Cooperate and Conquer*

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March 5, 2014

Abstract

The idea that cooperation can be a source of power has been extensively discussed in various disciplines. We study this idea from an economic viewpoint by analyzing the effect of intragroup cooperation in a static group rent-seeking model. Intragroup cooperation means that agents' rent-seeking efforts maximize their group's expected payoff. Although intragroup cooperation always reduces average welfare in the economy, it significantly increases the winning probabilities and expected payoffs of the cooperative group. Our basic model reveals that this advantage is proportional to group sizes. We study which contexts (technologies, preferences) tend to amplify or mitigate this advantage.

Keywords: Cooperation, Group Rent-Seeking, Conflict, Group Size Paradox, Altruism

JEL Codes: D03, D72, D74

*We thank Ingela Alger, Guillaume Cheikbossian, Charles Figuières, David Myatt and Karine Van Der Straeten for helpful discussions and comments. We also thank conference participants at EAERE 2011, EEA-ESEM 2011, IMEBE 2012 and the Toulouse Symposium on Identity and Incentives in Organizations in 2012.

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1 Introduction

We spend much of our lives in groups. Sometimes, our group competes with other groups to obtain a rent (e.g., a political favor or a victory prestige). In this competition process, there is a tension between acting cooperatively for the group in order to increase the chances that our group wins the rent, and just acting for ourself without wasting time, money or energy for the group. This paper considers a simple economic game of intergroup rent-seeking that captures this tension. It studies the impact of cooperation within a group on the equilibrium outcomes of the game given that the rival group may, or may not, cooperate.

Scholars in different disciplines (see below in 1.1) have suggested that intragroup cooperation plays an important role in situations of intergroup conflicts. Military rhetoric for instance repeatedly appeals to group interests, group loyalty and group identity, suggesting that cooperation can play a decisive role in wars (Janowitz 1960, Mc Adams 1995). Old maxims like "divide and conquer" seem consistent with this idea that intragroup cooperation is important for a group, or a country, to be powerful.[1] As Hardin (1995, p.19) puts it: "widespread identification with a group, such as an ethnic group, can be the source of great power". Bowles and Gintis (2011) argue that this idea explains "why humans became extraordinarily group-minded, often favouring cooperation with 'insiders' and expressing hostility towards others". For Diamond (1997), the key factor that explains such a large-scale cooperation among unrelated humans was religion, for it manipulates group members to become obedient and peaceful inside the group, and brave and aggressive outside the group.

It is intuitive that the ability for a group to cooperate should confer an advantage in a context of group conflict. But, can we define and measure precisely this advantage? And how does this advantage depend on the context? Does it depend for instance on the size of the groups or on the efficiency of the conflict technology? And ultimately, is intragroup cooperation good or bad for welfare in the society? Our objective in this paper is to study formally these questions.

The effect of intragroup cooperation in intergroup conflicts has received little attention in economics. The most relevant literature is that in strategic contests and rent-seeking games (e.g., Garfinkel and Skaperdas 2007, Congleton, Hillman and Konrad 2010). But the majority of this vast literature

[1] See for instance some countries' mottos like "united we stand, divided we fall", "one for all, all for one", "e pluribus unum" (i.e., out of many, one), or "l'union fait la force" (i.e., unity makes strength).

has studied situations of conflict among individuals, not among groups. For the limited number of papers on intergroup conflicts, games are in general played in the noncooperative way by selfish agents (Nitzan 1991, Esteban and Ray 2001, Nitzan and Ueda 2009), except that in some cases agents have the opportunity to collectively decide their intragroup sharing rule in a pre-contest stage (Lee 1995, Baik and Lee 1997, Nitzan and Ueda 2011). In this paper, we partially fill this gap in the economic literature by developing a comparative statics analysis of the effect of cooperation in a group rent-seeking game. More precisely, we study and compare the equilibrium of the game in three cases: i) a common case in which all agents are free-riders in each group, ii) an intragroup cooperation case in which agents fully cooperate in their own group, in the sense that they maximize the aggregate group's payoff, and iii) a unilateral cooperation case in which agents free-ride in one group and cooperate in the other.

A first insight from this analysis is that intragroup cooperation is always detrimental to welfare (i.e., the sum of expected payoffs across all groups). This simple result in section 2 arises because cooperation intensifies wasteful rent-seeking behavior. This negative effect of cooperation in one group on overall welfare holds whether or not the other group cooperates. Yet, we show in section 3 that there exist strong incentives to cooperate within a group. Cooperation in a group has indeed a dramatic positive effect on its agents' efforts, and as a result on the winning probability and on the expected payoffs of agents in this group. In particular, for the case iii) of unilateral cooperation, we derive a general rule when there are n agents per group, the so-called "proportionality rule". This rule states that the aggregate effort, the probability of winning and the average expected payoff of the cooperative group are all exactly n times as large as those of the free-riding group.

We consider the "proportionality rule" as a benchmark result in the paper. Indeed this rule can be easily obtained due to various linearity assumptions in the basic model, i.e., a linear CSF, a constant marginal cost of effort and risk neutrality. These assumptions reflect severe constraints on technologies or preferences in the model, but are common in the literature. We relax these assumptions in section 4 in order to study the effect of the context on the effectiveness of intragroup cooperation. In particular, we show that the proportionality rule is partially preserved for the standard class of power logit CSF. Yet the rule is mitigated for convex cost of effort, and can be exacerbated under risk aversion.

We then generalize in section 5 the model to allow different group sizes. In doing so, we study the Olson (1965)'s well-known group size paradox that small groups are more efficient to advance their interests. We interpret the paradox as that the smaller group can stimulate more aggregate effort compared to the larger group, and has therefore a higher probability to win. We show that the paradox always holds under no cooperation, but that it never holds under intragroup cooperation. Moreover, in the unilateral cooperation case, we show that the cooperative group has always a higher probability of winning no matter the relative size between the groups.

The main assumption in this paper is that agents cooperate in their own group. This assumption is "ad hoc" in the sense that there is no reason why selfish agents in our static model would fully cooperate with other group members. Although this assumption is arguably non standard in economics, we recall that there is a wealth of evidence in psychology (Batson 1998), in experimental economics (Ledyard 1995, Fehr and Gächter 2000) as well as in field studies (Ostrom 2000) that people often do behave cooperatively. Many economic models in the last decades have introduced other-regarding preferences like (e.g.) altruism (Becker 1974), warm glow (Andreoni 1989), reciprocity (Rabin 1993), inequity aversion (Fehr and Schmidt 1999, Bolton and Ockenfels 2000), or concerns for social efficiency (Charness and Rabin 2002) and norms (Bénabou and Tirole 2011). Our assumption of full and exogenous cooperation may be viewed as an extreme shortcut for capturing such social preferences.[2]

Nevertheless, we study in section 6 a different model in which cooperation is not "ad hoc", but is induced by intragroup altruism. In this model, we show that a group's probability to win the rent is always positively related to its degree of altruism. Specifically, under identical group sizes, we show that a unilateral increase in the degree of a group's altruism always increases effort in this group. Moreover, it also increases the rival group's effort if and only if this rival group has a higher degree of altruism. We then assume the same degree of altruism in both groups but allow for different group sizes. A uniform increase in altruism is shown to increase the aggregate efforts of both groups. We finally show that the smaller group has a higher probability of winning if and only if the degree of altruism is low enough.

[2]We emphasize that we assume cooperation within the group instead of in the whole economy. In fact, most experimental data in economics indicating high level of cooperation can be related to group membership (McAdams 1995). In psychology, this idea is often gathered under the term collectivism that "the ultimate goal is the welfare of the group" (Batson 1998). Consistent with this idea, it has been shown that subjects often tend to confuse self-interest and group-interest (Baron 2001). Also, cooperation increases with group identity (Kramer and Brewer 1984) and when punishment and communication is allowed in the group (Dawes et al. 1977).

1.1 Related literature

This subsection presents some studies in different disciplines focusing on intragroup cooperation in situations of intergroup conflicts.

- Political science

The main argument in Mancur Olson's "logic of collective action" is that self-interest typically acts against group interest. For instance, Olson (1965, p. 105) emphasizes that "class-oriented action will not occur if the individuals that make up a class act rationally." Yet, casual evidence shows that many collective actions do occur. In strikes, international or ethnic conflicts for instance, people can even risk their lives for the sake of the group. Karl Marx's theory, which is often presented as driven by self-interest, has also set aside the thorny issue of collective action (Lash and Urry 1984). Fearon and Laitin (2000) see this issue as a "major puzzle" to explain ethnic conflicts. In an essay on cooperation in group conflicts, Russel Hardin suggests that collective action may be explained by ideological or altruistic motivations, arguing that "something about the natures of the members of the group and its alter group makes group identification workable and therefore overcome the logic of collective action" (Hardin 1995, p.5). According to Hardin, group identification would be the key factor explaining collective actions. This seems consistent with empirical data on ethnic conflicts and wars. For instance, Costa and Kahn (2008), using life histories of more than 40,000 Civil War soldiers, find that loyalty (as opposed to desertion) is positively correlated to soldiers' group homogeneity in terms of ethnicity, age and social status. More generally, there exists a lot of empirical and experimental evidence of a positive effect of group homogeneity on community participation as diverse as (e.g.) public goods investments, voluntary fundraising or car-pooling (Alesina and La Ferrara 2000, Putnam 2007, Chen and Li 2009). On a different issue, in elections, voter turnout also provides a striking example of collective action (Downs 1957). It has been argued that the decision to vote is "expressive" in the sense that it reflects ethical or ideological principles (Brennan and Lomasky 1993). Also, Schram and Van Winden (1991) observe that a positive level of turnout may be optimal for a group as a whole. They propose a theory in which group leaders produce social pressure to persuade agents in their group to go to vote. Interestingly, there also exists experimental evidence that voter turnout increases with group identification (Schram and Sonnemans 1996). A different class of arguments justifies collective actions based on irrationality. For instance, individuals within groups or states could be controlled by charismatic leaders subject to pathological biases and extravagant beliefs. In environmental lobbying, Sunstein (2003) refers to group leaders as "availability entrepreneurs", who attempt to exploit citizens' cognitive biases in order to mobilize them, and then to use them politically. Summarizing case studies on ethnic conflicts, Esteban and Ray (2008) make a similar point about citizens' beliefs manipulation that "elites might seek to achieve [political power] by the vested use of history, legend or myth, or by the framing of incidents (such as murder or rape) in explicit ethnic terms."

- Biology and anthropology

It has long been observed that many species engage in group projects, like joining defensive patrols and collective hunting. Group behavior in organisms like ants, fish or birds is prevalent, often driven by individual contagious behavior and influential leaders (Couzin 2008). Moreover, group behavior can be aggressive towards other groups, as shown by Jane Goodall and collaborators for male chimpanzees. Interestingly, the importance of intragroup cooperation in intergroup conflicts was stressed at the origin of evolution theory. Charles Darwin writes: "A tribe possessing (...) a greater number of courageous, sympathetic and faithful members (...) would spread and be victorious over other tribes. (...) Thus the social and moral qualities would tend slowly to advance and be diffused throughout the world." (Darwin 1871, p.162). The theory of evolution has thus long recognized that there is a selective pressure at the level of a group. There exist several biological explanations for intragroup cooperation among species, such as kin selection and reciprocal altruism. But this can be interpreted as a form of self-interest, and the difficulty to explain how cooperators can survive in most biological contexts has largely discredited the idea of group selection some decades ago. Nevertheless, this idea got a revival in biology when it was approached from an intergroup perspective. As Wilson and Wilson (2008) explain, "within-group selection is opposed by between-group selection, and (...) rather than categorically rejecting group selection and 'for the good of the group' thinking, we need to evaluate the balance between levels of selection on a case-by-case basis". A case in point is humans, as Bowles and Gintis (2011) argue that cooperation among Homo Sapiens is unique in its scale. Between-group competition for resources and survival in ancestral human societies played a decisive role to explain this tendency. This hypothesis is consistent with the works of many anthropologists which account for high levels of violence between groups of hunter-gatherers (see, e.g., the references in Seabright 2010), as well as for high levels of equity within groups, for example through an egalitarian meat sharing process (Wilson and Wilson 2008). Along similar lines, Turchin (2003) argues that within-group cooperation is the basis of between-group conflict, and Lehmann and Feldman (2008) provide support to this idea using a population-genetic model. In a recent review paper, Levin (2010) notes that cooperation is widespread in the biological world, but especially in human societies. Interestingly, he observes that "groups with common interests and common norms often owe their existence to the fact that these coalitions provide advantages in competition with other groups".

- Psychology

The classical Robers Cave experiment conducted in the 50s in a boys' summer camp by Muzafer Sherif and colleagues showed marked ingroup favoritism and outgroup hostility. Research in experimental social psychology has long suggested that there is a universal tendency of human beings to differentiate themselves according to group membership (Brewer and Brown 1998). Whenever experimental subjects are divided into groups, even arbitrarily, they consistently evaluate members in their own group more favorably than those in other groups. Gary Bornstein and colleagues have developed a set of experiments to specifically study problems of cooperation and competition that arise within and between groups. A general finding is that group cohesion enhances intergroup competition, and conversely that intergroup competition enhances group cohesion. Bornstein and Ben-Yossef (1994) show that subjects are much more likely to cooperate in their group when the game is framed as an intergroup conflict, suggesting that subjects become more concerned with the collective group goal in a situation of conflict. The motivational explanation of Bornstein and Ben-Yossef is that "conflict serves as a unit-forming factor that enhances group identification". This can perhaps be related to the association principle that people want their group (or their home-team in sports for example) to win in order to demonstrate their own superiority (Cialdini 2007). McAdams (1995)'s hypothesis, based on his interpretation of many experiments in social psychology and economics, is that individuals cooperate because they seek status from strangers based on visible group membership, and also because they seek esteem from fellow members in their own's group. This idea is related to the notion of parochialism, namely the tendency of people to favor a group that includes them while underweighing the harm to outsiders. Examples of parochialism are nationalism, racism or sexism. As Baron (2008) states, "parochialism is almost always involved in 'rent seeking' by groups that lobby governments for special privileges, to the general detriment of others. It is generally not in each individual's interest to contribute to this group effort, yet individuals end up acting against their self-interest and against the general interest in order to support their group's lobbying efforts." Baron argues that parochialism is mostly the result of fallacious thinking and of framing effects, inducing for instance the depersonalization of people in other groups.

- Economics

The economics literature on intergroup conflict is fairly limited. In a basic group rent-seeking model (Katz, Nitzan and Rosenberg 1990, Nitzan 1991, Esteban and Ray 2001), selfish agents make nonzero rent-seeking efforts for the group because they are nonatomic with respect to the group size. But the specific effect of cooperation has not been studied in this literature. Nevertheless, there are two recent and important exceptions. First, Cheikbossian (2012) studies an infinite horizon group rent-seeking model. In this model, intragroup cooperation is possible due to the use of trigger strategies by selfish agents. The main result is that, unlike the Olson's paradox, the larger group can more easily sustain cooperation. A key aspect of Cheikbossian's model is that the degree of cooperation is endogenous; indeed it is affected by the size of the group and by the behavior of the rival group. Second, Esteban and Ray (2011) posit a static so-called "behavioral" model in which there is a parameter of intragroup "cohesion". This cohesion parameter can be interpreted as the level of exogenous cooperation. Esteban and Ray obtain an expression of the equilibrium level of conflict that depends on the parameter of cohesion. We finally mention a recent economic experiment (Abbink, Brandts, Herrmann and Orzen 2010). This experiment considers different treatments with conflicts between either individuals or groups, and the design is such that the conflict expenditure per rival (being a group or an individual) should be identical in all treatments. It shows however that conflict expenditures of groups are much larger than those of individuals, and both are substantially above equilibrium. Moreover, allowing group members to punish each other leads to even larger conflict expenditures. Abbink and coauthors therefore conclude that "(t)hese results contrast with those from public goods experiments where punishment enhances efficiency."

2 The Basic Model

Two groups, labeled A and B, compete for a rent. The probability for a group to win the rent depends on the aggregate effort of agents in each group. Specifically, we assume that this probability is equal to the standard Tullock (1980)'s "linear" contest success function (CSF) for which the probability that group A wins the rent is equal to

$$p_{_A} = \frac{\sum_{i\in \mathrm{A}} a_i}{\sum_{i\in \mathrm{A}} a_i + \sum_{i\in \mathrm{B}} b_i}, \tag{1}$$

where a_i and b_i indicate an individual agent's effort in either group. To simplify notations, we will denote in the following $A = \sum_{i\in \mathrm{A}} a_i$ and $B = \sum_{i\in \mathrm{B}} b_i$ for the aggregate effort in each respective group. We also define $p_{_A} = 1/2$ when $A = B = 0$. In this case, the rent is shared (or just goes to one group randomly). We will assume throughout that the rent is enjoyed equally by all agents in the winning group.

We start our study with the simplest setup in which groups are assumed to have an equal size of n agents, and where the cost of effort and the utilities are assumed to be linear. The expected payoff of player i in group A is therefore equal to

$$u_i = p_{_A} w(n) - a_i \tag{2}$$

in which the rent is denoted by $w(n)$. This notation is to remember that the payoff of an agent when her group gets the rent may depend on the number of agents in the group. Our results do not require to be explicit about this dependence until section 5 where we consider different group sizes.

In the following sections, we will adapt the model to different cases regarding cooperation, and to more general functional forms. We stress that the main text of the paper will usually present the comparative statics results without detailed proofs. The mathematical proofs, as well as a discussion of the properties of the equilibria can be found in the Appendices.

2.1 The standard equilibrium with no cooperation

We first study the equilibrium when all agents are free-riders. This is the standard case studied in the economics literature on group rent-seeking games (Katz et al. 1990, Nitzan 1991, Esteban and Ray 2001). In that case, everyone chooses her effort taking others' decisions as given. The equilibrium is a vector $\{a_1^*, ..., a_i^*, ..., a_n^*, b_1^*, ..., b_i^*, ..., b_n^*\}$ with each element satisfying

$$a_i^* = \arg\max_{a_i} p_{_A} w(n) - a_i \tag{3}$$

and

$$b_i^* = \arg\max_{b_i}(1 - p_{_A})w(n) - b_i \tag{4}$$

where $p_{_A}$ is defined as in (1).

The objective functions are concave, so we can use first order conditions to find the equilibrium efforts. Combining the two groups' first order conditions leads to the following condition:

$$\frac{B}{(A+B)^2}w(n) = \frac{A}{(A+B)^2}w(n) = 1. \tag{5}$$

This condition reflects that the marginal benefit of effort for every agent in each group must equal the marginal cost of effort (equal to one). This condition leads to equilibrium efforts $A = B = \frac{w(n)}{4}$, implying an equal winning probability for each group, i.e., $p_{_A} = (1 - p_{_A}) = \frac{1}{2}$. When $w(n)$ is independent from n, this shows a neutrality result, first identified by Katz et al. (1990), that the total level of expenditures in the economy is independent from group sizes. The average effort per agent $\frac{w(n)}{4n}$ therefore becomes negligible for large groups, indicating a collective action problem. Moreover, the average expected payoff in this economy is

$$u^{ff} = \frac{w(n)}{2} - \frac{w(n)}{4n}, \tag{6}$$

where the superscript *ff* indicates free-riding in both groups.

2.2 The equilibrium with intragroup cooperation

Now suppose that agents in the same group fully cooperate with each other. It is like if there is an imaginary group planner in each group who assigns individual agents' efforts for the benefit of the group. More precisely, we assume that the group planner maximizes the sum of expected payoffs of agents belonging to the group. The aggregate value of the rent to a group is $nw(n)$, and the aggregate cost is simply the sum of efforts in the group. The planner of group A then chooses A to maximize

$$u_{_A} = p_{_A} \cdot nw(n) - A \tag{7}$$

and a similar problem applies to group B. The intragroup symmetric equilibrium is then characterized by:

$$A = B = \frac{nw(n)}{4}, \quad p_{_A} = (1 - p_{_A}) = 1/2;$$

$$u^{cc} = \frac{w(n)}{4}, \tag{8}$$

with the superscript cc indicating that agents are cooperative in both groups.

Comparing to the free-riding case, intragroup cooperation pushes efforts up by n folds. This is because any individual agent now takes into account the effect of her actions on all n members of her group. As a result, the marginal return of effort is n times as large as in the no cooperation case. The free-riding problem internal to each group is perfectly overcome in this case, and the neutrality result no longer holds. However, as both groups increase effort with the same intensity, each group's winning probability remains unchanged. The total rent-seeking efforts simply increase to cause a larger damage to welfare in this economy.

3 The Effect of Unilateral Cooperation

In this section, we allow groups to be different in terms of their ability to cooperate. We assume that agents in group A cooperate whereas agents in group B do not. The optimization problems now follow

$$\begin{array}{cc} \max_A p_A nw(n) - A, & \text{and} \\ \max_{b_i} (1 - p_A) w(n) - b_i & \forall i \in \mathrm{B} \end{array} \tag{9}$$

The aggregate efforts in the equilibrium are no longer symmetric between the groups:

$$\frac{A}{B} = \frac{nw(n)}{w(n)} = n. \tag{10}$$

This shows that group A's aggregate effort is n times that of group B. The winning probability also dramatically shifts in favor of the cooperative group A, as now

$$p_A = \frac{n}{n+1}; (1 - p_A) = \frac{1}{n+1}. \tag{11}$$

Consequently, when n is large, group A is almost certain to secure the rent. Notice that the proportional relationship not only extends to the winning probabilities, but also to the average expected individual payoffs. Indeed we have at the equilibrium

$$\begin{aligned} A &= \frac{n^2}{(n+1)^2} w(n), \quad & B &= \frac{n}{(n+1)^2} w(n); \\ u^{cf} &= \frac{n^2}{(n+1)^2} w(n), \quad & u^{fc} &= \frac{n}{(n+1)^2} w(n). \end{aligned} \tag{12}$$

Consistent with previous notations, the superscripts indicate the cooperative characteristic of groups, with the first letter for own group and the second for the rival. These results are compacted into the following proposition, in which we emphasize the "proportionality rule".[3]

Proposition 1. *("The proportionality rule"). In the case with unilateral cooperation, the cooperative group has a probability of winning which is n times that of the free-riding group, i.e., $p_A = n(1 - p_A)$. Moreover, the sum of efforts and the average expected payoff are also n times as high, i.e., $A = nB$ and $u^{cf} = nu^{fc}$.*

The proportionality rule therefore illustrates the power of cooperation in this game. When every agent acts as "one for all" in the group, the efforts, the probability of winning and the expected payoffs increase by as much as n times in the cooperative group compared to the free-riding group. We must add however that this strictly proportional relationship relies on the linear functional forms that are used in the basic model. In the next section, we will see how it varies when a more general CSF and other nonlinearities are introduced into the utility function.

To summarize, Figure 1 depicts the equilibria in the three cases studied above. The curves represent the best response functions of a group in aggregate efforts:[4]

$$\begin{aligned} BR_c : A &= \max\{\sqrt{nw(n)B} - B, 0\} \\ BR_f : A &= \max\{\sqrt{w(n)B} - B, 0\} \end{aligned} \tag{13}$$

The subscripts "c" and "f" indicate respectively the cooperative group and the free-riding group. The point *ff* on the Figure therefore corresponds to the symmetric equilibrium in the no cooperation case, namely when the best response functions BR_f of both groups A and B cross on the 45 degree line. Starting from this point, suppose that group A is replaced by a cooperative one. Because a cooperative group reacts everywhere more aggressively to the rival's effort (see (13)), the best response of group A now crosses that of group B at a point above *ff*. As a result, the unilateral cooperation equilibrium locates at point *cf*, with group A exerting more effort than under

[3]It can be shown that when there are more than 2 groups, the rule holds in a general form as long as only one group is cooperative. Specifically, the efforts, the probability of winning and the expected payoffs of the cooperative group are all $[(n-1)f+1]$ times those of a free-riding group, where f indicates the number of free-riding groups.

[4]Notice that $(0, 0)$ is not an equilibrium, because the best response functions are discontinuous at this point.

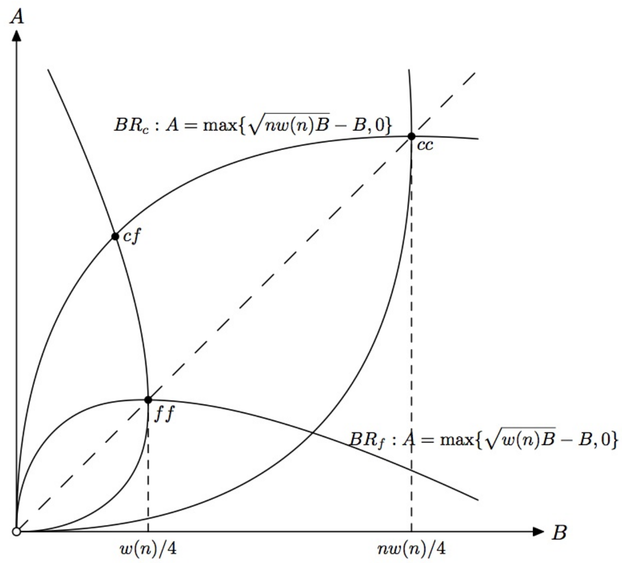

Figure 1: Best Response Functions ($n = 3$)

no cooperation. Note that, in contrast, noncooperative group B exerts less effort than at *ff*. This is because group A's effort is a strategic substitute to group B's in this region, where group B's effort is lower. Finally, when group B is also cooperative, the equilibrium is back to the 45 degree line at point *cc*. The level of efforts at this equilibrium is higher in both groups compared to previous equilibria, which illustrates a phenomenon of conflict escalation due to intragroup cooperation.

These observations suggest that intragroup cooperation destroys efficiency. Indeed, the average expected payoff under intragroup cooperation lies between that of no cooperation and that of intragroup cooperation, i.e. $u^{ff} \geq 0.5u^{cf} + 0.5u^{fc} \geq u^{cc}$, with equality when $n = 1$. These results about welfare comparison are summarized in the following proposition and are displayed in Figure 2.[5]

Proposition 2. *Intragroup cooperation has a negative impact on overall welfare: the highest average expected payoff is reached in the economy with no cooperation, then with unilateral cooperation and finally with intragroup cooperation, i.e.* $u^{ff} \geq 0.5u^{cf} + 0.5u^{fc} \geq u^{cc}$. *Nevertheless, the highest (resp. lowest) group member's expected payoff is reached under unilateral*

[5]We must remember here that rent-seeking efforts have no social value in our model, This is a key assumption for our welfare analysis.

cooperation when one's own group cooperates, i.e. u^{cf} *(resp. does not cooperate, i.e.* u^{fc}*).*

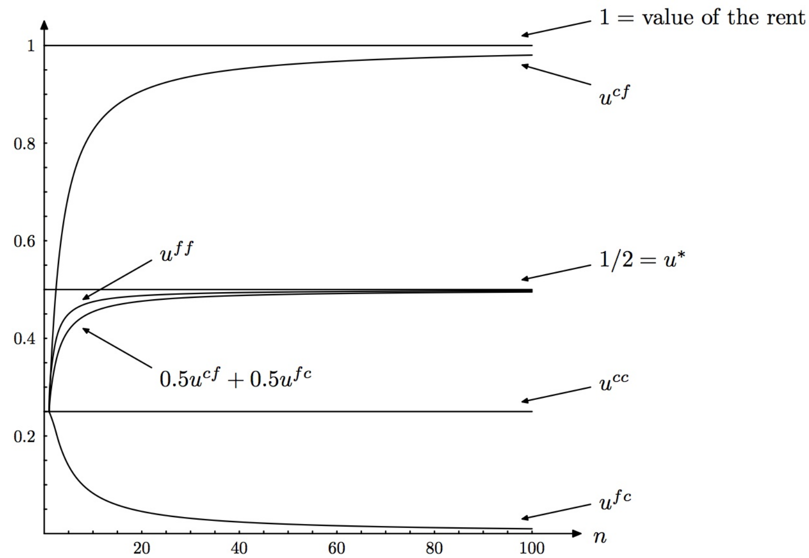

Figure 2: Average Expected Payoffs in Different Cases ($w(n) = 1$)

Finally, we discuss a more drastic form of cooperation. Since efforts have no social value, the best outcome for the society is of course the one in which groups settle down for a peaceful solution instead of fighting for the rent. All wasteful efforts will then be saved under such an intergroup cooperation, that is, $A = B = 0$. Let us assume then that the two homogeneous groups of equal size equally share the rent in a settlement (or allocate it randomly to one group). The individual (expected) payoff with a settlement is then equal to

$$u^* = \frac{w(n)}{2}. \tag{14}$$

Comparing with previous utility levels, we obviously have $u^* > u^{ff}$: it is always better to settle peace. Interestingly, we also notice that $u^{cf} > u^*$ (which holds for $n > 2$); this reveals the value of entering into conflict when the own group cooperates while the other does not.[6]

[6]This insight may be consistent with the following story. Consider a society initially composed of free-riders, where conflicts between groups can be settled in peace through negotiations. Settlement is a preferable outcome given that payoffs are higher for agents

4 General Technologies and Preferences

In this section we relax various linearity assumptions in model (9) and examine the impact of a more general CSF, cost convexity and risk aversion. Our main objective is to study whether our benchmark result, i.e. the proportionality rule (see Proposition 1), is robust to different contexts. Therefore we focus on the case of unilateral cooperation, and the main results are summarized in a proposition at the end of the section. Other results concerning the effect of nonlinearities in the no cooperation and intragroup cooperation cases are presented in the Appendices.

4.1 Logit CSF

We first introduce a more general CSF of the form

$$p_{_A} = \frac{f(A)}{f(A) + f(B)} \tag{15}$$

with $f(0) = 0$, $f'(\cdot) > 0$ and $f''(\cdot) \leq 0$. This so-called logit CSF encompasses most CSF that have been considered in the rent-seeking literature (Congleton et al. 2010).[7] Skaperdas (1996) provides an axiomatic foundation for the logit CSF. The most commonly used functional form is the specific power function $f(x) = x^r$ (Tullock 1980, Garfinkel and Skaperdas 2007). With this specification, r represents the "decisiveness" parameter, namely the efficiency of the conflict technology (Hirshleifer 1995). At the limit when $r = 0$, efforts would make no difference and the groups simply win with equal chances.

Similar as before, in the unilateral cooperation case, the group A planner maximizes $p_{_A} nw(n) - A$ over A, and agents in group B maximize $(1 -$

in either group (i.e., $u^* > u^{ff}$). Yet, suppose that cooperation emerges in one group (e.g., because of a norm or a charismatic group planner). This group then prefers conflict to settlement (because $u^{cf} > u^*$), leaving the other group with no choice but to fight. As a result, agents' expected payoffs in the group of free-riders are significantly reduced (since u^{fc} is the lowest utility level). One possibility is that the free-riding group also finds a way to become cooperative. In this case, the conflict will be further intensified, and either group then prefers the settlement again. However, at this point, the peaceful society has become one with only intragroup cooperators.

[7]In a group rent-seeking model, there could be an alternative modeling assumption: $p_{_A} = \sum_{i\in \mathrm{A}} f(a_i)/(\sum_{i\in \mathrm{A}} f(a_i) + \sum_{i\in \mathrm{B}} f(b_i))$. The CSF (15) highlights groups as conflict entities. It implies that individual agents' efforts first aggregate within the group, and then the conflict technology operates on this aggregate effort.

$p_A)w(n) - b_i$ over b_i. In the equilibrium the efforts will satisfy

$$\frac{1}{n}\frac{f(A)}{f'(A)} = \frac{f(B)}{f'(B)}. \tag{16}$$

It can be shown that the proportionality rule is perfectly preserved (i.e., $A = nB$, $p_A = n(1 - p_A)$, $u^{cf} = nu^{fc}$) if $f(\cdot)$ is homogeneous of degree one. A less strict requirement is that the function $f(\cdot)/f'(\cdot)$ is homogeneous of degree one.[8] In this case, the effort of the cooperative group A is still n times that of the free-riding group B. Yet, it is easy to show that there exist concave $f(\cdot)$ functions for which the latter property is not satisfied. The proportionality rule therefore need not be satisfied, and both $A > nB$ and $A < nB$ are possible situations depending on the form of $f(\cdot)$.

Finally, we derive closed-form solutions using the power form $f(x) = x^r$ introduced above. With intragroup symmetry assumed, we obtain

$$\begin{aligned} A &= \frac{rn^{r+1}w(n)}{(n^r+1)^2}, & B &= \frac{rn^r w(n)}{(n^r+1)^2}; \\ p_A &= \frac{n^r}{n^r+1}, & 1 - p_A &= \frac{1}{n^r+1} \\ u^{cf} &= \frac{n^r w(n)}{n^r+1} - \frac{rn^r w(n)}{(n^r+1)^2}, & u^{fc} &= \frac{w(n)}{n^r+1} - \frac{rn^{r-1}w(n)}{(n^r+1)^2}. \end{aligned} \tag{17}$$

Here $A = nB$, because the power function implies that $f(\cdot)/f'(\cdot)$ is homogenous of degree one. Nevertheless, the proportionality rule does not hold neither for the probability of winning nor for the payoff difference: indeed we find $p_A < n(1 - p_A)$ and $u^{cf} < n \cdot u^{fc}$ if $r < 1$.

4.2 Convex cost of effort

It is common in economics to assume that the marginal cost of effort (e.g. time, money or energy) is increasing. Esteban and Ray (2001) stress this point, and analyze a group rent-seeking model with a convex cost of effort. Suppose that the utility function of an agent i in group A is now

$$u_i = p_A w(n) - c(a_i) \tag{18}$$

where $p_A = \frac{A}{A+B}$, $c'(a_i) > 0$ and $c''(a_i) > 0$. The same $c(\cdot)$ applies to agents in group B.

[8] This holds when $f(\cdot)$ is homogeneous of degree k, $\forall k > 0$.

In the unilateral cooperation case, the planner of group A chooses $\{a_1, a_2, ..., a_n\}$ such that

$$\max_{\{a\}} \frac{\sum_{i=1}^{n} a_i}{\sum_{i=1}^{n} a_i + B} nw(n) - \sum_{i=1}^{n} c(a_i), \forall i \in \text{A} \tag{19}$$

As she optimally assigns equal efforts to agents (because this objective is concave in a_i), in the equilibrium efforts satisfy

$$\frac{a}{nb} = \frac{c'(b)}{c'(a)}, \tag{20}$$

or equivalently $ac'(a) = n \cdot bc'(b)$ in which a and b are the individual efforts in either group. Because $xc'(x)$ increases in x, a will be larger than b for $n > 1$. In contrast to the linear cost case (where the proportionality rule leads to $a = nb$), the difference in efforts between the cooperative and the free-riding groups is now less than n times since $c'(b)/c'(a)$ is less than 1. Cost convexity therefore dampens the difference between cooperators' and free-riders' efforts. This is an intuitive result: with convex costs, it becomes increasingly costly to invest in efforts. It is therefore no longer optimal for one cooperative agent to invest as much as all free-riding agents do in the other group.

4.3 Risk aversion

Rent-seeking is a risky activity. Indeed an individual or a group wins the rent only with some probability. In this subsection, we introduce risk aversion in our basic model. Formally, an agent's utility is represented by $u(x_i)$, where $x_i = w(n) - a_i$ if her group wins the rent, and $x_i = -a_i$ otherwise. We assume that $u(x_i)$ is thrice differentiable, increasing and concave.

It is easy to see that there is intragroup symmetry in the cooperative group A, and we again denote $a = A/n$. The equilibrium of the game is now given by

$$\begin{aligned} &\max_{a} \frac{na}{na+B} u(w(n) - a) \quad + \quad \frac{B}{na+B} u(-a), \\ &\max_{b_i} \frac{B}{A+B} u(w(n) - b_i) \quad + \quad \frac{A}{A+B} u(-b_i). \end{aligned} \tag{21}$$

We also assume intragroup symmetry for the free-riding group B, and denote

$b = B/n$. The following first order conditions are satisfied at the equilibrium:

$$\frac{b}{(a+b)^2}\Big[u(w(n)-a)-u(-a)\Big]-\Big[\frac{a}{a+b}u'(w(n)-a)+\frac{b}{a+b}u'(-a)\Big]=0$$
$$\frac{a}{n(a+b)^2}\Big[u(w(n)-b)-u(-b)\Big]-\Big[\frac{b}{a+b}u'(w(n)-b)+\frac{a}{a+b}u'(-b)\Big]=0 \quad (22)$$

Assuming a constant absolute risk aversion (CARA) utility function, namely $u(x) = -e^{-x}$, we can show that a solution of the system (22) implies $a > nb$, for $a > 0$ and $b > 0$. Comparing to the equilibrium under risk neutrality, such a result indicates that risk aversion can amplify the proportionality rule.

We finally condense some results obtained in this section in the following proposition.

Proposition 3. *In the case with unilateral cooperation and under the common power logit CSF, the aggregate equilibrium effort of the cooperative group is equal to n times that of the free-riding group, i.e. $A = nB$. However, we have $A < nB$ under convex cost and may have $A > nB$ under risk aversion.*

5 The Group Size Paradox

Olson (1965) argues that larger groups are less effective to obtain rents than smaller groups. This is often referred to as the group size paradox. It has been shown that this result may hold or not depending on the degree of publicness of the rent (Chamberlin 1974), and on the nonlinearity of the effort cost function (Esteban and Ray 2001, Nitzan and Ueda 2009, 2011) In this section, we will relax our previous assumption that the groups are of equal size, and examine under which conditions the group size paradox holds in our basic model.

Our results in this section will depend on the relationship between the rent and the size of group, that is, on the function $w(\cdot)$. We thus need to be more specific about this function. Following Esteban and Ray (2001), we assume that the rent can have both a pure and an impure public goods component. Namely, for each agent in a winning group of size n, $n \in \mathbb{Z}^+$, the value from winning the rent is equal to

$$w(n) = \lambda P + (1-\lambda)\frac{R}{n}. \quad (23)$$

This indicates that a proportion λ of the rent is both non-excludable and non-rivalrous. This pure public goods component has a value of P for every agent. The remaining private (or impure public) component R has to be divided equally among all the agents in the winning group. We will refer to $\lambda \in [0, 1]$ as the degree of publicness of the rent.

5.1 The equilibrium under no cooperation and under intragroup cooperation

Suppose that there are m agents in group A and n agents in group B. Without loss of generality, we assume in this subsection $m < n$. The value of the rent to an individual agent and to a group are respectively

$$\begin{aligned} w(m) &= \lambda P + (1-\lambda)R/m, \quad mw(m) = \lambda mP + (1-\lambda)R; \\ w(n) &= \lambda P + (1-\lambda)R/n, \quad\;\; nw(n) = \lambda nP + (1-\lambda)R. \end{aligned} \tag{24}$$

Observe that, if $\lambda \in (0, 1)$, then $w(m) > w(n)$ but $mw(m) < nw(n)$. That is, the rent brings more individual value in a smaller group, and more "group value" in a larger group. Moreover, notice that $w(m) - w(n)$ is decreasing in λ whilst $nw(n) - mw(m)$ is increasing in λ, with $w(m) = w(n)$ when $\lambda = 1$ and $mw(m) = nw(n)$ when $\lambda = 0$.

We first look at groups with free-riders, for which the equilibrium is

$$\frac{A}{B} = \frac{w(m)}{w(n)}. \tag{25}$$

This ratio decreases from n/m to 1 as λ increases from 0 to 1. The smaller group hence wins the rent with a higher probability than the larger group, though this advantage gradually diminishes as the rent becomes more public. A smaller group is then advantaged, which is consistent with the group size paradox.[9]

Next we consider the case of intragroup cooperation. The result goes in the opposite direction: the smaller group wins with a lower probability, for in the equilibrium

$$\frac{A}{B} = \frac{mw(m)}{nw(n)} < 1 \Rightarrow p_A = \frac{mw(m)}{mw(m) + nw(n)} < 1/2 \tag{26}$$

[9] Esteban and Ray (2001) obtain a different result because they consider a convex cost of effort.

The intuition is simply that the rent has more value for a larger group, which is what matters for a group planner.

We now study the effect of the group size on individual expected payoffs under no cooperation and under intragroup cooperation. To see this, observe first that the expected payoff ratios can be simplified to

$$\frac{u_a^{cc}}{u_b^{cc}} = \frac{m^2 w(m)^3}{n^2 w(n)^3}; \quad \frac{u_a^{ff}}{u_b^{ff}} = \frac{w(m)^2}{w(n)^2} \frac{mn[w(m) + w(n)] - nw(n)}{mn[w(m) + w(n)] - mw(m)}, \tag{27}$$

using obvious notations. As $w(m)$ and $w(n)$ are both functions of λ, we find that there exist a pair of $\lambda^{cc} \in (0,1)$ and $\lambda^{ff} \in (0,1)$, such that for $m < n$:

$$\begin{aligned} u_a^{cc} > u_b^{cc} &\iff \lambda < \lambda^{cc} \\ u_a^{ff} > u_b^{ff} &\iff \lambda < \lambda^{ff} \end{aligned} \tag{28}$$

This shows that if the publicness of the rent is low enough, agents reach a higher expected payoff in the smaller group, whether there is free-riding or intragroup cooperation. A small λ means that the rent value is largely divided among group members, which indeed favors the smaller group.[10] We summarize these results into the following proposition.

Proposition 4. *In the case with no (resp. with intragroup) cooperation, the smaller group has a higher (resp. lower) probability of winning. This result holds no matter the degree of publicness of the rent λ. However, in each case, agents in the smaller group are always better-off than the ones in the larger group when λ is low enough.*

5.2 The equilibrium under unilateral cooperation

In this subsection, we assume that agents cooperate in group A and free-ride in group B. The equilibrium features a similar condition to (10):

$$\frac{A}{B} = \frac{mw(m)}{w(n)}, \tag{29}$$

indicating that the cooperative group wins the rent with a higher probability. This holds no matter the respective size of groups. Note that (29) implies a "strength in numbers" result for the cooperative group: its probability of winning increases both in its own's as well as in its rival's sizes. Moreover,

[10] Numerical examples show that λ^{cc} is rather small, usually around 0.14; whereas λ^{ff} lies relatively large between 0.75 and 0.8.

observe that the aggregate effort of the cooperative group is greater than m times that of the free-riding group if and only if $w(m) > w(n)$, or equivalently when there are less members in the cooperative group than in the free-riding group. The equilibrium efforts and individual payoffs are given by

$$\begin{aligned} A &= \frac{m^2 w(m)^2 w(n)}{[mw(m)+w(n)]^2}, \quad u_a^{cf} = \frac{mw(m)^2}{mw(m)+w(n)} - \frac{mw(m)^2 w(n)}{[mw(m)+w(n)]^2}, \\ B &= \frac{mw(m)w(n)^2}{[mw(m)+w(n)]^2}, \quad u_b^{fc} = \frac{w(n)^2}{mw(m)+w(n)} - \frac{mw(m)w(n)^2}{n[mw(m)+w(n)]^2}. \end{aligned} \tag{30}$$

We have seen in the previous subsection that the degree of publicness of the rent, λ, is instrumental for comparing expected individual payoffs across groups of different sizes. This is still the case here when $m \geq n^2$. However, when $1 < m < n^2$, we always have $u_a^{cf} > u_b^{fc}$ for all λ. This means that unless the cooperative group is "too" big, its agents are always better-off than those in the noncooperative group no matter the degree of publicness of the rent. These results are summarized in the following proposition.

Proposition 5. *In the case with unilateral cooperation, the cooperative group has always a higher probability of winning than the free-riding group no matter the respective size of each group, and no matter the degree of publicness of the rent λ. Moreover, the agents' expected payoffs are also higher in the cooperative group, i.e., $u_a^{cf} > u_b^{fc}$, when $1 < m < n^2$, or $m \geq n^2$ with λ large enough.*

6 Intragroup Altruism

In this section, we assume intragroup (pure) altruism, namely that every agent cares about the expected payoff of the other agents in the same group. As a result, cooperation is no more exogenous, but will be the consequence of this assumption over preferences.

For analytical simplicity, we suppose members of the same group share the same degree of altruism. Denoting it by $\alpha_A \in [0,1]$ for group A and $\alpha_B \in [0,1]$ for group B, the utility functions of agent i in group A and in group B are respectively:

$$u_i = p_A w(m) - a_i + \alpha_A \sum_{j \in \mathrm{A}, j \neq i} [p_A w(m) - a_j], i \in \mathrm{A} \tag{31}$$

$$u_i = (1-p_A) w(n) - b_i + \alpha_B \sum_{j \in \mathrm{B}, j \neq i} [(1-p_A) w(n) - b_j], i \in \mathrm{B}$$

where m and n again indicate respective group sizes. By varying the altruism parameters, we notice that (31) are identical to the objective functions in the no cooperation case when $\alpha_A = \alpha_B = 0$, and to those in the intragroup cooperation case when $\alpha_A = \alpha_B = 1$. One can interpret other combinations of α_A and α_B as intermediate cases between full free-riding and full intragroup cooperation.

In the same way as $w(m)$ and $w(n)$ indicate the value of the rent to a selfish agent in either group, we can regard $[1 + \alpha_A(m-1)]w(m)$ and $[1 + \alpha_B(n-1)]w(n)$ as the "perceived" value of the rent to an altruistic agent in the respective group.[11] Indeed compare the following reorganized utility function of group A with (2):

$$u_i = p_A[1 + \alpha_A(m-1)]w(m) - a_i - \alpha_A \sum_{j \in \mathrm{A}, j \neq i} a_j, i \in \mathrm{A} \tag{32}$$

It is immediate then that the first order conditions of (31) lead to the following condition at the equilibrium:

$$\frac{A}{B} = \frac{[1 + \alpha_A(m-1)]w(m)}{[1 + \alpha_B(n-1)]w(n)}. \tag{33}$$

This condition shows that equilibrium efforts are affected by the sizes of the groups as well as by the degrees of altruism. To investigate the specific impact of either factor, in the following we consider two cases in turn: (i) $m = n$, and (ii) $\alpha_m = \alpha_n$. More details on the general case are provided in the Appendices.

(i) When $m = n$, (33) shows that the more altruistic group collects more effort and wins with a higher probability. This is simply because the "perceived" value of the rent is higher for the more altruistic group. Furthermore, it can be shown that a group's effort always increases in its own degree of altruism but it increases (resp. decreases) in the rival's degree of altruism when the rival is relatively less (resp. more) altruistic. Moreover, the aggregate effort $A + B$ always increases in the degree of altruism. These results are summarized in the following proposition.

Proposition 6. *In model (31) with identical group sizes, i.e. $m = n$, a unilateral increase in altruism in one group increases efforts in this group, and decreases (resp. increases) efforts in the rival group if the rival group has a lower (resp. higher) degree of altruism. Moreover, the aggregate efforts exerted by both groups also increase with a unilateral increase of altruism.*

[11] Alger (2010) uses a similar interpretation in a public goods game in which agents are altruistic.

(ii) Now let $\alpha_A = \alpha_B \equiv \alpha$, and suppose $m < n$ without loss of generality. We know that $\alpha = 0$ is the noncooperative case where the smaller group A exerts more effort and that $\alpha = 1$ is the cooperative case where the larger group B exerts more effort. For all intermediate values of $\alpha \in (0, 1)$, we use (33) to obtain

$$p_A = \frac{w(m)}{w(m) + \frac{1+\alpha(n-1)}{1+\alpha(m-1)} w(n)}. \tag{34}$$

It can be shown that p_A is decreasing in α given $m < n$. Combined with observations above concerning extreme values of α, there must be a critical value α^* such that the smaller group exerts more effort than the larger one if and only if $\alpha < \alpha^*$. As both groups become more altruistic, the advantage tilts from the smaller group to the larger group. This is another illustration of a "strength in numbers" result that is induced by intragroup cooperation. Nevertheless, as before, agents in the smaller group can reach higher expected payoff than those in the larger group when the degree of publicness of the rent is low enough. Finally, the effect of altruism on aggregate efforts is positive. Some of these results are summarized in the following proposition.

Proposition 7. *In model (31) with identical degree of altruism, i.e., $\alpha_A = \alpha_B$, the smaller group exerts more effort than the larger one if and only if the degree of altruism is low enough. Moreover, the aggregate efforts exerted by both groups increase with altruism.*

7 Conclusion

Whether at the workplace, the club, the corporation or the polity, people belong to groups. In some situations, groups are in conflict with each other. Moreover, group members may behave in a more or less cooperative way towards other members, depending for instance on the homogeneity of the group, or on the group's norms, institutions or leaders' characteristics. The main objective of this paper was to study the consequences (and not the causes) of the degree of internal cooperativeness of a group when this group is in conflict with another group. In doing so, we have formalized the idea, extensively documented in other disciplines, of "cooperate and conquer".

More specifically, we have used a basic group rent-seeking model in which two groups compete for a rent. In this model, agents exerting effort induce both a positive externality for their group members and a negative externality for the rival group members. As a simplifying assumption, we have considered an exogenous and ultimate form of intragroup cooperation, namely that group members maximize their group's payoff. We have especially focused on the case of unilateral cooperation, in which members free-ride in one group and cooperate in the other group. In this case, we have put forward a benchmark result: the proportionality rule. This rule states that the probability of winning, the aggregate efforts and the expected payoffs of the members in the cooperative group are exactly n times as high as those in the noncooperative group, where n is the number of members in each group. Therefore if the groups are sufficiently large, this simple rule implies that the cooperative group is almost sure to win the rent.

Moreover, we have shown that, regardless of relative group sizes, a cooperative group always has a higher winning probability than a noncooperative group. This last result offers a new perspective on the celebrated Olson (1965)'s group size paradox. This paradox suggests that a smaller group has a higher probability of winning. Instead, our result suggests that a larger group may have a higher probability of winning provided its members are able to cooperate more than the members of the smaller group. Obviously, this result should be interpreted with caution as the degree of cooperativeness within a group may depend on the number of people in this group. Another point that we have overlooked in this paper is that the value of the rent may differ depending on whether the group cooperates or not. Consider for instance the case of a common property resource with free access, as in Nitzan and Ueda (2009). In that case, cooperation would increase the value of the rent and this would generate an additional incentive to exert effort for the cooperative members.

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[Not for Publication]

Appendices

The following appendices contain computational details that are not included in the main text. They also contain some additional results. In order to facilitate reading, we present separately the computations for each of the three cases, namely first the case of no cooperation, then the case of intragroup cooperation, and finally the case of unilateral cooperation. A last section then presents the case of intragroup altruism.

A The No Cooperation Case

A.1 The basic model

With the linear CSF, linear cost of effort, risk neutrality and equal group sizes ($= n$), individual agents choose efforts to maximize

$$p_A w(n) - a_i, \text{ or } (1 - p_A) w(n) - b_i, i \in \text{A or B} \tag{A.1}$$

where $p_A = \frac{a_i + \sum_{j \neq i} a_j}{a_i + \sum_{j \neq i} a_j + \sum_i b_i} \equiv \frac{A}{A+B}$. Since objective functions are concave, the first order conditions

$$\frac{B}{(A+B)^2} w(n) - 1 = 0, \text{ and } \frac{A}{(A+B)^2} w(n) - 1 = 0 \tag{A.2}$$

indicate that the optimal aggregate efforts are

$$A = B = \frac{w(n)}{4}. \tag{A.3}$$

So the groups have equal winning probabilities. By assuming intragroup symmetry, we have $a_i = b_i = A/n = w(n)/4n$, and the expected individual payoffs: $u^{ff} = \frac{w(n)}{2} - \frac{w(n)}{4n}$.

A.2 General CSF

Now consider $p_A = \frac{f(A)}{f(A)+f(B)}$, with $f(0) = 0$, $f'(\cdot) > 0$, and $f''(\cdot) \leq 0$. The objective functions in (A.1) remain concave under these conditions. We can still use the first order conditions to find the equilibrium.

The first order conditions are

$$\begin{cases} \frac{f'(A)f(B)}{[f(A)+f(B)]^2} w(n) - 1 = 0 \\ \frac{f(A)f'(B)}{[f(A)+f(B)]^2} w(n) - 1 = 0 \end{cases} \tag{A.4}$$

Notice that they are identical to the first order conditions of a two-agent rent-seeking game, where A and B represent respective individual effort. The aggregate efforts in the current group game therefore have the same properties as the equilibrium efforts in that individual game. As previous studies (e.g., Szidarovszky and Okuguchi 1997, Cornes and Hartley 2005) have shown that there exists a unique equilibrium in an individual rent-seeking game with such a CSF, the same is true for the aggregate efforts in the current group game.

(A.4) implies that $f(A)/f'(A) = f(B)/f'(B)$ in the equilibrium. Because $f(\cdot)/f'(\cdot)$ is an increasing function under our assumptions about $f(\cdot)$, the equilibrium must be symmetric, i.e., $A = B$. We solve (A.4) to have

$$\frac{f(A)}{f'(A)} = \frac{f(B)}{f'(B)} = \frac{w(n)}{4}. \tag{A.5}$$

We can now compare the equilibrium efforts with those under the linear technology (i.e., $A = B = w(n)/4$). For $f(\cdot)/f'(\cdot)$ being increasing, the aggregate efforts that satisfy (A.5) are lower than $w(n)/4$ if and only if

$$\frac{f(w(n)/4)}{f'(w(n)/4)} \geq \frac{w(n)}{4}, \text{ or } \frac{f(w(n)/4)}{w(n)/4} \geq f'(w(n)/4). \tag{A.6}$$

In words, it requires that the slope of the tangent to $f(\cdot)$ at $w(n)/4$ is no larger than the slope of the line connecting 0 and this point. This is always true given our assumptions about $f(\cdot)$, because for all $A \geq 0$:

$$\begin{aligned} \frac{f(A)}{f'(A)} &= \frac{1}{f'(A)}(f(0) + \int_0^A f'(x)dx) \\ &= \frac{1}{f'(A)}([f'(x)x]_0^A - \int_0^A f''(x)xdx) \\ &\geq A, \text{ for } f(0) = 0 \text{ and } f''(\cdot) \leq 0, \end{aligned} \tag{A.7}$$

with an equality when $f''(\cdot) = 0$. We therefore state the following:

Result 1. *In the no cooperation case, the equilibrium efforts under CSF (15) are lower than those under the linear CSF.*

A.3 Convex cost of effort

Now we keep the linear CSF but consider a convex cost of effort $c(\cdot)$, with $c'(\cdot) > 0$ and $c''(\cdot) > 0$. According to Esteban and Ray (2001), there exists a unique equilibrium in this case. This equilibrium also contains equal efforts within either group, because for agents in the same group, e.g., group A, efforts are all determined by the same first order condition:

$$\frac{B}{(A+B)^2} w(n) - c'(a_i) = 0, \forall i \in \text{A}. \tag{A.8}$$

Denoting $a_i = a = A/n$ and $b_i = b = B/n$, we use (A.8) and the corresponding condition of group B to find the following in the equilibrium:

$$\frac{A}{B} = \frac{c'(b)}{c'(a)}, \text{ or equivalently } ac'(a) = bc'(b). \tag{A.9}$$

The function $xc'(x)$ is increasing in x given that $c(\cdot)$ is convex. (A.9) then implies that $a = b$ in the equilibrium. Bringing $A = B = na$ back to (A.8), we have

$$a \cdot c'(a) = b \cdot c'(b) = \frac{w(n)}{4n}. \tag{A.10}$$

Also because $xc'(x)$ is an increasing function, the equilibrium efforts defined by (A.10) are lower than those in the basic model (where $a = b = w(n)/4n$) if and only if

$$w(n)/4n \cdot c'(w(n)/4n) > w(n)/4n, \text{ or } c'(w(n)/4n) > 1. \tag{A.11}$$

Recall that at $w(n)/4n$ the marginal return of effort equals the unity marginal cost in the basic model. (A.11) simply states that an agent will choose a lower effort under cost convexity when the effort level of $w(n)/4n$ is associated with a marginal cost above one.

We notice that the impact of cost convexity may depend on the size of groups. Especially, if $w(n)$ is nonincreasing in n, $c'(w(n)/4n)$ will be a decreasing function of n. It is therefore possible that, although cost convexity reduces the effort of a smaller group, everything else equal, it increases the effort of a larger group.

We summarize these results below:

Result 2. *In the no cooperation case, the equilibrium efforts under a convex cost function are lower (resp. higher) than those under a linear cost function if and only if $c'(w(n)/4n) > 1$ (resp. $c'(w(n)/4n) < 1$).*

A.4 Risk aversion

When the agents are risk averse, they solve

$$\begin{aligned}\max_{a_i} \frac{A}{A+B}u(w(n)-a_i) \quad &+ \quad \frac{B}{A+B}u(-a_i),\\ \max_{b_i} \frac{B}{A+B}u(w(n)-b_i) \quad &+ \quad \frac{A}{A+B}u(-b_i). \end{aligned} \tag{A.12}$$

Assuming that $u(x_i)$ is increasing, concave and thrice differentiable, Cornes and Hartley (2009) prove that an equilibrium exists in an individual rent-seeking game, and that it is unique if all agents exert positive efforts. Their proof of existence applies to our group game when we assume intragroup symmetry, i.e., $a_i = a = A/n$ and $b_i = b = B/n$. Here the equilibrium is also unique, because neither group exerts zero effort.

Treich (2010) shows that risk aversion reduces efforts in the individual rent-seeking game if and only if agents are prudent. In the following we prove our Result 3 in a similar fashion, which says that the same is true for the group game, and hence generalize Katz et al. (1990)'s conclusion based on exponential utility functions.

Result 3. *In the no cooperation case, the equilibrium efforts under risk aversion are lower than those under risk neutrality if and only if agents are prudent, i.e.,* $u''' \geq 0$.

Proof. Denote the (symmetric) equilibrium condition under risk aversion and under risk neutrality by F^{ff} and f^{ff}, respectively:

$$\begin{aligned} F^{ff}(a) &= \frac{1}{4na}[u(w(n)-a) - u(-a)] - \frac{1}{2}[u'(w(n)-a) + u'(-a)] = 0\\ f^{ff}(a) &= \frac{1}{4na}w(n) - 1 = 0 \end{aligned} \tag{A.13}$$

$f^{ff}(a)$ is clearly decreasing in a. The equilibrium efforts under risk aversion (denoted by x) are therefore lower than those under risk neutrality if and only if $f^{ff}(x) \geq 0$, or $\frac{1}{4nx} \geq \frac{1}{w(n)}$. Since $F^{ff}(x) = 0$, this inequality is equivalent to

$$\frac{1}{2}[u'(w(n)-x) + u'(-x)]w(n) - [u(w(n)-x) - u(-x)] \geq 0 \tag{A.14}$$

We then apply Eeckhoudt and Gollier (2005)'s Lemma 1 to show that (A.14) is satisfied if and only if u' is convex, i.e., when agents are prudent. Specifically, in (A.14) the second part on the left hand side satisfies the following:

$$
\begin{aligned}
u(w(n)-x)-u(-x) &= \int_{-x}^{w(n)-x} u'(a)da \qquad \text{(A.15)}\\
\leq \quad & \int_{-x}^{w(n)-x} \left[\frac{a+x}{w(n)}u'(w(n)-x)+\frac{w(n)-x-a}{w(n)}u'(-x)\right] da
\end{aligned}
$$

if and only if u' is convex. The last inequality is obtained using the definition of a convex curve, by realizing that $a = \frac{a+x}{w(n)}(w(n)-x)+\frac{w(n)-x-a}{w(n)}(-x)$. Solving the integral yields

$$u(w(n)-x)-u(-x) \leq \frac{1}{2}[u'(w(n)-x)+u'(-x)]w(n),$$

which can be rearranged into (A.14). □ □

A.5 Group size paradox

We now elaborate Proposition 4's claims that correspond to asymmetric group sizes under no cooperation. We suppose that group A has m members and group B has n members, with $m < n$.

The first order conditions

$$\frac{B}{(A+B)^2}w(m)-1=0, \frac{A}{(A+B)^2}w(n)-1=0 \qquad \text{(A.16)}$$

imply that in the equilibrium $A/B = w(m)/w(n)$ and $p_A = \frac{w(m)}{w(m)+w(n)}$. Because the value of the rent is higher for agents in the smaller group (i.e., $\lambda P+(1-\lambda)R/m \geq \lambda P+(1-\lambda)R/n$ for $m < n$), we have $A \geq B$ and $p_A \geq 1/2$, with equality when $\lambda = 1$. The smaller group exerts a higher effort and has a higher probability to win.

We solve (A.16) for the aggregate efforts, and then assume intragroup symmetry to have the expected payoffs as following:

$$
\begin{aligned}
A &= \frac{w(m)^2 w(n)}{[w(m)+w(n)]^2}, \qquad B = \frac{w(n)^2 w(m)}{[w(m)+w(n)]^2}\\
u_a^{ff} &= \frac{w(m)^2}{w(m)+w(n)} - \frac{w(m)^2 w(n)}{m[w(m)+w(n)]^2},\\
u_b^{ff} &= \frac{w(n)^2}{w(m)+w(n)} - \frac{w(n)^2 w(m)}{n[w(m)+w(n)]^2}. \qquad \text{(A.17)}
\end{aligned}
$$

A direct comparison reveals that $u_a^{ff} > u_b^{ff}$ is equivalent to

$$w(m)^3 - w(n)^3 + \frac{m-1}{m} w(m)^2 w(n) - \frac{n-1}{n} w(n)^2 w(m) > 0. \tag{A.18}$$

We define the left hand side of (A.18) as $Q^{ff}(\lambda)$, and notice that it is a continuous and cubic function. We also identify that only one out of the three solutions of $Q^{ff}(\lambda) = 0$ is real. This means that the sign of $Q^{ff}(\lambda)$ changes at most once when λ increases from 0 to 1. So we check the two boundary values of $Q^{ff}(\lambda)$ on $[0, 1]$:

$$\begin{aligned} Q^{ff}(\lambda = 0) &= \frac{(n-m)R^3}{m^3 n^3}(n+m)(n+m-1) > 0, \\ Q^{ff}(\lambda = 1) &= P^3 \left(\frac{m-1}{m} - \frac{n-1}{n} \right) < 0. \end{aligned}$$

Hence we claim that there must exist a $\lambda^{ff} \in (0, 1)$, such that $u_a^{ff} > u_b^{ff}$ if and only if $\lambda < \lambda^{ff}$.

B The Intragroup Cooperation Case

We assume now there is intragroup cooperation in both groups. As we have shown in the text, this case amounts to study a two-agent rent-seeking game. The existence and uniqueness of equilibrium have thus been proved in the literature.

B.1 The basic model

In the linear basic model, planners choose A and B respectively to maximize either group's aggregate payoff. Specifically, they solve

$$\max_A p_{_A} \cdot nw(n) - A, \text{ and } \max_B (1 - p_{_A}) \cdot nw(n) - B. \tag{B.1}$$

The first order conditions are:

$$\frac{B}{(A+B)^2} nw(n) - 1 = 0, \text{ and } \frac{A}{(A+B)^2} nw(n) - 1 = 0. \tag{B.2}$$

Comparing to (A.2), the marginal return of effort is now n times as large. Solving (B.2) returns $A = B = nw(n)/4$, $p_{_A} = 1/2$, and $a_i = b_i = A/n = w(n)/4$ with intragroup symmetry being assumed. Agents in different groups have the same expected payoff, indicated by $u^{cc} = \frac{1}{2}w(n) - \frac{w(n)}{4} = \frac{w(n)}{4}$.

B.2 General CSF

With the general CSF, the problem has the following first order conditions:

$$\begin{cases} \frac{f'(A)f(B)}{[f(A)+f(B)]^2} nw(n) - 1 = 0 \\ \frac{f'(B)f(A)}{[f(A)+f(B)]^2} nw(n) - 1 = 0 \end{cases} \tag{B.3}$$

which yield in the equilibrium

$$\frac{f(A)}{f'(A)} = \frac{f(B)}{f'(B)} = \frac{nw(n)}{4}. \tag{B.4}$$

Like in section A.2, the equilibrium efforts are lower than those in the basic model if and only if

$$\frac{f(nw(n)/4)}{f'(nw(n)/4)} \geq \frac{nw(n)}{4}. \tag{B.5}$$

Again this is always true under our assumptions about $f(\cdot)$. So we claim:

Result 4. *In the intragroup cooperation case, the equilibrium efforts under CSF (15) are lower than those under the linear CSF.*

B.3 Convex cost of effort

When individual agent's cost of effort follows the convex function $c(\cdot)$, the planner of group A (symmetrically for the planner of group B) chooses $\{a_1, a_2, ...a_i, ...a_n\}$ to achieve

$$\max_{\{a\}} \frac{\sum_{i=1}^n a_i}{\sum_{i=1}^n a_i + B} nw(n) - \sum_{i=1}^n c(a_i), \forall i \in \text{A}. \tag{B.6}$$

By Jensen's inequality, there is $\frac{1}{n}\sum_{i=1}^n c(a_i) \geq c(\frac{\sum_{i=1}^n a_i}{n})$, meaning for any level of aggregate effort, it is most economic to assign equal efforts within a group. So we replace a_i by $a \equiv A/n$ and b_i by $b \equiv B/n$ and reduce the problem to that of a two-agent game:

$$\begin{aligned} &\max_a \frac{a}{a+b} w(n) - c(a) \\ &\max_b \frac{b}{a+b} w(n) - c(b). \end{aligned} \tag{B.7}$$

Like in section A.3, equilibrium efforts are equal in both groups, with

$$a \cdot c'(a) = b \cdot c'(b) = \frac{w(n)}{4}. \tag{B.8}$$

The impact of cost convexity is then captured following the same argumentation in section A.3, and is stated in the following Result:

Result 5. *In the intragroup cooperation case, the equilibrium efforts under a convex cost function are lower (resp. higher) than those under a linear cost function if and only if $c'(w(n)/4) > 1$ (resp. $c'(w(n)/4) < 1$).*

If we compare Result 5 with Result 2, we would notice that $c'(w(n)/4) > c'(w(n)/4n)$ for any $n > 1$. Therefore everything else equal, cost convexity is more likely to reduce efforts in the presence of ingragroup cooperation.

B.4 Risk aversion

When agents are risk averse, a planner assigns equal efforts within the group. This is optimal because by Jensen's inequality, a concave utility function $u(.)$ implies $\sum_{i=1}^{n} u(w(n) - a_i) \leq nu(\frac{nw(n)-A}{n})$ and $\sum_{i=1}^{n} u(-a_i) \leq nu(\frac{-A}{n})$. So following previous notations, the optimization problems are

$$\begin{aligned} \max_a \frac{na}{na+nb} u(w(n) - a) \quad &+ \quad \frac{nb}{na+nb} u(-a) \\ \max_b \frac{nb}{na+nb} u(w(n) - b) \quad &+ \quad \frac{na}{na+nb} u(-b) \end{aligned} \tag{B.9}$$

with the following symmetric equilibrium condition:

$$F^{cc}(a) = \frac{1}{4a}\Big[u(w(n)-a) - u(-a)\Big] - \frac{1}{2}\Big[u'(w(n)-a) + u'(-a)\Big] = 0. \tag{B.10}$$

We now compare the equilibrium effort with that under risk neutrality. Noticing that $F^{cc}(a)$ has a similar structure as $F^{ff}(a)$ in (A.13), one can develop a similar analysis as in the no cooperation case. We thus directly present Result 6 below without repeating the proof.

Result 6. *In the intragroup cooperation case, the equilibrium efforts under risk aversion are lower than those under risk neutrality if and only if agents are prudent, i.e., $u''' \geq 0$.*

One shall also notice that $F^{cc}(a) > F^{ff}(a)$ for all a and $n > 1$, so that $F^{cc}(x) > 0$ for any x that satisfies $F^{ff}(x) = 0$. Because $F^{cc}(a)$ can be found as decreasing in a, risk averse agents' efforts under intragroup cooperation must be higher than those in the no cooperation case.

B.5 Group size paradox

When groups' sizes differ, we find in the equilibrium $\frac{A}{B} = \frac{mw(m)}{nw(n)}$, and

$$A = \frac{m^2 nw(m)^2 w(n)}{[mw(m) + nw(n)]^2}, \quad B = \frac{mn^2 w(m)w(n)^2}{[mw(m) + nw(n)]^2};$$
$$u_a^{cc} = \frac{m^2 w(m)^3}{[mw(m) + nw(n)]^2}, \quad u_b^{cc} = \frac{n^2 w(n)^3}{[mw(m) + nw(n)]^2}.$$

For $m < n$, there is $mw(m) \leq nw(n)$ given $\lambda \geq 0$, so $A \leq B$. The larger group hence has a higher probability to win the rent. Comparing the two expected payoffs reveals that $u_a^{cc} > u_b^{cc}$ is equivalent to

$$Q^{cc}(\lambda) \equiv m^2 w(m)^3 - n^2 w(n)^3 > 0.$$

Like in section A.5, $Q^{cc}(\lambda)$ is a continuous cubic function of λ with which we find only one real solution for $Q^{cc}(\lambda) = 0$. For $\lambda \in [0, 1]$, its boundary values

$$Q^{cc}(\lambda = 0) = \frac{(n - m)R^3}{mn} > 0, \text{ and } Q^{cc}(\lambda = 1) = (m^2 - n^2)P^3 < 0$$

imply that there exists a unique threshold $\lambda^{cc} \in (0, 1)$, such that $u_a^{cc} > u_b^{cc}$ if and only if $\lambda < \lambda^{cc}$.

The above findings are also included in Proposition 4.

C The Unilateral Cooperation Case

In this case, we suppose that there is cooperation in group A and free-riding in group B. It is a mixture of the two previous cases, and we will assume that the existence and uniqueness properties carry over to this case.

C.1 The basic model

For the basic model

$$\begin{cases} \max_A p_A nw(n) - A \\ \max_{b_i} (1 - p_A)w(n) - b_i, \forall i \in \text{B} \end{cases}$$

the first order conditions are the same as those in equations (B.2) and (A.2) for the respective group. The equilibrium efforts will then satisfy

$\frac{A}{B} = \frac{nw(n)}{w(n)} = n$, and in turn $p_A = \frac{n}{n+1} = n(1-p_A)$. By further assuming intragroup symmetry in both groups, we derive the following equilibrium efforts and expected payoffs that support the "proportionality rule" in Proposition 1:

$$\begin{aligned} A &= \frac{n^2}{(n+1)^2}w(n), \qquad B = \frac{n}{(n+1)^2}w(n); \\ u^{cf} &= \frac{n}{n+1}w(n) - \frac{n}{(n+1)^2}w(n) = \frac{n^2}{(n+1)^2}w(n), \\ u^{fc} &= \frac{1}{n+1}w(n) - \frac{1}{(n+1)^2}w(n) = \frac{n}{(n+1)^2}w(n) \end{aligned}$$

Now we have u^{ff}, u^{cc}, u^{cf} and u^{fc}. The claim of Proposition 2 is obtained through a direct comparison of these expected payoffs. Besides, with $u^* = \frac{w(n)}{2}$ representing the expected payoff under settlement, $u^{cf} > u^*$ is satisfied when $n > 1 + \sqrt{2}$.

C.2 General CSF

When $p_A = \frac{f(A)}{f(A)+f(B)}$, we solve

$$\begin{cases} \max_A \frac{f(A)}{f(A)+f(B)} nw(n) - A \\ \max_{b_i} \frac{f(B)}{f(A)+f(B)} w(n) - b_i \end{cases} \tag{C.1}$$

The first order conditions of this problem are

$$\begin{cases} \frac{f'(A)f(b)}{[f(A)+f(B)]^2} nw(n) - 1 = 0 \\ \frac{f(A)f'(B)}{[f(A)+f(B)]^2} w(n) - 1 = 0 \end{cases} \tag{C.2}$$

which lead to the following equilibrium condition:

$$\frac{1}{n}\frac{f(A)}{f'(A)} = \frac{f(B)}{f'(B)} \tag{C.3}$$

Suppose $f(\cdot)$ is homogeneous of degree one. Differentiating both sides of $f(nB) = nf(B)$ with regard to B we get $nf'(nB) = nf'(B)$, which indicates that $f'(\cdot)$ is homogeneous of degree zero. It then follows that $f(\cdot)/f'(\cdot)$ is homogeneous of degree one. The equilibrium condition (C.3) therefore determines that $A = nB$. We further obtain $f(A) = nf(B)$ and

$$p_A = \frac{n}{n+1} = n(1-p_A). \tag{C.4}$$

By assuming intragroup symmetry, the expected payoffs simply satisfy

$$u^{cf} = p_{_A} w(n) - a = n(1 - p_{_A})w(n) - nb = nu^{fc}.$$

The proportionality rule is therefore perfectly preserved.

In the example that we give in the text, $f(x) = x^r$ is homogeneous of degree r, with $r \in (0, 1]$ to comply with our assumptions about $f(\cdot)$. $f'(x)$ is homogeneous of degree $r - 1$ and then $\frac{f(x)}{f'(x)}$ is still homogeneous of degree one. The first order conditions

$$\begin{cases} \frac{rA^{r-1}B^r}{(A^r+B^r)^2} nw(n) - 1 = 0 \\ \frac{rA^r B^{r-1}}{(A^r+B^r)^2} w(n) - 1 = 0 \end{cases} \tag{C.5}$$

then yield $A = nB$. The remaining parts of the proportionality rule however no longer hold, with:

$$\begin{aligned} A &= \frac{rn^{r+1}w(n)}{(n^r+1)^2}, & B &= \frac{rn^r w(n)}{(n^r+1)^2}; \\ p_{_A} &= \frac{n^r}{n^r+1}, & 1 - p_{_A} &= \frac{1}{n^r+1}; \\ u^{cf} &= \frac{n^r w(n)}{n^r+1} - \frac{rn^r w(n)}{(n^r+1)^2}, & u^{fc} &= \frac{w(n)}{n^r+1} - \frac{rn^{r-1}w(n)}{(n^r+1)^2}. \end{aligned} \tag{C.6}$$

Notice that $p_{_A} \leq n(1 - p_{_A})$ and $u^{cf} \leq n \cdot u^{fc}$, with equality only for $r = 1$.

C.3 Convex cost of effort

The planner of group A certainly assigns equal efforts to agents under cost convexity. Using the following objective functions:

$$\max_a \frac{na}{na+B} w(n) - c(a), \qquad \max_{b_i} \frac{B}{na+B} w(n) - c(b_i) \tag{C.7}$$

we derive the following first order conditions

$$\frac{n^2 b}{(na+B)^2} w(n) - c'(a) = 0, \qquad \frac{na}{(na+B)^2} w(n) - c'(b_i) = 0. \tag{C.8}$$

To proceed, we assume intragroup symmetry in group B. The equilibrium condition is then the following:

$$a \cdot c'(a) = nb \cdot c'(b). \tag{C.9}$$

If $n > 1$, then $ac'(a) > bc'(b)$. This implies that $a > b$ and $c'(a) > c'(b)$, because $xc'(x)$ is increasing in x. By rewriting (C.9) into $a/b = n \cdot c'(b)/c'(a)$, we clearly see that $a/b < n$, namely cost convexity dampens the proportionality rule.

C.4 Risk aversion

Like in the previous case, in group A agents are optimally assigned with equal efforts, and in group B we shall assume intragroup symmetry. We use the CARA utility function to show that risk aversion can amplify the proportionality rule.

Applying $u(x) = -e^{-x}$ to the first order conditions in (22), we obtain:

$$\begin{aligned} \frac{b}{a+b}(-e^{a-w(n)} + e^{a}) - (ae^{a-w(n)} + be^{a}) &= 0 \\ \frac{a/n}{a+b}(-e^{b-w(n)} + e^{b}) - (be^{b-w(n)} + ae^{b}) &= 0 \end{aligned} \tag{C.10}$$

This system has two pairs of solutions. Using numerical examples, we find that one solution may be non-positive while the other features $a > nb$. For example, for $(n, e^{w}) = (2, 5)$, we have $(a, b)_1 = (-0.513, 1.377)$ and $(a, b)_2 = (0.325, 0.061)$. We therefore claim that risk aversion may amplify the proportionality rule at least for the exponential utility.

Findings in section C.2, C.3 and C.4 support Proposition 3 in the text.

C.5 Group size paradox

Suppose now that group A consists of m cooperative agents and group B consists of n free-riding agents, with $m \geq 2$. We show how expected payoffs are higher in group A than in group B as Proposition 5 claims.

Using the following first order conditions:

$$\begin{cases} \frac{B}{(A+B)^2} mw(m) - 1 = 0 \\ \frac{A}{(A+B)^2} w(n) - 1 = 0 \end{cases} \tag{C.11}$$

we obtain the equilibrium condition $A/B = mw(m)/w(n)$, and in turn the groups' winning probabilities

$$p_{A} = \frac{mw(m)}{mw(m) + w(n)}, \qquad 1 - p_{A} = \frac{w(n)}{mw(m) + w(n)}.$$

Agents' expected payoffs under intragroup symmetry are

$$u_a^{cf} = \frac{m^2 w(m)^3}{[mw(m) + w(n)]^2}, \qquad u_b^{fc} = \frac{w(n)^3 + m(n-1)w(m)w(n)^2/n}{[mw(m) + w(n)]^2}. \tag{C.12}$$

Comparing u_a^{cf} and u_b^{fc} reveals that $u_a^{cf} > u_b^{fc}$ is equivalent to

$$Q^{cf}(\lambda) \equiv m^2 w(m)^3 - w(n)^3 - m(n-1)w(m)w(n)^2/n > 0, \tag{C.13}$$

where $Q^{cf}(\lambda)$ is a continuous cubic function of λ. Like in the other cases, we find only one real solution for $Q^{cf}(\lambda) = 0$. To see if this solution lies in $[0,1]$, we examine the boundary values of $Q^{cf}(\lambda)$ on this domain

$$Q^{cf}(\lambda = 0) = \frac{(n^2 - m)}{mn}, \text{ and } Q^{cf}(\lambda = 1) = m^2 - m(n-1)/n - 1 \tag{C.14}$$

For $m \geq 2$, $Q^{cf}(\lambda = 1)$ is always positive, and:

i) if $m < n^2$, then $Q^{cf}(\lambda = 0) > 0$. In this case, $u_a^{cf} > u_b^{fc}, \forall \lambda \in [0,1]$. Agents in the cooperative group always have a higher expected payoff;

ii) if $m \geq n^2$, then $Q^{cf}(\lambda = 0) \leq 0$. In this case there exists a unique threshold λ^{cf}, such that $u_a^{cf} > u_b^{fc}$ if and only if $\lambda > \lambda^{cf}$.

D Intragroup Altruism

We start this section by comparing groups' efforts in the general case, where groups may differ both in sizes and degrees of altruism. Following the notations in the text, agents' objective functions (31) are reorganized into

$$\begin{aligned} &\max_{a_i} p_A[1 + \alpha_A(m-1)]w(m) - a_i - \alpha_A \sum_{j \in A, j \neq i} a_j \\ &\max_{b_i} (1 - p_A)[1 + \alpha_B(n-1)]w(n) - a_i - \alpha_B \sum_{j \in B, j \neq i} b_j \end{aligned} \tag{D.1}$$

The first order conditions

$$\begin{aligned} \frac{B}{(A+B)^2}[1 + \alpha_A(m-1)]w(m) - 1 = 0 \\ \frac{A}{(A+B)^2}[1 + \alpha_B(n-1)]w(n) - 1 = 0 \end{aligned} \tag{D.2}$$

imply that in the equilibrium there is

$$\frac{A}{B} = \frac{[1 + \alpha_A(m-1)]w(m)}{[1 + \alpha_B(n-1)]w(n)}. \tag{D.3}$$

We then derive the following equilibrium aggregate efforts:

$$\begin{aligned} A &= \frac{[1 + \alpha_A(m-1)]^2 w(m)^2 [1 + \alpha_B(n-1)]w(n)}{\{w(m)[1 + \alpha_A(m-1)] + w(n)[1 + \alpha_B(n-1)]\}^2} \\ B &= \frac{[1 + \alpha_A(m-1)]w(m)[1 + \alpha_B(n-1)]^2 w(n)^2}{\{w(m)[1 + \alpha_A(m-1)] + w(n)[1 + \alpha_B(n-1)]\}^2} \end{aligned} \tag{D.4}$$

By comparing A and B, Result 7 summarizes how the efforts are affected by group sizes, degrees of altruism, and the degree of publicness of the rent.

Result 7. *Group A exerts more efforts than group B does when* $[1+\alpha_A(m-1)]w(m) > [1+\alpha_B(n-1)]w(n)$. *If* $m < n$, *this is equivalent to:*

(i) $\alpha_A \geq \frac{n-1}{m-1}\alpha_B$, *or*

(ii) $\alpha_A \in (\frac{m(n-1)\alpha_B-(n-m)}{n(m-1)}, \frac{n-1}{m-1}\alpha_B)$, *and*
$\lambda < \frac{R[\frac{1+\alpha_A(m-1)}{m}-\frac{1+\alpha_B(n-1)}{n}]}{P[\alpha_B(n-1)-\alpha_A(m-1)]+R[\frac{1+\alpha_A(m-1)}{m}-\frac{1+\alpha_B(n-1)}{n}]}$.

So the group with a higher perceived value of rent exerts more effort. This means that a smaller group exerts more effort if its degree of altruism is higher enough, or if it is not too low meanwhile the degree of publicness of the rent is low enough. We now prove this Result.

Proof. Comparing A and B in (D.4) reveals that $A > B$ if and only if

$$[1+\alpha_A(m-1)]w(m) > [1+\alpha_B(n-1)]w(n), \tag{D.5}$$

or after replacing $w(m)$ and $w(n)$:

$$[\alpha_A(m-1)-\alpha_B(n-1)]\lambda P > (1-\lambda)R\left[\frac{1+\alpha_B(n-1)}{n} - \frac{1+\alpha_A(m-1)}{m}\right] \tag{D.6}$$

- If $\alpha_A \geq \alpha_B\frac{(n-1)}{(m-1)}$, then $\alpha_A(m-1) \geq \alpha_B(n-1)$ and $\frac{1+\alpha_A(m-1)}{m} > \frac{1+\alpha_B(n-1)}{n}$. The left hand side of (D.6) is non-negative whilst the right hand side is negative. (D.6) is hence satisfied for any λ. Otherwise:

- If $\frac{m\alpha_B(n-1)-(n-m)}{n(m-1)} < \alpha_A < \alpha_B\frac{(n-1)}{(m-1)}$, then both sides of (D.6) are negative. (D.6) is satisfied only when $\lambda < \lambda^*$, where
$$\lambda^* = \frac{R[\frac{1+\alpha_A(m-1)}{m} - \frac{1+\alpha_B(n-1)}{n}]}{P[\alpha_B(n-1)-\alpha_A(m-1)] + R[\frac{1+\alpha_A(m-1)}{m} - \frac{1+\alpha_B(n-1)}{n}]};$$

- If $\alpha_A \leq \frac{m\alpha_B(n-1)-(n-m)}{n(m-1)}$, the left hand side of (D.6) is negative whilst the right hand side is positive. There then exists no λ that satisfies (D.6). $\square$

□

To finish this section, we prove Proposition 6 and Proposition 7, and in addition compare the expected material payoffs in either case.

(i) For Proposition 6: taking $m = n$ to (D.4) we find:

$$\begin{aligned} A &= \frac{[1+\alpha_A(m-1)]^2[1+\alpha_B(m-1)]}{\{[1+\alpha_A(m-1)]+[1+\alpha_B(m-1)]\}^2} w(m) \\ B &= \frac{[1+\alpha_A(m-1)][1+\alpha_B(m-1)]^2}{\{[1+\alpha_A(m-1)]+[1+\alpha_B(m-1)]\}^2} w(m) \end{aligned} \tag{D.7}$$

Clearly $A > B$ if and only if $\alpha_A > \alpha_B$. We then study how equilibrium efforts vary with the degrees of altruism:

$$\begin{aligned} \frac{\partial A}{\partial \alpha_A} &= \frac{2(m-1)\bar{\alpha}_A \cdot \bar{\alpha}_B^2}{(\bar{\alpha}_A + \bar{\alpha}_B)^3} \\ \frac{\partial A}{\partial \alpha_B} &= \frac{(m-1)^2\bar{\alpha}_A^2 \cdot (\alpha_A - \alpha_B)}{(\bar{\alpha}_A + \bar{\alpha}_B)^3} \\ \frac{\partial (A+B)}{\partial \alpha_A} &= \frac{(m-1)\bar{\alpha}_B^2}{(\bar{\alpha}_A + \bar{\alpha}_B)^2} \end{aligned} \tag{D.8}$$

In above $\bar{\alpha}_A = 1 + \alpha_A(m-1)$ and $\bar{\alpha}_B = 1 + \alpha_B(m-1)$. The claims of Proposition 6 are then obvious, that is, a group's effort increases in its degree of altruism, increases (decreases) in the rival group's degree of altruism if the rival has a lower (higher) degree of altruism, and the total effort exerted by both groups increases with *either* group's degree of altruism.

We want to study whether the more altruistic group also has a higher expected material payoff. To do so, we assume intragroup symmetry and denote an agent's expected *material* payoff by u_a in group A and by u_b in group B. Using (D.7), we get

$$\begin{aligned} u_a &= \frac{[1+\alpha_A(m-1)]w(m)}{[2+\alpha_A(m-1)+\alpha_B(m-1)]} - \frac{[1+\alpha_A(m-1)]^2[1+\alpha_B(m-1)]}{[2+\alpha_A(m-1)+\alpha_B(m-1)]^2}\frac{w(m)}{m}; \\ u_b &= \frac{[1+\alpha_B(m-1)]w(m)}{[2+\alpha_A(m-1)+\alpha_B(m-1)]} - \frac{[1+\alpha_A(m-1)][1+\alpha_B(m-1)]^2}{[2+\alpha_A(m-1)+\alpha_B(m-1)]^2}\frac{w(m)}{m}, \end{aligned}$$

or

$$u_a = p_A U, \qquad u_b = (1-p_A)U,$$

where $p_A = \frac{\bar{\alpha}_A}{\bar{\alpha}_A + \bar{\alpha}_B}$ and $U = w(m) - \frac{\bar{\alpha}_A\bar{\alpha}_B}{\bar{\alpha}_A+\bar{\alpha}_B}\frac{w(m)}{m}$. So $u_a > u_b$ if and only if $p_A > 1/2$. Result 8 highlights this finding:

Result 8. *With equal sizes, the more altruistic group wins the rent with a higher probability, and its agents also have a higher expected material payoff.*

(ii) For Proposition 7: with $\alpha_A = \alpha_B = \alpha$ and $m < n$, (D.3) leads to

$$p_A = \frac{w(m)}{w(m) + \frac{1+\alpha(n-1)}{1+\alpha(m-1)} w(n)}, \tag{D.9}$$

which is decreasing in α. After replacing $w(m)$ and $w(n)$ we have $p_A > 1/2$ if and only if

$$\alpha < \alpha^* \equiv \frac{(1-\lambda)R}{\lambda P + (1-\lambda)R/(mn)}, \tag{D.10}$$

where α^* is clearly decreasing in λ. Therefore for the smaller group to have a higher winning probability than its rival, a low degree of altruism and a low degree of publicness of the rent are necessary.

Now using $\alpha_A = \alpha_B = \alpha$ and (D.4), the total effort exerted by both groups can be expressed as

$$A + B = \frac{W(m)W(n)}{W(m) + W(n)} = \frac{1}{1/W(m) + 1/W(n)}, \tag{D.11}$$

where $W(m) = w(m)[1 + \alpha(m-1)]$ and $W(n) = w(n)[1 + \alpha(n-1)]$. It is obvious that $A + B$ increases in both $W(m)$ and $W(n)$, which in turn both increase in α. Therefore we claim in Proposition 7 that the aggregate efforts exerted by both groups increase with altruism.

Finally we look at the expected material payoffs. Following the previous notations, we have

$$\begin{aligned} u_a &= \frac{W(m)w(m)}{W(m) + W(n)} - \frac{W(m)^2 W(n)}{[W(m) + W(n)]^2} \frac{1}{m} \\ u_b &= \frac{W(n)w(n)}{W(m) + W(n)} - \frac{W(n)^2 W(m)}{[W(m) + W(n)]^2} \frac{1}{n} \end{aligned} \tag{D.12}$$

$u_a > u_b$ is then equivalent to

$$W(m)w(m) - W(n)w(n) - \frac{W(m)W(n)}{W(m) + W(n)} \left[\frac{W(m)}{m} - \frac{W(n)}{n} \right] > 0 \tag{D.13}$$

By replacing $W(m)$, $W(n)$, $w(m)$ and $w(n)$ with their respective expressions, we obtain from (D.13) once more an inequality $Q(\lambda) > 0$, where $Q(\lambda)$ is continuous and cubic in λ, and has only one real solution for $Q(\lambda) = 0$. We examine the boundary values of $Q(\lambda)$ on $[0, 1]$ as following:

$$
\begin{aligned}
Q(\lambda = 0) &= \left[\tfrac{1+\alpha(m-1)}{m^2} - \tfrac{1+\alpha(n-1)}{n^2}\right]\left[1 - \tfrac{[1+\alpha(m-1)][1+\alpha(n-1)]}{n[1+\alpha(m-1)]+m[1+\alpha(n-1)]}\right], \\
Q(\lambda = 1) &= -\alpha - \tfrac{[1+\alpha(m-1)][1+\alpha(n-1)]}{2+\alpha(m-1)+\alpha(n-1)} \tfrac{1-\alpha}{mn}.
\end{aligned}
$$

For $m < n$ and any $\alpha \in [0, 1]$, one can always find that $Q(\lambda = 0) > 0$ and $Q(\lambda = 1) < 0$. Therefore there exists a $\lambda^* \in (0, 1)$, such that $u_a > u_b$ if and only if $\lambda < \lambda^*$. Result 9 states this finding:

Result 9. *With equal degrees of altruism, agents in the smaller group have a higher expected material payoff than the ones in the larger group when the degree of publicness of the rent is low enough.*