor provides an answer and then gives a probability that an answer is correct.

• Most pervasive finding in miscalibration is overconfidence (Fischhoff and al, 1977, J.Exp.Psych).

• Overconfidence is more extreme with tasks of great difficulty; Underconfidence may be found for easy tasks.
Illusion of Control

• People behave as if chance events are subject to control
• Dice players play softly (strongly) if they want a small (large) number
• Illusion of control: Expectancy of a personal success probability is inappropriately higher than the objective probability under conditions of perceived control (Langer, 1975, JPSP)
• Car accidents vs. plane accidents
Bounded Rationality

General points made by Simon (1997, collected papers):

• The largest fraction of decision-making is spent on searching for possible courses of action; much less time is spent in making final choices
• The generation of alternatives is a lengthy and costly process
• Computation ability is limited, and attention is a scarce resource
• People do not maximize but want to attain goals

=> “Substantive” vs. “procedural” rationality
Bounded Rationality (cont’d)

Main behavioral hypotheses suggested by Simon:

• Instead of assuming a fixed set of alternatives, we may postulate a process generating alternatives
• Instead of assuming known probabilities, we may introduce estimations procedures for them
• Instead of assuming a utility maximization, we may postulate a “satisficing” strategy
Ambiguity: Ellsberg’s Experiment

• Ellsberg (1961, QJE)’s experiment is one the most popular experiment ever
• A decision-maker chooses from an urn that contains 30 red balls and 60 balls in some combination of black and yellow.
• The probability of winning is thus unknown, or ambiguous.
# Ellsberg’s Experiment

<table>
<thead>
<tr>
<th>Act</th>
<th>30 red balls</th>
<th>black balls</th>
<th>yellow balls</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$</td>
<td>$W$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$Y$</td>
<td>0</td>
<td>$W$</td>
<td>0</td>
</tr>
<tr>
<td>$X'$</td>
<td>$W$</td>
<td>0</td>
<td>$W$</td>
</tr>
<tr>
<td>$Y'$</td>
<td>0</td>
<td>$W$</td>
<td>$W$</td>
</tr>
</tbody>
</table>

Many people choose $X > Y$ and $Y' > X'$.
Inconsistency?

- Contradicts first order stochastic dominance
- Contradicts the « sure-thing principle »: let \( f, f', g \) and \( g' \) be lotteries and \( S \) an event. If \( f=g \) and \( f'=g' \) on \( S \), and on \( \sim S \) (not \( S \)), then \( f \) is preferred to \( f' \) iff \( g \) is preferred to \( g' \) (Let \( S \) equals yellow, \( f=X, f'=Y, g=X' \) and \( g'=Y' \))
- Raiffa (2001, QJE)’s example: suppose option A delivers X or Y’ with equal probability, and option B delivers Y or X’ with equal probability
Ambiguity

• Defining ambiguity is a popular pastime.
• Ambiguity is “known-to-be-missing information” (Camerer, 1995, Handbook Exp. Econ.)
• Ambiguity aversion is a robust finding in experiments (e.g., Slovik and Tversky, BS)
Ambiguity in Markets

- Camerer and Kunreuther (1989, JRU) found ambiguity aversion in insurance markets
- And Sarin and Weber (1993, MS) in double-oral and sealed-bid auctions
- Other potential implications:
  - Global preference for home-country investments (French and Poterba, 1991, AER)
  - Entrepreneurship and labor contracting (Bewley, 1986)
  - Main explain the equity premium puzzle (Chen and Epstein, 2002)
Ambiguity: Theory

- Gilboa and Schmeider (1989) provides an axiomatic for the maxmin criterion
- Assume F and G are two prior cumulative distributions (associated to X)
- Denote $u(d,X)$ for the utility function
- For each possible decision $d$ ex ante, individuals compute expected utility conditional to each prior: $E_F u(d,X)$ and $E_G u(d,X)$
- Then they compute $U(d) = \text{Min} \ (E_F u(d,X), \ E_G u(d,X))$
- Finally, $\text{Max}_d \ U(d)$
- Main problem: only focus is the worst-case scenario
Smooth ambiguity

- Klibanoff, Marinacci and Mukerji (Etca, 2005)
- Assume that the decision-maker sets a probability $p$ for $F$ to be the true prior
- Let $V(d) = \Phi^{-1}(p\Phi(E_F u(d,X)) + (1-p)\Phi(E_G u(d,X)))$
  in which $\Phi$ is the attitude towards ambiguity
- $\Phi(x) = x$: EU model
- $\Phi(x) = -n^{-1}\text{Exp}(-nx)$: constant ambiguity aversion, and when $n$ tends to infinity one has the Gilboa-Shmeidler’s ambiguity model
9. Behavioral Economics

Outline
- Decision weights
- Rank-dependent expected utility
- Prospect theory, loss aversion
- Regret and disappointment
- Anticipatory feelings
Non-Expected Utility

- A decade ago, this chapter would have been labeled non-expected utility models
- These models were mainly influenced by early “experimental failures” of EU and, more recently, by Kahneman and Tversky’s work
- The term “behavioral” suggests that models have become more “psychologically realistic”
- Remark: Many models simply encompass EU, so necessarily perform better
- I will present a portfolio of models, but little applications
Outcomes and Probabilities

- Main feature of EU: separation between outcomes and probabilities, and linearity in probabilities
- Let $p_i$ and $x_i$ be resp. probabilities and outcomes
- EU is a special case of $\sum_i f(p_i,x_i)$ in which $f(p,x)=pu(x)$
- Observe that we might also consider:
  - $f(p,x)=w(p)x$: «distortion» of probabilities alone, e.g. Handa (1977, JPE) or Yaari (1987, Etca)’s dual theory
  - $f(p, x) =w(p)u(x)$: «distortion» of both probabilities and outcomes, e.g., Viscusi (1989, JRU)’s prospective theory, or weighted utility (Starmer, 2000)
Outcomes and Probabilities (Cont’d)

• \( w(p) \) is usually called the “decision weight” function

• Remark: decision weights \( w(p) \) are different from misperception of probabilities (i.e., they are judgments as opposed to choices) under uncertainty conditions (instead often coined “probability weights”)

• Non-linear \( w(p) \) can explain most of the experimental anomalies presented in chapter 7

• Problem: non-linear \( w(p) \) lead to unappealing normative properties like dominance violations
Dominance Violation

• Suppose \( f(x,p+q) > f(x,p) + f(x,q) \)
• Take \( X = (x,p+q; 0,1-p-q) \)
• Take \( Y = (x+k,p; x,q; 0,1-p-q) \)
• Note that \( Y \) (first-order) stochastically dominates \( X \)
• However \( X \) may induce a higher utility with such preferences for \( k \) sufficiently small
Rank-Dependent EU

- One way to avoid dominance violation is to let decision weights transform the entire probability distributions of outcomes

\[ \text{RDEU} = \sum_{i=1}^{n} \left[ g\left( \sum_{j=i}^{n} p_j \right) - g\left( \sum_{j=i+1}^{n} p_j \right) \right] u(x_i) \]

with ranked outcomes \( x_1 \leq x_2 \leq \ldots \leq x_n \) with \( g \) increasing and \( g(0)=0 \) and \( g(1)=1 \).

- Concave \( u \) and convex \( g \) induces aversion to mean-preserving spreads (Chew, Karni and Safra, 1987, JET; Chateauneuf and Cohen, 1994, JRU)
Example: 3 Outcomes

- RDEU = (1-g(p₂+p₃))u(x₁) + (g(p₂+p₃)-g(p₃))u(x₂) + g(p₃)u(x₃)
- Transforms *cumulative* probabilities; the decision weight varies according to how good or bad it is compared to other outcomes
- = u(x₁) + [u(x₂)-u(x₁)]g(p₂+p₃) + [u(x₃)-u(x₂)]g(p₃))
- Interpretation: Obtaining the minimum satisfaction u(x₁) with certainty, and evaluates successive additional utility differences using the decision weights
RDEU and Ambiguity

• Ellsberg’s experiment (90 balls with 30 R balls, and 60 B and Y; bet on B) may be viewed as a two-stage lottery
  • In the first stage, there is an imaginary lottery $P$, where $P$ is the proportion of B balls (i.e., there is a sampling from a set of possible urns and $P$ is the belief about the sampling process)
  • In the second stage, there is a real lottery with $P=p$ the probability to get $w$ and 0 otherwise
• Important ingredient needed to explain the experimental outcome: the reduction of compound lotteries axiom must not apply
RDEU and Ambiguity (cont’d)

- Bet on R: \( g(1/3)u(w) \)
- Assume that \( P \) is such that \( P \) equals 1/6 (15 B) or 3/6 (45 B) with equal probability
- Bet on B: \( g(1/2)g(3/6)u(w) + (1-g(1/2))g(1/6)u(w) \)
- Lower than \( g(1/3)u(w) \) (e.g.) for \( g(p)=p^2 \)
- Hence explain aversion to ambiguity
- See Segal (1987, IER, 1990, Etca)
Prospect Theory (Kahneman and Tversky, Etca, 1979)

Differ in three ways from EU:

- People « edit » lotteries
- The utility function is replaced by a psychological value function
- Decision weights $w(p)$
Prospect Theory

« Edit » lotteries:

- Coding: e.g., people perceive outcomes as gains or losses rather than final outcomes
- Gains and losses are defined w/r to a reference point
- Other aspects: combination, segregation, cancellation, rounding probabilities…
- Importance of framing
Prospect Theory: \( v(.) \)

Psychological value function \( v(.) \)

- Diminishing sensitivity principle (concave for gains convex for losses)
- Loss aversion (steeper for losses)
Prospect Theory: \( w(.) \)

Decision weights \( w(.) \)
- Should be interpreted as beliefs, not probabilities
- Discontinuities around certainty 1 and impossibility 0 (1979, Etca)
- Diminishing sensitivity principle from 0 and 1

![Inverted S-shape curve](image)
Prospect Theory

=> A simple form for Prospect Theory (PT):

$$PT = \sum_{i=1}^{n} w(p_i)v(x_i - r)$$

with:

• $r$: reference point (e.g., status quo wealth)
• $v(z)$: convex (concave) if $z < (>) 0$ (and steeper for losses than for gains)
• $w(.)$: inverted S-shaped (with possible discontinuities around 0 and 1)
Cumulative Prospect Theory

Kahneman and Tversky (1992, JRU):
• Cumulative weighting of probabilities in order to satisfy stochastic dominance (similar to RDEU)
• Any number of outcomes (not only two)
• Two different continuous inverted S-shaped \( w(.) \), one for losses, one for gains
• Extensions to subjective beliefs
Prospect Theory: A Few Implications

- People hold losing stocks too long, and sell winning stocks too early
- Disparity WTA/WTP
- New York city cab drivers quit around daily targets (Camerer et al., 1997, QJE)
Myopic Loss Aversion (MLA)

Benartzi and Thaler (1995, QJE)

• MLA is the combination of two factors, loss aversion and « myopia »
• Depends on how often people evaluate their portfolio
• Idea: if evaluation every day, stock investments not attractive because stock prices fall too often
MLA (cont’d)

- Experimental evidence on MLA (Gneezy and Potters, 1997, QJE)
- Can potentially explain the Samuelson’s fallacy of the law of large number (cf Chapter 3)

- Let $X = ($200, 50%; -$100, 50%)$ every period
- Assume $v(x) = x$ if $x > 0$ and $2.5x$ if $x < 0$
- One-period evaluation: $0.5(200) + 0.5(-250) < 0$
- Two-period: $0.25(400) + 0.5(100) + 0.25(-500) > 0$
Benartzi and Thaler (1995, QJE)

- Benartzi and Thaler take $v(x) = x^a$ if $x > 0$ and $bx^a$ if $x < 0$ using KT (1992)’s estimates $a = 0.88$ and $b = 2.25$
- They can explain the equity premium puzzle for an evaluation period of 13 months
Reference Points

How to select reference points? Not clear! Based on the previous examples, they could be:

• Wealth at the beginning of the period, endowment
• Purchase price
• Expectation (target performance…)
• «Reference» consumption level…

Kahneman and Tversky (1991, QJE) : « The question of the origin and the determinants of the reference state lies beyond the scope of the present paper. »
Disappointment

- Let $X=($1000, 99%; $0, 1\%)$ or $Y=($0, 99%; $1000, 1\%)$
- Suppose you win 0. Do you feel the same if played $X$ versus $Y$?
- May feel disappointment if played $X$
- Idea: capture the psychological feeling that an outcome falls short or exceeds some «expectation»
Disappointment (cont’d)

- Bell (1985, OR), Loomes and Sugden (1986, RES)
- Let $u^* = \sum p_i u(x_i)$ be the «expectation», and may be understood as a reference point here
- Let $v(x_i) = u(x_i) + D(u(x_i)-u^*)$ in which $D(.)$ is «disappointment» or «elation»
- $\text{DET} = \sum p_i v(x_i) = \sum p_i [u(x_i) + D(u(x_i)-u^*)]$
- $D$ linear: EU
- Bell assumes $D(.)$ linear but kinked at the origin with different slopes
- Loomes and Sugden assume $D(x)$ convex (concave) if $x$ is positive (negative)
Regret

• Loomes and Sugden (1982, EJ), Bell (1982, OR)
• Let $d_1$ or $d_2$ two decisions and let $x_{1s}$ and $x_{2s}$ be resp. outcomes in states of the world $s$
• Let $v(d_1, d_2, s) = u(x_{1s}) + R(u(x_{1s}) - u(x_{2s}))$ with $u$ and $R$ increasing and concave
• $d_1$ «chosen», and $d_2$ «foregone»
• $R$ captures «regret» or «rejoicing»
• $RT = \Sigma_i p_i v(d_1, d_2, i)$
• The «reference point» here is the foregone utility
Anticipatory Feelings

- People experience feelings of anticipation prior to the resolution of uncertainty
- Hopefulness, anxiety, suspense...
- Not properly captured in EU: e.g., information about medical tests cannot change welfare ex ante
- Remark: Recursive preferences (Kreps and Porteus, 1978, Epstein and Zin, 1989, Etca) allow for a preference w/r to uncertainty resolution
- Idea: beliefs should be an argument of the vNM utility, and/or could even be endogenously selected (cf., cognitive dissonance literature in psychology)
- In economics, see e.g. Caplin and Leahy (2001, QJE), Brunnermeir and Parker (2005, AER)
Exercise 1 (6 points)

An expected utility maximizer derives utility from the consumption of a cake (e.g., a resource, a bequest) over two periods. He has a lifetime utility \( u(c_t) + \beta u(c_{t+1}) \) in which \( c_t \) is consumption in period \( t \), \( \beta \) a discount factor and \( u \) is an increasing, concave and three times differentiable utility function.

Consider two cases. Under certainty, the size of the cake is \( y \). Under uncertainty, the size is either \( y + x \) (good state) or \( y - x \) (bad state) with equal probability.

i) Show that the certainty case is preferred.

ii) Suppose from now that the consumer is prudent (i.e., \( u'' \geq 0 \)). Show that optimal consumption in the first period is lower in the uncertainty case than in the certainty case. (Assume interior solutions.)

iii) Consider only the uncertainty case. But suppose that the consumer will face another (independent) risk, say plus or minus \( z \) with equal probability. Suppose also that he can choose in which state he will face this additional risk. Which state do you think he will choose, the good or the bad one? Detail your answer.

Exercise 2 (6 points)

Due to a legal issue, an expected utility maximizer is currently involved in a trial. He will loose the monetary amount \( L \) if he looses this trial. However, he may objectively reduce his chances of losing the trial by spending an amount \( e \) in some expertise (e.g., hiring a lawyer). Formally, the probability of winning the trial, \( p(e) \), is increasing in \( e \).

Answer formally the following questions (unique and interior solutions will be assumed throughout):

i) Show that the optimal amount spent \( e \) is larger if this amount is reimbursed in case of victory in the trial (as it is the case in many jurisdictions) than if it is not reimbursed.

ii) Assuming this amount is actually reimbursed if victory, show that more risk-aversion decreases, and not increases, the optimal amount \( e \)? Give an intuition.

Hint. You may use the following property: let \( T \) increasing, differentiable and concave, then for any \( b > a \), \( T'(a)(b - a) \geq T(b) - T(a) \).
Exercise 3 (3 points)

Here follows one famous example from Tversky and Kahneman (1981, *Science*) that was presented to two groups of subjects.

“Imagine that the US is preparing for the outbreak of an unusual asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed”.

Then options A or B were presented to group 1.

“If program A is adopted, 200 people will be saved.
If program B is adopted, there is a 1/3 probability that 600 people will be saved and a 2/3 probability that no people will be saved”

Options C or D were presented to group 2.

“If program C is adopted, 400 people will die.
If program D is adopted, there is a 1/3 probability that nobody will die and a 2/3 probability that 600 people will die”

Tversky and Kahneman found that 72% of subjects in group 1 preferred option A to B while only 22% in group 2 preferred C to D. Could you explain this result?

Exercise 4 (5 points)
Answer one of the two following problems. (Deeqa students: Answer both problems)

i) It is well-known that expected utility maximizers always like (costless) information. Build a formal example in which the value of information is negative for an individual who does not maximize expected utility.

ii) How would you test experimentally for the “irreversibility effect”?

Bonus question (Deeqa students only)

Discuss the following statement: “Scoring rules cannot be used with risk-averse subjects”.


Exercise 1 (3 points)

Consider four lotteries that are given to participants in an experiment: $X_1 = (6000, 45%; 0, 55%), X_2 = (3000, 90%; 0, 10%), X_3 = (6000, 0.1%; 0, 99.9%)$ and $X_4 = (3000, 0.2%; 0, 99.8%)$.

Participants prefer $X_2$ to $X_1$, and prefer $X_3$ to $X_4$. Show formally that this experimental result is inconsistent with expected utility theory. Build an example using an alternative theory that could be consistent with this result.

Exercise 2 (4 points)

An agent faces an investment opportunity. If she invests today she gets $B$ for sure today, and she gets either +$2000 or -$2000 with equal probability tomorrow. She may also delay her decision up to tomorrow, and decide whether to invest then. In this case, she has to give up the sure $B$ today, but she will learn before having to make her investment decision whether she gets +$2000 or -$2000 tomorrow.

Assume risk aversion. What can you say about the optimal investment decision rule? (Hint: discuss for various values of $B$)

Exercise 4 (6 points)

Examine formally two of the following three statements about insurance demand. (Deeqa students: please examine all statements)

i) “An agent who demands more insurance than another agent, should also demand less risky asset”

ii) “If the price of insurance (that is, the loading factor) is high enough, it is better not to buy any insurance at all”.

iii) “If wealth increases, insurance demand decreases”.

Exercise 5 (7 points)

Please answer two of the following four questions. (Deeqa students: please answer three questions)

i) Prove formally that “less variance” is not equivalent to a “decrease in risk”.

ii) Some data suggest that self-employed and sales persons, those typically thought to have the most risky income, save less than other groups. Is this empirical result necessarily inconsistent with the hypothesis of “prudence”?

iii) Explain (briefly) why the result of the Ellsberg (1961)’s experiment on ambiguity over probabilities is inconsistent with (subjective) expected utility.

iv) Build an example in which Prospect Theory violates first-order stochastic dominance.