Coalition Formation and Environmental Policies in International Oligopoly Markets

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April 06, 2012

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The usual disclaimer applies.
Abstract: This paper analyzes the problem of international environmental cooperation as a coalition formation game. For this purpose, we develop a simple model with three countries of unequal size. Strategic interactions between those countries come from the imperfect competition among producers in global markets and from the transboundary pollution generated by the firms. To capture efficiency gains from coordinating policies, countries can join a coalition and sign an international environmental agreement. The equilibrium coalition structure then depends on the country-size asymmetry and on the marginal environmental damage. Interestingly, we show that the grand coalition is less likely to emerge as an equilibrium outcome once two countries form a subcoalition. Furthermore, the further enlargement of the initial subcoalition can be blocked either by the outsider or by the insiders.

Keywords: Tax coordination, Transboundary Pollution, International Trade, Oligopoly, Coalition Formation

JEL Classification: F55, H23, Q56
1 Introduction

Some of the most serious environmental problems that urgently call for solutions are those related to trans-boundary pollution, greenhouse gas emissions and climate change. The Kyoto Protocol was symbolically an important step, but it failed to deliver a global effort toward greenhouse gas reductions. More recently, the Copenhagen summit on climate change in December 2009 involved representatives of 193 countries with the aim to obtain a general agreement on how to cut global emission of greenhouse gases. Unfortunately, Copenhagen summit ended in a failure. The so-called Copenhagen accord recognizes the scientific case for keeping temperature rises to no more than 2°C Celsius, but does not contain any commitments to emissions reductions to achieve that goal. The Swedish EU’s presidency then termed the general outcome of that summit as a ‘disaster’ and declared both China and the United States, the world’s number one and two polluters, responsible for the result. Even at the European level, obtaining a general environmental agreement is quite difficult. Indeed, since the early 1990s, there have been several attempts to introduce a unitary carbon tax across all EU member states. But it has never materialized, as countries like the UK were unwilling to render national competencies on taxation to Brussels. Another example is given by France in autumn 2009 which outlined a plan to set up a national carbon tax, following the example of Scandinavian countries which introduced such a tax on fuels back in early 1990s. But, the bill was rejected by the country’s highest court on the grounds that the bill contained (too many) unconstitutional exceptions. Finally, French president did not support the bill, saying that France needed support from the rest of the European Union before it would proceed with a carbon tax.

The difficulty of coordinating environmental policies among sovereign countries is inherent to the global trend of trade liberalization. Indeed, in the context of free-trade, governments are reluctant to abandon sovereignty on environmental policies because they can use these policies for trade related goals. For example, it is often argued that governments may relax their environmental standards in order to gain a competitive advantage over their trading partners. Conrad (1993) and Barrett (1994a) indeed show that under imperfect competition, governments have an incentive to use environmental policies in order to implicitly subsidize their exports. This lead to standards or pollution taxes below the Pigouvian level, and to what has become called as the “environmental dumping” effect. However as emphasized by Kennedy (1994), strategic distortions may also play in opposite direction. Government may indeed raise environmental standards in order to export polluting production since goods can be imported. A critical factor to determine which strategic distortion is likely to dominate the other is then the degree to

\footnote{Note though that in conjunction with the Kyoto protocol, the European Union Emission Trading Scheme (or EU ETS), which is the largest multi-national greenhouse gas emissions system, was launched in January 2005. Nevertheless, this system covers only half of the European CO2 emissions.}
which pollution is transboundary. When pollution is perfectly transboundary, as in the case of greenhouse
gas emission, the pollution-shifting effect vanishes because foreign pollution causes as much damage to
the domestic environment as does domestic pollution. In this case, trade liberalization therefore leads to
environmental dumping.\footnote{As shown by Barrett (1994a), this conclusion holds provided competition in international markets is Cournot. Otherwise, if competition is Bertrand, then governments will impose strong environmental standards.}

This paper examines the problem of coordination of environmental tax policies between several het-
erogenous countries in the context of free trade, and extends the standard analysis of strategic environ-
mental policies into several complementary directions. First, unlike most analysis of environmental policies
under imperfect competition, we introduce domestic consumption within each production region. Second,
we assume that countries are of unequal size. This, we believe, better represent real-world situations since
most international agreements involve a small country (or a few small countries) and a large partner as
the US or the EU. Last but not least, we model the emergence of a stable international environmental
agreement as a coalition formation game.

More specifically, the model employed in this paper is the following. There are three countries of
unequal population size and in each one of them, there is a single firm producing a homogeneous good
$X$. Production of $X$ generates pollution emissions that spread perfectly across the national borders. We
also assume that governments cannot use import tariff or export subsidies and that there is no shipping
costs. In a word, free-trade prevails and the three monopolists compete à la Cournot in all three countries.
Finally, a production – or emission – tax is the sole policy instrument at governments’ disposal to deal
with the market failure of imperfect competition (on domestic and international markets) and that due to
polluting production. In this setting, there are thus gains to coordinating tax policies and our objective is
to determine whether a stable cooperative arrangement can be reached to exploit these gains.

Our analysis of such arrangements is framed by a three stage game. In the first stage, countries choose
their coalition partners. A coalition forms if there is unanimity with respect to a partnership plan, that
is each country would like to join precisely the other(s) in a coalition. In the second stage, each coalition
commit to tax polluting production at a rate that maximizes the coalition’s aggregate welfare. Finally, in
the third stage, firms decide (non-cooperatively) on quantities, taxes are levied, international trade occurs
and consumption takes place. We also assume that the cooperative arrangement (if any) prescribes that the
fiscal revenues raised remain in the country of origin. In other words, there is no transfer payments between
countries, which seems reasonable to assume as (very) few environmental agreements have provisions for
transfers. In this set-up, we then define a stable cooperative arrangement as an equilibrium coalition
structure which is immune to any – unilateral or multilateral – deviations.
The main result is that if two countries can benefit from being part of a subcoalition, then the grand coalition is less likely to emerge as an equilibrium coalition structure, whilst the three countries would have agreed to join the grand coalition if the sole alternative was the singleton coalition. Furthermore, the grand coalition can be blocked not only by one country – called the outsider – but also by a subcoalition of two countries.

Intuitively, when a subcoalition of two countries is formed, it gives rise to a strategic advantage for the outsider. Indeed, this last can take advantage of the internalization of the pollution externality by the coalition – resulting in higher taxes and then in increased production costs for the firms of the coalition – to make its own firm more competitive on international markets. In turn, this makes the grand coalition less likely to emerge because the outsider is now more demanding to give up its competitive advantage created by the formation of the subcoalition. In other words, the pollution externality and the benefits of internalizing this externality must be higher than when the only alternative to the singleton coalition is the grand coalition. However, when both the spillover parameter and the population size of the outsider become ‘very’ large, the two countries of the subcoalition prefer the largest country to stay outside the coalition and free-ride on them. This is because, in this case, the formation of the grand coalition would lead to a significant decrease in produced and consumed quantities – due to the internalization of the pollution externality over all three countries – which in turn would lead to a strong decrease in export revenues for the two (small) countries of the coalition. Certainly, with a two-coalition structure, the two members of the coalition loose markets shares to the benefit of the outsider, but the market size is also much larger than under the grand coalition and a large market size mostly benefit to small countries.

This paper contributes to and connects two different strands of the literature: the one on the use of the relationship between international trade and environmental policies and the other on international environmental agreements (IEAs). In general, since Barrett (1994b), the first strand of literature considers that countries compete on a third market. This greatly simplifies the analysis because, in this case, there is no consumer surplus in domestic welfare. It follows that environmental policy of each country only takes into account environmental damage and the competitiveness – in terms of production costs – of the domestic firms. When firms compete in domestic markets, as here, governments have an incentive to deal with the imperfect competition effect because it harms consumer surplus. Kennedy (1994) or Tanguay (2001) also consider imperfect competition and that trade occurs between countries and not in a third market. But in their analysis, and in contrast to the present one, there is no heterogeneity across countries. There is however a paper by Duval and Hamilton (2002) who analyze strategic environmental policy in a two-country model with asymmetric number of firms and asymmetric pollution diffusion across countries, but they do not investigate the issue of environmental policy coordination. We investigate another source of
heterogeneity across countries – in terms of population and market sizes – and principally focus on the stability of international environmental agreements in a coalition formation game.\textsuperscript{3}

The earliest works (Hoel, 1992; Carraro and Siniscalco, 1993; Barrett, 1994b) analyze a global emission-abatement game and characterize equilibrium IEAs by applying the internal-external stability concept developed for cartel theory (d’Aspremont et al., 1983). These works have been extended in many directions including that of heterogeneous countries participating in an IEA (e.g. Finus and Rundshagen, 1998; Barrett, 2001). In this paper, we also investigate the stability of IEAs but in a context of imperfect competition in global markets. In addition, drawing on the recent literature on endogenous coalition formation (see, Bloch, 1996; Yi, 1996; Ray and Vohra, 1999), we consider deviations not only by single countries but also by subgroup of countries to determine which IEA emerges as an equilibrium coalition structure.\textsuperscript{4}

It is worth pointing out that these extensions were made possible by assuming that the world is composed of only three countries, thereby substantially limiting the number of possible coalition structures. In addition, we will assume at some point that two of the three countries are of equal size which further restrains the number of coalition structures to be analyzed. Yet, the model used in this paper is rich (and complex) enough to put forward the idea – in a single and coherent analytical framework – that when a subset of countries sign an IEA, then a further extension of the coalition may be blocked either by the outsider or by the insiders. In fact, the result that the outsider may refuse to join the (grand) coalition may seem expected (and quite intuitive) because sub-global agreements, in general, give rise to a strategic advantage to the outsider(s). An IEA corresponds in many aspects to the provision of a public good generating positive externalities for the outsider so that this last is better off free-riding on the coalition. But, in our analysis, the members of a subcoalition may also refuse the entry of the outsider in the (grand) coalition. That result is rather unexpected and much less intuitive. In a word, when the pollution externality and the country-size asymmetry (between insiders and the outsider) are relatively strong, then the benefits of the new entry (for the two countries of the initial coalition) may be too low to compensate the loss – in term of export revenues – due to the drastic change in the environmental policy of the grand coalition.\textsuperscript{5}

\textsuperscript{3}In a recent paper (Cheikbossian, 2010), we analyzed the problem of coordinating environmental policies among two asymmetric countries. But, in a two-country model, we cannot investigate the formation and the stability of global and sub-global agreements.

\textsuperscript{4}Recently, some authors have applied approach of coalition formation to the analysis of IEAs (see, e.g., Carraro and Marchiori, 2003, or Finus and Rundshagen, 2009). But, again, most authors use a global emission-abatement game which often needs to be solved numerically.

\textsuperscript{5}In the same spirit, Alesina and Grilli (1993) show that the twelve potential members (in 1993) of the European monetary
In Section 2, we present the general framework. Section 3, 4 and 5 derive the outcome of the international policy game under respectively the singleton coalition, the grand coalition and the coalition structure with only two countries forming a coalition. In Section 6, we give a precise definition of an equilibrium coalition structure and then derive the outcome of the coalition formation game, depending on the parameter reflecting the pollution externality and that reflecting the asymmetry in population sizes.

2 The Model

2.1 The Households

We consider a world composed of three countries, $N = \{A, B, C\}$. There are $n_i$ consumers in country $i$ and we normalize total population to 1, i.e. $n_A + n_B + n_C = 1$. Two goods are consumed in each country. The numeraire good $Y$ is produced by competitive firms, while good $X$ is produced under conditions of imperfect competition. Pollution is generated as a by-product of the production of $X$ and this pollution cross borders. The $Y$ industry does not pollute. An individual consumer in country $i$ has the quadratic utility function:

$$u_i = \alpha x_i - (1/2) x_i^2 + y_i - D(X), \quad i = A, B, C, \quad (1)$$

where $x$ and $y$ are per capita consumption levels of $X$ and $Y$ respectively, and where $D(X)$ is the environmental damage incurred by the consumer. This damage is a function of world production of $X$ (which is identically denoted as $X$). Finally, $\alpha$ corresponds to the maximal marginal utility of $X$.

An individual is assumed to own $L$ units of labor, and the production function for $Y$ is simply $Y = L_y$, where $L_y$ is the labor allocated to the production of $Y$. Again, $Y$ (or $L$) is the numeraire and $p_i$ is the consumer price of good $X$ in country $i$ in terms of $Y$ (or $L$). The production of good $X$ under conditions of imperfect competition will be more precise in the next sub-section.

In addition, in each country, all revenues that the government obtains from taxation are distributed equally and in a lump-sum fashion across the population. (If these revenues are negative, this implies that each government can impose lump-sum taxes on its population). Denoting the per capita tax revenues by $T_i$, the budget constraint facing a representative household in each country is:

$$L + T_i = y_i + p_i x_i, \quad i = A, B, C. \quad (2)$$

Maximization of (1) subject to the budget constraint (2) yields the representative household’s inverse demand of good $X$: union (EMU) may not agree to enlarge it once it is formed, due to the heterogeneity in preferences (and in economic conditions) of the potential new entrants. It has to be said that it was not the case.
\[ \alpha - x_i = p_i, \quad i = A, B, C. \] (3)

A convenient feature of the quadratic utility function (1) is that the individual’s tax return do not enter the demand for good \( X \) since at the margin, income changes affect only the demand for the numeraire good \( Y \). Aggregating over households in each country yields the following market demand curves for each of the three countries:

\[ X_i = n_i x_i = n_i (\alpha - p_i), \quad i = A, B, C, \] (4)

where \( X_i \) represents aggregate consumption of good \( X \) in country \( i \).

### 2.2 The Firms and the Environmental Damage

There is in each country a monopolist that produces good \( X \).\(^6\) More specifically, the \( X \) firm produces with a constant marginal cost \( m \) and with fixed costs \( F \) (all in terms of units of labor).

For all firms, production leads to the emission of the same pollutant which is transboundary. We assume that one unit of pollution is produced by one unit of \( X \) whether production takes place in the home country or in the foreign countries. We also assume that pollution is a pure public bad so that consumers in all countries are equally harmed by the pollution released from any country. More specifically, pollution generates environmental damage in country \( i \) (for \( i = A, B, C \)) according to the following form

\[ D(X) = \beta [X_A + X_B + X_C], \] (5)

where \( X^i \) (for \( i = A, B, C \)) represents the production of the firm located in country \( i \) and where \( \beta \in (0, \alpha) \) is a parameter that captures the marginal environmental damage caused by the production of \( X \). We suppose that this marginal environmental damage cannot be higher than the maximal marginal utility of \( X \) given by \( \alpha \), otherwise production of \( X \) would not be socially beneficial.

The three countries constitute a common market. Each monopolist can export in the two other markets at no shipping costs. \( X_i \) being the demand for good \( X \) in country \( i \) (for \( i = A, B, C \)), the aggregate demand in the integrated economy is

\[ X_A + X_B + X_C = \alpha - p. \] (6)

In equilibrium, aggregate demand equals aggregate supply, i.e. \( X^A + X^B + X^C \).

\(^6\) Monopoly can emerge as an equilibrium market structure if firm-specific fixed costs are sufficiently high to make entry for a second firm unprofitable in each market (see, Horstman and Markunsen 1992).
In order to reduce emissions, each government can charge a tax on the production of $X$ that is produced by the domestic firm. Let $t_i$ be the per-unit production tax set by the government of country $i$. Profits for the firm located in country $i$ are

$$\pi^i = \left[ \alpha - (X^i + X^j + X^k) \right] X^i - m X^i - t_i X^i - F, \quad i = A, B, C, \quad i \neq j \neq k. \tag{7}$$

Since an increase in the marginal cost $m$ is equivalent to a decrease in $\alpha$, we set $m = 0$ for sake of simplicity.\(^7\)

Differentiating this profit expression with respect to $X^i$, and setting the derivative equal to zero yields the following best response functions of the firms,

$$X^i = \frac{\alpha - t_i - (X^j + X^k)}{2}, \quad i = A, B, C, \quad i \neq j \neq k. \tag{8}$$

The Nash equilibrium in quantities is then given by

$$X^i = \frac{\alpha - 3t_i + t_j + t_k}{4}, \quad i = A, B, C, \quad i \neq j \neq k. \tag{9}$$

Firms $i$’s production is decreasing in the domestic pollution tax rate and increasing in the pollution tax rate faced by its competitors. In equilibrium, aggregate supply, i.e. $X^A + X^B + X^C$, equals aggregate demand, i.e. $X_A + X_B + X_C$.\(^8\) Since total population is normalized to 1, aggregate supply or aggregate demand also corresponds to individual consumption of good $X$ in the three countries, that is

$$x_A = x_B = x_C = x = \frac{3\alpha - (t_A + t_B + t_C)}{4}. \tag{10}$$

Expressions (6), (9) and (10) complete the output stage of the model.

We can now express the domestic welfare of each country as the sum of net consumer surplus which includes the environmental damage, plus profits, plus tax revenues, that is

$$W_i = n_i \left[ \frac{1}{2} x_i^2 + L - \beta \left( X^A + X^B + X^C \right) \right] + \pi^i + t_i X^i, \quad i = A, B, C. \tag{11}$$

To simplify the interpretation of the results, we shall use another expression of the domestic welfare of each country. First, recall that aggregate production equals aggregate and individual consumption, and so we denote by $p(x) = \alpha - x$, the market price. Now, adding and subtracting the term $n_i p(x) x$ - i.e. the value of domestic consumption in country $i$ - into (11), we can express the welfare level of country $i$ as follows

\(^7\)We also assume, throughout the analysis, that profits are always strictly positive in all three countries, i.e. profit margin implied by the choice of the (exogenous) parameters $\alpha$ and $\beta$ is sufficiently large to cover the fixed costs $F$.

\(^8\)We assume throughout that $L$ is sufficiently large such that the consumers are able to pay for the $X$ produced.
The first term corresponds to the gross consumer surplus denoted by $CS_i = n_i \left[ \frac{1}{2}x^2 + (p(x) - \beta)x + L \right] + p(x)(X^i - n_i x) - F$, $i = A, B, C$, and the second term corresponds to the value of external trade denoted by $EX_i = p(x)(X^i - n_i x)$, which can be positive (negative) if country $i$ is a net exporter (importer).\footnote{Equivalently, each country’s welfare can be written as the difference between gross surplus $CS_i$ and total costs denoted $TC_i$, i.e. $W_i = CS_i - TC_i$. These costs are the sum of production costs, i.e. $mX^i + F = F$ since $m = 0$, plus the costs of imports, i.e. $p(x)(n_i x - X^i)$.}

The objective in this paper is to determine whether there exists stable cooperative arrangements among the three countries to deal with the two market failures: the pollution externality and the imperfect competition in global markets. To this end, we consider the following three-stage game. In the first stage, countries decide on their membership in a coalition. In the second stage, each coalition sets a tax rate on polluting production. Finally, in the third stage, firms play the Cournot-Nash game described just above, international trade occurs and consumption takes place.

We now turn to the second stage at which countries are already aligned into coalition structures. A coalition structure is a partition of the set of countries and there are three types of possible coalition structures: the singleton coalition denoted $B_S$, the grand coalition denoted $B_G$ and the structure which involves a coalition between only two countries $i$ and $j$, while country $k$ remains a singleton. This coalition structure will be denoted $B_{(i,j)} = \{(i,j), \{k\}\}$. We first analyze the case where no arrangement has been agreed in the first stage of the game, what corresponds to the singleton coalition $B_S$.\footnote{Equivalently, each country’s welfare can be written as the difference between gross surplus $CS_i$ and total costs denoted $TC_i$, i.e. $W_i = CS_i - TC_i$. These costs are the sum of production costs, i.e. $mX^i + F = F$ since $m = 0$, plus the costs of imports, i.e. $p(x)(n_i x - X^i)$.}

### 3 The Singleton Coalition Structure $B_S$

The singleton coalition structure corresponds to the situation where the three countries play a Nash game in tax rates. In other words, each country chooses its tax rate on production so as to maximize domestic welfare, taking as given the other countries’ tax rates and anticipating the behavior of both the firms and the consumers.

Substituting (7), (9) and (10) into (11), differentiating this expression with respect to $t_i$, and setting the derivative equal to zero yields the following countries’ best response functions in tax rates,

$$t_i(t_j, t_k) = \frac{4\beta n_i - \alpha(2 + 3n_i)(2 - n_i)(t_j + t_k)}{6 - n_i}, \quad i = A, B, C \quad i \neq j \neq k.$$  \hspace{1cm} (13)

Best response functions are downward sloping. Indeed, as it is typically the case in a Cournot oligopoly model of international trade à la Brander and Spencer (1985), when one country increases its tax rate, the other countries have an incentive to lower their tax rates which enables the domestic firms to further increase their shares of international markets.
Solving this system of best response functions, we obtain the following equilibrium tax rates under the singleton coalition $\mathcal{B}_S$,

$$t_i(\mathcal{B}_S) = \frac{(10\beta - 9\alpha)n_i - 2\beta}{9} \quad i = A, B, C.$$  

(14)

Using (10) and (14), the per capita consumption level of $X$ in each country is $x(\mathcal{B}_S) = \alpha - \beta/9$ and the equilibrium price is $p(\mathcal{B}_S) = \beta/9$. In equilibrium, aggregate demand (i.e. $x(\mathcal{B}_S)$) is equal to aggregate production, and substituting (14) into (9), we obtain the production of the firm located in country $i$ (for $i = A, B, C$), i.e. $X^i(\mathcal{B}_S) = [(9\alpha - 10\beta)n_i + 3\beta]/9$. The gross surplus for country $i$ is $CS_i(\mathcal{B}_S) = [(1/162)(9\alpha - 17\beta)(9\alpha - \beta + L)n_i$, and the value of its external trade is $EX_i(\mathcal{B}_S) = p(\mathcal{B}_S)(X^i(\mathcal{B}_S) - n_ix(\mathcal{B}_S))$ or $EX_i(\mathcal{B}_S) = (\beta^2/27)(1 - 3n_i)$. Substituting these expressions into the welfare function given by (12), we obtain the welfare of each country when the singleton coalition structure prevails, that is

$$W_i(\mathcal{B}_S) = \frac{1}{102} \left[(6 - n_i)\beta^2 + 81\alpha n_i(\alpha - 2\beta)\right] + n_iL - F.$$  

(15)

To conclude this section, we consider the difference in tax rates between countries at the Nash equilibrium. Suppose first that the marginal cost of environmental damage is not too high, i.e. $\beta < (9/10)\alpha$, and that, for example, $n_A > n_B > n_C$. This implies from (14) that $t_A(\mathcal{B}_S) < t_B(\mathcal{B}_S) < t_C(\mathcal{B}_S)$. The factor explaining why, when $\beta < (9/10)\alpha$, a small country has an incentive to set a higher tax rate than the other(s) is related to the openness of the economies. Indeed, the national monopolists compete against each other in a common market and without governmental regulation, each would get an equal share of the market, and so a relatively small country would be a net exporter. A small country is thus at some intrinsic advantage to capture oligopoly rents from foreign consumers and, consequently, has less need to reduce its tax rate in order to gain a competitive advantage over its trading partner.\textsuperscript{10} Put it differently, an exporting country has an incentive to set a higher tax rate because a portion of the tax is shifted into world prices, which disproportionately affects consumers of the importing country. Inversely, a large country has an incentive to reduce its imports and hence to set a lower tax rate than smaller countries so as to increase domestic production.

However, increasing production also reduces the gross consumer surplus to the extent of both the marginal cost of pollution $\beta$ and the size of the population. Indeed, the marginal gross consumer surplus in country $i$ is negative, that is $CS_i'' = n_i[p(\mathcal{B}_S) - \beta] = -n_i(8\beta/9)$ since $p(\mathcal{B}_S) = \beta/9$. Therefore, when the marginal cost of environmental damage is relatively high, i.e. $\beta \geq (9/10)\alpha$, a large country has no longer incentives to undercut its competitor(s) and hence it sets a higher tax rate at the Nash equilibrium (see (14)).

\textsuperscript{10}This effect is similar to the terms-of-trade effect identified by (among others) Krutilla (1991).
4 The Grand Coalition $B_G$

Suppose now that the three countries are willing to form the grand coalition which means that they jointly decide to tax polluting production in all three countries at rates that maximizes the sum of the welfare functions given by (11). Note however that coalition members retain the responsibility to levy taxes and that all revenues raised remain in the country of origin.

Since the countries form a common market and since environmental damage is linear in pollution, there is a unique (linear) tax rate that maximizes the (grand) coalition’s aggregate welfare. Actually, uniform (tax) solutions are frequently viewed as efficient means to tackle a (pure) global environmental problem and furthermore often constitutes a typical feature of many IEAs to reduce the emission of pollutants (see, e.g., Hoel 1992; Finus and Rundshagen, 1998).

Therefore, we can set $t_A = t_B = t_C = t$ which implies that $X^A = X^B = X^C$ and $\pi^A = \pi^B = \pi^C = \pi$. Then, the aggregate welfare of the grand coalition is given by

$$ W = \left[ \frac{1}{2} x^2 + L - \beta x \right] + 3\pi + tx, \quad (16) $$

since $X^A + X^B + X^C = x$. Using (10), we have $x = \frac{3}{4} (\alpha - t)$ and the profit of each firm is then $\pi = (\alpha - t)^2 / 16 - F$. Differentiating $W$ with respect to $t$ and setting the derivative equal to 0, we obtain the following pollution tax rate under the grand coalition $B_G$,

$$ t(B_G) = \frac{4\beta - \alpha}{3}. \quad (17) $$

This common pollution tax rate is set to correct for two distortions as the market is characterized by both over-production due to the negative externality and under-production due to oligopoly pricing. Then, the policy of the grand coalition levies a second-best tax below the marginal cost of pollution (equal to $\beta$) which can be, in principle, either positive or negative.\footnote{The marginal variation of the gross consumer surplus is $\alpha - x - \beta$ and is positive as long $x < \alpha - \beta$. Without environmental regulation, the aggregate supply - which equals $x$ - is $3\alpha / 4$ (see (10)). Hence, the common tax rate is negative as long $3\alpha / 4 < \alpha - \beta$ i.e. $4\beta - \alpha < 0$.}

From (10), individual consumption of good $X$ in each country is $x(B_G) = \alpha - \beta$ and the equilibrium price is $p(B_G) = \beta$. In equilibrium, aggregate demand is equal to aggregate supply and the three firms have an equal market share of the integrated economy (i.e. $1/3$), and then each produces $X^i(B_G) = (\alpha - \beta) / 3$ (for $i = A, B, C$). The gross surplus for country $i$ is $CS_i(B_G) = \left[ (1/2) (\alpha - \beta)^2 + L \right] n_i$, and the value of its external trade is $EX_i(B_G) = p(B_G)(X^i(B_G) - n_i x(B_G))$ or $EX_i(B_G) = \left[ \beta (\alpha - \beta) / 3 \right] (1 - 3n_i)$.

Substituting the above expressions into (12), we obtain the aggregate welfare of country $i$ in the grand coalition, i.e.
\[ W_i(B_G) = \frac{1}{6} (\alpha - \beta) [2\beta + 3n_i(\alpha - 3\beta)] + n_iL - F, \quad i = A, B, C. \] (18)

5 The Coalition Structure \( B_{(i,j)} \)

Suppose now that two countries – say countries \( i \) and \( j \) – form a coalition and choose a common tax rate, denoted \( t_{ij} \), so as to maximize their joint-welfare \( W_i + W_j \), where \( W_i \) and \( W_j \) are given by (11). As for the grand coalition, each coalition partner still has the responsibility to levy and collect taxes on its own territory and there is a unique tax rate maximizing the subcoalition’s aggregate welfare.

The third country – i.e. country \( k \) – remains a singleton. In the last stage of the game, each firm still maximizes its profits independently of the other firms, given \( t_{ij} \) and \( t_k \). From (9) with \( t_i = t_j = t_{ij} \), we have the following equilibrium quantities

\[
X^i = X^j = \frac{\alpha - 2t_{ij} + t_k}{4}, \quad X^k = \frac{\alpha - 3t_k + 2t_{ij}}{4}. \] (19)

Again, since the market is common, there is only one price \( p \) and therefore individual consumption of good \( X \) is identical in the three countries and is given (using (10)) by

\[
x = \left[3\alpha - (2t_{ij} + t_k)\right]/4. \] (20)

It follows that the equilibrium market price is \( p = (\alpha + 2t_{ij} + t_k)/4 \). The profit of each firm of the coalition is \( \pi^i = \pi^j = (\alpha - 2t_{ij} + t_k)^2/16 - F \), while the profit of the firm located in country \( k \) is \( \pi^k = (\alpha - 3t_k + 2t_{ij})^2/16 - F \).

In the second stage of the game, the coalition and country \( k \) choose their pollution tax rates independently of each other so as to maximize their respective welfare. Substituting (19), (20) and profits into (11), and maximizing \( W_i + W_j \) with respect to \( t_{ij} \) yields

\[
t_{ij}(t_k) = \frac{(n_i + n_j)(4\beta - 3\alpha + t_k)}{2[4 - (n_i + n_j)]}. \] (21)

Observe that the best response function of the coalition is upward sloping. The explanation is the following. Suppose that country \( k \) increases its tax rate by \( dt_k \). Then, the monopolist located in country \( k \) reduces its production level while each monopolist of the coalition increases its production (see eq. (19)). The key point is that the two monopolists of the coalition act independently of each other in the oligopoly game. In order to internalize the increased competition between the two monopolists, the coalition best reacts to an increase in tax rate in the third country by increasing its own tax rate so as to internalize the market externality between the two monopolists located on its territory. In turn, it makes the two firms of the coalition act as if they were a single firm.

Similarly, substituting (19), (20) and profits into (11), and maximizing \( W_k \) with respect to \( t_k \) yields
In contrast to the best response function of the coalition, that of the country outside the coalition is still downward sloping.

Solving this system, we obtain the following equilibrium tax rates under the coalition structure $\mathcal{B}_{(i,j)}$:

$$t_{ij} (\mathcal{B}_{(i,j)}) = -\frac{(5\alpha - 6\beta)(n_i + n_j)}{10},$$

$$t_k (\mathcal{B}_{(i,j)}) = -\frac{(5\alpha - 6\beta)n_k + 2\beta}{5}. \quad (23)$$

To conclude this section, we now determine the welfare level of each country. Using (20) and (23), the per capita consumption level of $X$ in each country is $x (\mathcal{B}_{(i,j)}) = \alpha - \beta/5$, while the market price is $p (\mathcal{B}_{(i,j)}) = \beta/5$. In equilibrium, aggregate demand (i.e. $x (\mathcal{B}_{(i,j)})$) is equal to aggregate production. Substituting (23) into (19), we obtain $X^i (\mathcal{B}_{(i,j)}) = X^j (\mathcal{B}_{(i,j)}) = \frac{[5\alpha (n_i + n_j) + 2\beta [1 - 3 (n_i + n_j)]]}{10}$ and $X^k (\mathcal{B}_{(i,j)}) = \frac{[5\alpha n_k + 3\beta (1 - 2n_k)]}{5}$.

The gross surplus for country $i = A, B, C$ is $CS_i (\mathcal{B}_{(i,j)}) = [(1/50) (5\alpha - 9\beta)(5\alpha - \beta) + L] n_i$. Finally, the value of exports for country $i$ is $EX_i (\mathcal{B}_{(i,j)}) = (\beta/50) [5\alpha (n_j - n_i) - 2\beta (2n_i + 3n_j - 1)]$, while that of country $j$, i.e. $EX_j (\mathcal{B}_{(i,j)})$, is given by permuting $n_i$ and $n_j$ into $EX_i (\mathcal{B}_{(i,j)})$. It follows that $EX_k (\mathcal{B}_{(i,j)}) = -(EX_i (\mathcal{B}_{(i,j)}) + EX_i (\mathcal{B}_{(i,j)})) = -(\beta^2/25) [5 (n_i + n_j) - 2]$. Substituting these expressions into the welfare functions given by (12), one can obtain

$$W_i (\mathcal{B}_{(i,j)}) = \frac{1}{50} [25\alpha^2 n_i - 5\alpha\beta (11n_i - n_j) + \beta^2 (5n_i - 6n_j + 2)] + n_i L - F, \quad i \neq j,$$

$$W_k (\mathcal{B}_{(i,j)}) = \frac{1}{50} [25\alpha^2 n_k - 5\alpha\beta n_k + \beta^2 (6 - n_k)] + n_k L - F; \quad (24)$$

and $W_j (\mathcal{B}_{(i,j)})$ is obtained by permuting $n_i$ and $n_j$ into $W_i (\mathcal{B}_{(i,j)})$. Calculating the difference between aggregate welfare for the two countries $i$ and $j$ when they form a subcoalition and when they do not, i.e. $(W_i (\mathcal{B}_{(i,j)}) + W_j (\mathcal{B}_{(i,j)})) - (W_i (\mathcal{B}_S) + W_j (\mathcal{B}_S))$ where $W_i (\mathcal{B}_S)$ and $W_j (\mathcal{B}_S)$ are given by (15), we obtain

$$(W_i (\mathcal{B}_{(i,j)}) + W_j (\mathcal{B}_{(i,j)})) - (W_i (\mathcal{B}_S) + W_j (\mathcal{B}_S)) = \left(\frac{2\beta}{50}\right)^2 [3 - 7 (n_i + n_j)]. \quad (25)$$

Therefore, a two-country coalition increases the joint-welfare of the two participating members with respect to the singleton coalition if and only if $n_i + n_j \leq 3/7$. In other words, two countries would collectively benefit from being part of a coalition only if total population size is lower than that of the country outside the coalition. The interpretation of this result is postponed to Section 6.3.1.
6 Coalition Formation

6.1 Preliminaries

We now characterize the outcome of first stage of the game at which each country decides on its membership in a coalition in cognizance of the subsequent stages. A strategy for country $i$ is a choice of a coalition $S_i$ to which $i$ wants to belong. Formally, the set of strategies for country $i$ is given by: $\forall i \in N, \Sigma_i = \{S_i | S_i \in N \text{ and } i \in S_i\}$. A strategy profile is denoted $s = (S_A, S_B, S_C) \in \Sigma$, where $\Sigma$ stand for the set of all strategy profiles (i.e. $\Sigma \equiv \Sigma_A \times \Sigma_B \times \Sigma_C$).

A coalition-structure rule is given by a function, $\Psi : \Sigma \rightarrow B$, that assigns to any $s \in \Sigma$ a coalition structure $B = \Psi(s)$. We restrict attention to the coalition-structure rule which prescribes that a coalition forms if and only if there is unanimity with respect to a partnership plan. For example, if $s = (\{A,B,C\}; \{A,B,C\}; \{C\})$, then no coalition is formed because countries $A$ and $B$ chooses country $C$ as a partner, but country $C$ is not available as a partner. If however $s = (\{A,B\}; \{A,B\}; \{A,B,C\})$, then countries $A$ and $B$ forms a coalition - since they agree on the partnership plan - but country $C$ remains a singleton. We identify a coalition structure $B$ as an equilibrium coalition structure if $B = \Psi(s)$ for an equilibrium strategy profile $s$ of the coalition-formation game.

The equilibrium concept used in this paper is that of coalition-proof Nash equilibrium (CPNE) due to Bernheim, Peleg and Whinston (1987). Roughly, a strategy profile is coalition-proof if it is immune to self-enforcing deviations by any coalition, and a deviation is self-enforcing if there is no further profitable deviation available to a subcoalition of players. In other words, a strategy profile is coalition proof if there does not exist a credible deviation for a subset of countries.

In the following, for expositional felicity, we distinguish between two cases. We first consider the case where the coalition structure $B(i,j)$ does not increase the joint-welfare of the two coalition partners, i.e. $n_i + n_j > 3/7 \forall i,j = A,B,C$ and $i \neq j$ (see eq. (25)). Subsequently, we will consider the situation where there exists a couple of countries $i$ and $j$ such that the coalition structure $B(i,j)$ does increase the joint-welfare of the two coalition partners ($n_i + n_j \leq 3/7$).

6.2 The Grand Coalition $B_G$ Versus the Singleton Coalition $B_S$

When $n_i + n_j > 3/7$, country $i$ or $j$ or both are worse off under the coalition structure $B(i,j)$ than under the singleton coalition $B_S$ and, hence, $B(i,j)$ cannot be supported by a CPNE as we will see in Proposition 1. Hence, in this section, we only determine the preference ordering of the three countries over the two alternatives $B_G$ and $B_S$. Calculating the difference between $W_i (B_G)$ given by (18) and $W_i (B_S)$ given by (15) for each country $i = A, B, C$, we obtain

$$W_i (B_G) - W_i (B_S) = \frac{\alpha}{61} [27\alpha (1 - 3n_i) - 2\beta(15 - 61n_i)], \quad i = A, B, C. \quad (26)$$
To simplify the exposition, let $\hat{\beta}(n_i)$ such that $W_i (B_G) = W_i (B_S)$ for $i = A, B, C$, i.e. $\hat{\beta}(n_i) = (27\alpha/2)(1 - 3n_i)/(15 - 61n_i)$.

We then have the following lemma which in fact holds independently of the country-size distribution.\footnote{All the proofs are in the Appendix.}

**Lemma 1:** The preference ordering for country $i = A, B, C$ over $B_G$ and $B_S$ is: (i) $W_i (B_S) \geq W_i (B_G)$ if $n_i \leq 3/41$ and $\beta \geq \hat{\beta}(n_i)$ or $n_i \geq 1/3$ and $\beta \leq \hat{\beta}(n_i)$; (ii) $W_i (B_G) \geq W_i (B_S)$ for all other configurations of the parameters.

**Corollary 1:** If $n_i \in [3/41, 1/3]$, then country $i$ (for $i = A, B, C$) prefers $B_G$ to $B_S$ independently of $\beta$.

To interpret these results, recall that each country’s welfare can be written as the sum of gross surplus plus the value of exports minus production costs as shown by (12). Under the singleton coalition, the marginal gross consumer surplus is $p (B_S) - \beta$, which is negative since $p (B_S) = \beta/9$, and becomes nil under the grand coalition since $p (B_G) = \beta$. Then, the increase in the market price due to the formation of the grand coalition leads to a decrease in produced and consumed quantities by $8\beta/9$. (Recall that the demand function is $x = \alpha - p$). This in turn increases the gross consumer surplus of each country in proportion to its population size.\footnote{Indeed, the difference in the gross surplus between $B_G$ and $B_S$ for country $i$ is given by $CS_i (B_G) - CS_i (B_S) = (32\beta^2/81) n_i$.}

Now consider the impact of the formation of the grand coalition on the external trade of each country and consider first that $n_i \leq 1/3$. We have seen that, under the singleton coalition $B_S$, a small country sets higher tax rates than larger countries provided that the parameter reflecting the environmental damage is not too high (i.e. $\beta < (9/10)\alpha$). In this case, since the grand coalition sets a common tax rate, the domestic firm of a relatively small country becomes more competitive and then benefits from increased export revenues.\footnote{We indeed have $EX_i (B_G) - EX_i (B_S) = (\beta/27)(9\alpha - 10\beta)(1 - 3n_i)$, which is positive when $\beta < (9/10)\alpha$ and $n_i \leq 1/3$.} Hence, the grand coalition increases both the gross surplus and the value of external trade of a net exporting country (whose relative size is lower than $1/3$) with respect to the singleton coalition.

Suppose now the spillover parameter is very high (i.e. $\beta \geq (9/10)\alpha$). In this case, a relatively small country (i.e. $n_i \leq 1/3$) undercuts larger countries under the singleton coalition and then becomes less competitive under the grand coalition with a common tax rate, thus losing market shares and export revenues. Therefore, for this country to prefer the grand coalition, it must be the case that the gross surplus gain due to the formation of the grand coalition is large enough to compensate losses in export revenues, implying that its population size must be sufficiently important. To be more specific, observe
that one must have $\beta \leq \alpha$ and that $\tilde{\beta}(n_i)$ is increasing in $n_i$ with $\tilde{\beta}(n_i = 3/41) = \alpha$. A sufficient condition for country $i$ to prefers $B_G$ to $B_S$ is then that $n_i \geq 3/41$ (since then $\beta \leq \alpha \leq \tilde{\beta}(n_i)$). In sum, country $i$ prefers to join the grand coalition than to remain a singleton whenever $n_i \in [3/41, 1/3]$ independently of the size of the spillover parameter as stated in corollary 1.

When $n_i \notin [3/41, 1/3]$, country $i$ may also prefer to join the grand coalition than to remain a singleton depending on the extent of the pollution externality, as stated in Lemma 1. Indeed, for country $i$ to prefer to remain a singleton when $n_i \leq 3/41$ (respectively, $n_i \geq 1/3$), it must be the case that the pollution externality is larger (respectively, lower) than $\tilde{\beta}(n_i)$. Consider first that $n_i \leq 3/41$ and $\beta \geq \tilde{\beta}(n_i)$, which implies that $\beta > (9/10)\alpha$. In this case, the significant decrease in consumed quantities resulting from the formation of the grand coalition - because of a high $\beta$ - has a strong negative impact on the external trade (i.e. on exports) of the small country, all the more this country cannot longer undercut larger countries.\footnote{Recall that under $B_S$ a small country undercuts its competitors when $\beta > (9/10)\alpha$.}

This negative impact on the external trade dominates the positive impact on the gross consumer surplus, and so in this case country $i$ prefers $B_S$ to $B_G$.

Consider now that $n_i \geq 1/3$ and $\beta \leq \tilde{\beta}(n_i)$, which implies $\beta < (9/10)\alpha$. In this case, under $B_G$, the large country $i$ cannot longer undercut smaller countries, and consumed quantities do not decrease sufficiently to avoid an increase in import costs. And this increased cost cannot be compensated by the gain in gross surplus when the grand coalition is formed compared to the singleton coalition.

We can now state the following Proposition.

**Proposition 1:** Suppose that $n_i + n_j > 3/7 \forall i, j = A, B, C$ and $i \neq j$. The unique equilibrium coalition structure is: (i) $B_S$ if there is at least one country $i$ for which $n_i \leq 3/41$ and $\beta \geq \tilde{\beta}(n_i)$ or $n_i \geq 1/3$ and $\beta \leq \tilde{\beta}(n_i)$; (ii) $B_G$ in all other cases.

The interpretation of this Proposition and that of Proposition 2 in the next sub-section is postponed to Section 6.4

### 6.3 The Possibility of Subcoalitions $B_{(i,j)}$

Suppose now that there exists a couple of countries $i$ and $j$ such that the coalition structure $B_{(i,j)}$ increases the joint-welfare of the two coalition members with respect to the singleton coalition, i.e. $n_i + n_j \leq 3/7$. It should be noted that this does not necessarily implies that both countries will agree with this partnership plan.

#### 6.3.1 The Coalition $B_{(i,j)}$ versus the Singleton Coalition $B_S$

We first compare for each country its preference ordering over the coalition structures $B_{(i,j)}$ and $B_S$. Using
(15) and (24), the welfare difference for country $i$ between $B_{(i,j)}$ and $B_S$ is given by

$$W_i (B_{(i,j)}) - W_i (B_S) = \frac{\beta}{405} [2\beta (6 + 215n_i - 243n_j) - 405\alpha(n_i - n_j)], \quad i \neq j. \quad (27)$$

The respective welfare difference for country $j$, i.e. $W_j (B_{(i,j)}) - W_j (B_S)$ is given by permuting $n_i$ and $n_j$ into (27).

This welfare difference can be positive or negative depending on the country size asymmetry as well as on the value of the marginal environmental damage. For example, if $\beta$ is relatively small and if $(n_i - n_j)$ is positive and relatively large, then the formation of the subcoalition would be detrimental to the larger country.

To simplify our analysis, we now assume throughout the rest of the paper that $n_i = n_j = n$ implying that $n_k = 1 - 2n$. In this case, the welfare difference for countries $i$ and $j$ when they form a coalition and when they do not is given by

$$W_i (B_{(i,j)}) - W_i (B_S) = W_j (B_{(i,j)}) - W_j (B_S) = 2 \left( \frac{\beta}{405} \right)^2 [3 - 14n]. \quad (28)$$

The welfare difference for the country outside the coalition is given by

$$W_k (B_{(i,j)}) - W_k (B_S) = 28 \left( \frac{\beta}{405} \right)^2 [5 + 2n]. \quad (29)$$

We can now state - directly from (28) and (29) - the following lemma.

**Lemma 2:** Suppose $n_i = n_j = n \leq 3/14$ which implies $n_k = 1 - 2n > 4/7$. Hence, all countries prefer the coalition structure $B_{(i,j)}$ to the singleton coalition $B_s$, independently of the extent of the pollution externality.

When the coalition structure $B_{(i,j)}$ forms, it increases the market price from $p (B_S) = \beta/9$ to $p (B_{(i,j)}) = \beta/5$, reducing total consumed quantities by $4\beta/45$. Again, this leads to an increase in the gross consumer surplus of each country in proportion to its population size, that is $CS_i (B_{(i,j)}) - CS_i (B_S) = 152 (\beta/45)^2 n$ for country $i$ (or $j$), while $n$ is replaced by $(1 - 2n)$ for country $k$.

Now, let evaluate the impact of the formation of the coalition structure $B_{(i,j)}$ on each country’s external trade. As shown before, the value of external trade for country $i$ (or $j$) under $B_S$ is $EX_i (B_S) = (\beta^2/27) (1 - 3n)$, while it amounts to $EX_i (B_{(i,j)}) = (\beta^2/25) (1 - 5n)$ under $B_{(i,j)}$. Calculating the difference, we have $EX_i (B_{(i,j)}) - EX_i (B_S) = -(2\beta^2/675) (30n - 1)$ implying that $EX_k (B_{(i,j)}) - EX_k (B_S) = (4\beta^2/675) (30n - 1)$.

It appears that the formation of the subcoalition has most often a negative impact on the value of the external trade of the two coalition members. This is indeed the case if population size in country $i$ (or
j) is not too small, i.e. \( n \geq 1/30 \). The reason is that the two coalition members set a higher tax rate, and then their domestic firms are less competitive, under the coalition structure \( B_{(i,j)} \) than under the singleton coalition \( B_S \), while the reverse holds for the outsider. Hence, when \( n \geq 1/30 \), the formation of the subcoalition induces for each coalition member a decrease in export revenues and an increase in gross surplus but the latter effect is stronger provided the population size of each coalition member is small enough (i.e. \( n \leq 3/14 \)).

This is somewhat surprising because the gain in gross surplus due to the formation of the subcoalition is increasing in country size. The explanation is the following. The larger the population size of each coalition member, the greater is the impact of the pollution externality on the joint consumer surplus and the higher is the incentive to tax polluting production so as to internalize pollution externalities. Country \( k \) best responds to an increasing tax rate in the coalition by decreasing its own tax rate to take advantage of the increased cost incurred by the firms of the coalition. In fact, the strategic response of country \( k \) is inversely related to its size, as one can infer from (22). It follows that the larger the population sizes of country \( i \) and \( j \) and the lower the size of country \( k \), the greater is the incentive for country \( k \) to reduce its tax rate, and this negative effect on the competitiveness of countries \( i \) and \( j \) overcomes their gains in gross surplus for \( n > 3/14 \).\(^{16}\)

Now, suppose that the population sizes of country \( i \) and \( j \) are very small, i.e. \( n < 1/30 < 3/14 \). In this case, the two coalition partners record an increase in export revenues following the formation of the subcoalition. Intuitively, when country-size is very small, export quantities are sufficiently large for the decrease in export quantities to be compensated by the increase in the export price. Since, coalition members also register an increase in gross surplus, they unambiguously prefer to form a subcoalition than to remain singletons.

Now consider the impact of the formation of the coalition on the value of external trade of the country that remains outside the coalition, i.e. country \( k \). If \( n < 1/30 \), then country \( k \) experiences an increase in import costs but it also experiences a huge gain in gross surplus since country size is very large \((1 - 2n > 14/15)\). If \( n \geq 1/30 \), then the formation of the coalition between countries \( i \) and \( j \) leads to both a decrease in import costs and to an increase in gross surplus for country \( k \). As a result, country \( k \) always benefit from the formation of the coalition structure \( B_{(i,j)} \).

### 6.3.2 Equilibrium Coalition Structures

We need to determine each country’s preference ordering over the three coalition structures. First, Lemma

\[^{16}\text{We indeed have } [CS_i (B_{(i,j)}) - CS_i (B_S)] + [EX_i (B_{(i,j)}) - EX_i (B_S)] = 2 \left( \frac{d}{3\pi} \right)^2 (3 - 14n), \text{ which is negative for } n > 3/14.\]
1 determines each country’s preference ordering over \( \mathcal{B}_G \) and \( \mathcal{B}_S \) independently of the country-size distribution, and so this lemma remains valid when \( n_i = n_j = n \leq 3/14 \) and \( n_k = 1 - 2n > 4/7 \). Second, Lemma 2 states that when \( n_i = n_j = n \leq 3/14 \) all countries prefer the coalition structure \( \mathcal{B}_{(i,j)} \) to the singleton coalition \( \mathcal{B}_S \). Therefore, we just need to calculate the welfare difference between \( \mathcal{B}_{(i,j)} \) and \( \mathcal{B}_G \) for each country.

When countries \( i \) and \( j \) form a subcoalition and \( n_i = n_j = n \), the welfare level of each country is given (from (24)) by

\[
W_i(\mathcal{B}_{(i,j)}) = W_j(\mathcal{B}_{(i,j)}) = \frac{1}{50} \left[ 25\alpha(\alpha - 2\beta)n + \beta^2(2 - n) \right] + nL - F,
\]

\[
W_k(\mathcal{B}_{(i,j)}) = \frac{1}{50} \left[ 25\alpha(\alpha - 2\beta)(1 - 2n) + \beta^2(5 + 2n) \right] + (1 - 2n)L - F. \quad (30)
\]

Using (18) and (30), the welfare difference between \( \mathcal{B}_G \) and \( \mathcal{B}_{(i,j)} \) for country \( i \) (or \( j \)) is thus given by

\[
W_i(\mathcal{B}_G) - W_i(\mathcal{B}_{(i,j)}) = W_j(\mathcal{B}_G) - W_j(\mathcal{B}_{(i,j)}) = \frac{2}{5} \left[ 25\alpha(1 - 3n) - 2\beta(14 - 57n) \right], \quad (31)
\]

while the difference in welfare between \( \mathcal{B}_G \) and \( \mathcal{B}_{(i,j)} \) for country \( k \) is given by

\[
W_k(\mathcal{B}_G) - W_k(\mathcal{B}_{(i,j)}) = \frac{2}{5} \left[ -25\alpha(1 - 3n) + 2\beta(20 - 57n) \right]. \quad (32)
\]

To simplify the exposition of the results, let \( \hat{\beta}_{ij}(n) \) such that \( W_i(\mathcal{B}_{(i,j)}) = W_i(\mathcal{B}_{(ij)}) \), i.e. \( \hat{\beta}_{ij}(n) = (25\alpha/2)(1 - 3n)/(14 - 57n) \), and \( \hat{\beta}_{k}(n) \) such that \( W_k(\mathcal{B}_G) = W_k(\mathcal{B}_{(i,j)}) \) i.e. \( \hat{\beta}_{k}(n) = (25\alpha/2)(1 - 3n)/(20 - 57n) \).

Observe that \( \hat{\beta}(n) > \hat{\beta}_{ij}(n) > \hat{\beta}_{k}(n) > \hat{\beta}(1 - 2n) \) \( \forall n \in [0, 3/14] \), where \( \hat{\beta}(n) \) (or \( \hat{\beta}(1 - 2n) \)) is given just above Lemma 1.

We first determine the preference ordering for countries \( i \) and \( j \). We have

**Lemma 3:** Suppose \( n_i = n_j = n \leq 3/14 \). Then, the preference ordering for country \( i \) - or country \( j \) - is: (i) \( W_i(\mathcal{B}_{(i,j)}) \geq W_i(\mathcal{B}_G) \geq W_i(\mathcal{B}_S) \) if \( n \leq 1/13 \) and \( \hat{\beta}_{ij}(n) \leq \beta \leq \hat{\beta}(n) \); (ii) \( W_i(\mathcal{B}_{(i,j)}) \geq W_i(\mathcal{B}_G) \geq W_i(\mathcal{B}_S) \) if \( \beta \geq \hat{\beta}(n) \) (implying \( n \leq 3/41 \)); (iii) \( W_i(\mathcal{B}_G) \geq W_i(\mathcal{B}_{(i,j)}) \geq W_i(\mathcal{B}_S) \) if \( n \leq 1/13 \) and \( \beta \leq \hat{\beta}_{ij}(n) \) or if \( n \geq 1/13 \).

The following lemma describes the preference ordering for country \( k \).

**Lemma 4:** Suppose \( n_i = n_j = n \leq 3/14 \) implying \( n_k = 1 - 2n > 4/7 \). Then, the preference ordering for country \( k \) is: (i) \( W_k(\mathcal{B}_{(i,j)}) \geq W_k(\mathcal{B}_G) \geq W_k(\mathcal{B}_S) \) if \( \hat{\beta}_{k}(n) \geq \beta \geq \hat{\beta}(1 - 2n) \); (ii) \( W_k(\mathcal{B}_{(i,j)}) \geq W_k(\mathcal{B}_G) \geq W_k(\mathcal{B}_S) \) if \( \beta \leq \hat{\beta}(1 - 2n) \); (iii) \( W_k(\mathcal{B}_G) \geq W_k(\mathcal{B}_{(i,j)}) \geq W_k(\mathcal{B}_S) \) if \( \beta \geq \hat{\beta}_{k}(n) \).

Again, we need to focus only on the preference ordering over \( \mathcal{B}_{(i,j)} \) and \( \mathcal{B}_G \) for each country. We mention
first that the market price under the coalition structure $B_{(i,j)}$, i.e. $p(B_{(i,j)}) = \beta/5$, is lower than that under the grand coalition where it is equal to the marginal cost of pollution, i.e. $p(B_G) = \beta$. This implies that total quantities produced and consumed are lower by $4\beta/5$ under the grand coalition compared to the coalition structure $B_{(i,j)}$, which leads to an increase in the gross surplus of each country in proportion to its population size.$^{17}$

Now, let evaluate the difference in the value of external trade between the two coalition structures $B_{(i,j)}$ and $B_G$ for each country, starting with country $k$. Again, when countries $i$ and $j$ form a coalition, country $k$ undercuts the tax rate of the coalition, i.e. $t_k(B_{(i,j)}) < t_{ij}(B_{(i,j)})$ for any $n_k > 4/7$, as one can infer from (23). Now, if country $k$ joins the grand coalition and sets the same tax rate than its partners, it becomes less competitive and then may suffer from increased import costs. We indeed have $EX_k(B_G) - EX_k(B_{(i,j)}) = -(2\beta/75)[25\alpha(1 - 3n) - 2\beta(14 - 45n)]$, which is negative for any $\beta \leq \tilde{\beta}(n)$ with $\tilde{\beta}(n) = (25\alpha/2)(1 - 3n)/(14 - 45n)$. If the pollution externality is even lower than $\tilde{\beta}(n)$, i.e. $\beta \leq \hat{\beta}_k(n) < \tilde{\beta}(n)$, then the increase in gross surplus following the accession to the grand coalition is not sufficient to compensate the increased import costs and, hence, country $k$ prefers to be an outsider under the coalition structure $B_{(i,j)}$ than to join the grand coalition (as stated in (i) and (ii) of Lemma 4). As $\beta$ increases it becomes more profitable for country $k$ to join the grand coalition. Indeed, the greater $\beta$ the lower are consumed quantities under $B_G$ compared to $B_{(i,j)}$, which alleviates the increased import costs and further contributes to the increase in gross surplus. Hence, when the spillover parameter is sufficiently large (i.e. $\beta \geq \hat{\beta}_k(n)$), preferences of country $k$ are reversed and it prefers to join the grand coalition than to be an outsider (as stated in (iii) of Lemma 4). If $\beta$ further increases and becomes larger than $\tilde{\beta}(n)$, country $k$ experiences both a large increase in gross surplus and a decrease in import costs by acceding to the grand coalition.

We now consider the preference ordering between $B_{(i,j)}$ and $B_G$ for country $i$ or $j$. The difference in the value of external trade between $B_{(i,j)}$ and $B_G$ for country $i$, or $j$, is $EX_i(B_G) - EX_i(B_{(i,j)}) = (\beta/75)[25\alpha(1 - 3n) - 2\beta(14 - 45n)]$, which is negative for any $\beta \geq \tilde{\beta}(n)$. The argument is symmetric to that for country $k$. When the pollution externality is relatively large, the decrease in produced and consumed quantities due to the formation of the grand coalition is sufficiently large for countries $i$ and $j$ to register a decrease in export revenues even though country $k$ can no longer undercut the tax rate of countries $i$ and $j$. If the pollution externality is even larger than $\tilde{\beta}(n)$, i.e. $\beta \geq \tilde{\beta}_{ij}(n) > \tilde{\beta}(n)$, the decrease in export revenues cannot be compensated by the increase in gross surplus, and then countries $i$ and $j$ prefer the coalition structure $B_{(i,j)}$ to the grand coalition, as stated in (i) and (ii) of Lemma 3.

$^{17}$We indeed have $CS_i(B_G) - CS_i(B_{(i,j)}) = (8\beta^2/25)n$ for country $i$ (or $j$) while for country $k$, $n$ must be replaced by $(1 - 2n)$. 
In fact, one can observe that $\tilde{\beta}_{ij}(n)$ is increasing in $n$ and that it is equal to $\alpha$ in $n = 1/13$. Since $\beta \leq \alpha$, $\beta \geq \tilde{\beta}_{ij}(n)$ can hold only if $n \leq 1/13$ - as stated in (i) of Lemma 3 - so that the formation of the grand coalition has a limited impact on the gross consumer surplus of country $i$ (or $j$), due to its small size. If $\beta \geq \tilde{\beta}(n)$, then $\beta \geq \tilde{\beta}_{ij}(n)$ and countries $i$ and $j$ still prefer $B_{(i,j)}$ to $B_G$, but this implies that the population size of each coalition partner is even lower. This is because $\tilde{\beta}(n)$ is increasing in $n$ and reaches $\alpha$ in $n = 3/41$, and so $\beta \geq \tilde{\beta}(n)$ can hold only if $n \leq 3/41 < 1/13$, as stated in (ii) of Lemma 3. For such a small country, the positive effect on consumer surplus due to the formation of $B_G$ is even more limited.

Now, when the pollution externality decreases below $\beta \leq \tilde{\beta}_{ij}(n)$, then the negative impact on export revenues due to the formation of the grand coalition is small enough to be compensated by the increase in gross surplus. In fact, if $\beta$ is even lower than $\tilde{\beta}_{ij}(n)$, i.e. $\beta \leq \tilde{\beta}(n) < \tilde{\beta}_{ij}(n)$, the formation of the grand coalition leads to an increase in export revenues for countries $i$ and $j$. Therefore, for $\beta \leq \tilde{\beta}_{ij}(n)$, countries $i$ and $j$ prefer $B_G$ to $B_{(i,j)}$, as stated in (iii) of Lemma 3. This is also the case if population size in country $i$ (or $j$) is large enough, i.e. $n \geq 1/13$.\footnote{Again, when $n \geq 1/13$, we have $\tilde{\beta}_{ij}(n) > \alpha$ and so $\beta \leq \tilde{\beta}_{ij}(n)$ since $\beta \leq \alpha$}

We can now state the following Proposition.

**Proposition 2:** Suppose $n_i = n_j = n \leq 3/14$ implying $n_k = 1 - 2n > 4/7$. The unique equilibrium coalition structure is: (i) $B_{(i,j)}$ if $\beta \leq \tilde{\beta}(n)$ or if $\beta \geq \tilde{\beta}_{ij}(n)$; (ii) $B_G$ if $\tilde{\beta}_k(n) \leq \beta \leq \tilde{\beta}_{ij}(n)$.

We now turn to the interpretation of Propositions 1 and 2 on the equilibrium outcome of the coalition formation game.

### 6.4 Interpretation of the results

To interpret Propositions 1 and 2 together, we consider increasing values of the parameter $\beta$ reflecting the pollution externality. Suppose first that the environmental damage due to production of $X$ is ‘very’ low. In this case, the larger country (i.e. country $k$) undercuts countries $i$ and $j$ under the singleton coalition $B_S$. Therefore, the formation of a grand coalition $B_G$ with a common tax rate, results in a loss (gain) of competitiveness for country $k$ (country $i$ or $j$). Moreover, since the spillover parameter is relatively low, the environmental policy under $B_G$ slightly decreases produced and consumed quantities compared to $B_S$.

In turn, for the larger country $k$, the significant increase in import costs cannot compensate the moderate gain in gross consumer surplus. Countries $i$ and $j$ would like to form the grand coalition than to remain singletons because $B_G$ increases both their gross surplus and their export revenues, but country $k$ is not available as a partner when $\beta$ is low.
As the spillover parameter increases, the grand coalition sets a more stringent environmental tax policy, and so produced and consumed quantities decrease more significantly. For the larger country, the formation of the grand coalition is then less costly in terms of imports costs, while the gain in gross surplus becomes more important. There is thus a threshold value of $\beta$, i.e. $\hat{\beta} (1 - 2n)$, above which the larger country becomes available as a partner in the grand coalition, provided countries $i$ and $j$ have no interest in forming a subcoalition (i.e. $n > 3/14$).

If, however, the coalition structure $B_{(i,j)}$ is welfare-improving for countries $i$ and $j$, then country $k$ may prefer to remain a singleton even though $\beta \geq \hat{\beta} (1 - 2n)$. The reason is that the coalition structure $B_{(i,j)}$ gives rise to a strategic advantage for country $k$. Indeed, recall that under the coalition structure $B_{(i,j)}$, the two coalition members set a higher tax rate than under $B_S$ so as to internalize the pollution externality and the market externality between the domestic firms. In turn, the larger country undercuts the coalition to take advantage of the increased costs incurred by the firms of the coalition and to make its own firm more competitive on international markets. As a result, for the larger country to resign its competitive advantage and to join the grand coalition, it requires greater benefits from internalizing the pollution externality and hence a higher $\beta$, i.e. $\beta \geq \hat{\beta} (1 - 2n)$. Indeed, when $\beta \geq \hat{\beta}_k(n)$, the decrease in produced and consumed quantities induced by $B_G$ as compared to $B_{(i,j)}$ - implying an increase in gross surplus and a slight increase in import costs - is large enough for the larger country to most prefer the grand coalition. As the spillover parameter further increases and becomes larger than $\tilde{\beta}(n)$ (with $\tilde{\beta}(n) > \tilde{\beta}_k(n)$), the larger country records both an increase in gross surplus and a decrease in import costs, although it resigned its competitive advantage.

Now let consider again that countries $i$ and $j$ do not have interest in forming a subcoalition. In this case, as just mentioned above, country $k$ agrees to form the grand coalition for any $\beta \geq \hat{\beta} (1 - 2n)$. Countries $i$ and $j$ also agree to join the grand coalition except if the spillover parameter is 'very' large. Indeed, recall that in that case, smaller countries undercut the larger country under the singleton coalition $B_S$. It follows that the formation of the grand coalition implies, for those countries, a decrease in competitiveness and export revenues, especially since produced and consumed quantities sharply decrease as a result of a large value of $\beta$. As the spillover parameter decreases, this negative impact on the external trade of countries $i$ and $j$ becomes less important compared to the gain gross surplus. There is thus a threshold value of $\beta$, i.e. $\hat{\beta}(n)$, below which countries $i$ and $j$ agree to form the grand coalition. Again, a sufficient condition for $\beta \leq \hat{\beta}(n)$ to be satisfied is that $n \geq 3/41$, which is rather intuitive because the benefit in gross surplus due to the formation of the grand coalition is increasing in group size.

Now, let consider that countries $i$ and $j$ prefer the coalition structure $B_{(i,j)}$ to the singleton coalition $B_S$ (i.e. $n \leq 3/14$). In this case - for countries $i$ and $j$ to join country $k$ in the grand coalition - the
environmental pollution externality must be even lower than $\hat{\beta}(n)$, i.e. $\beta \leq \hat{\beta}_{ij}(n) < \hat{\beta}(n)$. In other words, when $\beta > \hat{\beta}_{ij}(n)$, countries $i$ and $j$ are better off by letting country $k$ stay outside the coalition and free-ride on them. The explanation is the following. First, recall that $\beta > \hat{\beta}_{ij}(n)$ necessarily implies that $n < 1/13$. So, if the population size of the subcoalition is relatively small, $B_G$ has a limited positive impact on the gross consumer surplus of each coalition member and, if the environmental damage is relatively high, $B_G$ has a strong impact on their external trade. Indeed, the greater $\beta$ the lower are produced and consumed quantities under $B_G$ compared to $B_{(i,j)}$.

Obviously, under $B_G$, the common pollution tax rate implies that all firms have equal market shares, whereas letting country $k$ be an outsider (under $B_{(i,j)}$) implies a competitive framework. Indeed, we have seen that the formation of the subcoalition gives rise to a strategic response of the outsider that decreases its own tax rate to increase domestic production. However, for the outsider, increasing production also reduces the gross surplus especially when both $\beta$ and population size are large. Hence, for high values of $\beta$ and $n_k$, country $k$ is not willing to substantially reduce its tax rate so that it captures a moderate competitive advantage over the two coalition members. As a result, in this case, countries $i$ and $j$ sign up together without country $k$. The resulting coalition structure $B_{(i,j)}$, when $\beta > \hat{\beta}_{ij}(n)$, avoids the strong decrease in export markets that would arise with the grand coalition $B_G$, while the losses in market shares (for countries $i$ and $j$) remain moderate since the tax policy of country $k$ is slightly aggressive due to its large size.

In conclusion, when the coalition structure $B_{(i,j)}$ is a profitable alternative for countries $i$ and $j$ to the singleton coalition $B_S$, the parameter reflecting the pollution externality must lie within a restricted range (i.e. $[\hat{\beta}_k(n),\hat{\beta}_{ij}(n)]$) for the grand coalition $B_G$ to be the (unique) equilibrium coalition structure.

7 Conclusion

The main conclusion of this paper is that a multi-step process is less likely to give rise to a global international environmental agreement than a one-step process. Indeed, once a first group of countries sign an IEA, it modifies the incentives of all countries to sign a global agreement and, actually, it makes them more demanding than if a preliminary restricted agreement was not signed. In addition, the further enlargement of the initial coalition may be blocked, not only by the outsider(s), but also by the insiders depending on the size of the pollution externality and on the country-size asymmetry. Overall, the grand coalition is less likely to emerge when subcoalitions are profitable which implies – in the context of our model – a strong country-size asymmetry. To deal with this asymmetry problem, this suggests that the European Union should speak with one voice with the other big producers-polluters (such as US, China or India) to push forward a (real) global environmental agreement. Admittedly, one reason for the failure of
the Copenhagen climate change summit in December 2009 was that Europe spoke with many and, often, controversial voices.

The simplicity of the framework analyzed in this paper is attractive but might be criticized on several fronts. For example, we assumed that the marginal environmental damage caused by production is the same in each country. This assumption seems reasonable to analyze strategic interactions amongst advanced industrial countries. But it is less convincing to analyze strategic interactions between, for example, countries of the EU and emerging and developing countries. More importantly, we assumed that countries decide on whether to join a coalition once and for all and furthermore that side payments are not allowed. Repeated interactions and the possibility to make (or receive) transfers would certainly modify the incentives to sign an IEA, but not necessarily in the expected direction.\(^{19}\) Clearly, the paper leaves questions that need to be addressed in future research.

8 Appendix

8.1 Proof of Lemma 1

We have \(W_i(B_S) \geq W_i(B_G)\) whenever \(\Delta_i(n_i, \alpha, \beta) \equiv 27\alpha(1 - 3n_i) - 2\beta(15 - 61n_i) \leq 0\). Suppose first that \((15 - 61n_i) \geq 0\) or \(n_i \leq 15/61 < 1/3\). Then \(\Delta_i(n_i, \alpha, \beta) \leq 0\) for any \(\beta \geq \tilde{\beta}(n_i) \equiv (27\alpha/2)(1 - 3n_i)/(15 - 61n_i)\). \(\tilde{\beta}(n_i)\) is increasing in \(n_i\) and is equal to \(\alpha\) in \(n_i = 3/41\). Yet, one must have \(\beta \leq \alpha\). Therefore, when \(n_i \leq 15/61\), \(\beta \geq \tilde{\beta}(n_i)\) can be satisfied only if \(n_i \leq 3/41 < 15/61\). Suppose now that \((15 - 61n_i) \leq 0\) or \(n_i \geq 15/61\). In this case, \(\Delta_i(n_i, \alpha, \beta) \leq 0\) for any \(\beta \leq \tilde{\beta}(n_i)\). \(\tilde{\beta}(n_i)\) is negative for any \(n_i \in [15/61, 1/3]\) and becomes positive from \(n_i = 1/3\). Since \(\beta \geq 0\), \(\beta \leq \tilde{\beta}(n_i)\) for \(n_i \geq 15/61\) can be satisfied only if \(n_i \geq 1/3 > 15/61\). Corollary 1 directly follows from this.

8.2 Proof of Lemma 3

(i) \(W_i(B_{i,j}) \geq W_i(B_G)\) whenever \(25\alpha(1 - 3n) - 2\beta(14 - 57n) \leq 0\). One have \(n \leq 3/14 < 14/57\). Hence \(14 - 57n > 0\) and \(W_i(B_{i,j}) \geq W_i(B_G)\) for any \(\beta \geq \beta_{ij}(n) \equiv (25\alpha/2)(1 - 3n)/(14 - 57n)\). \(\beta_{ij}(n)\) is increasing in \(n\) and is equal to \(\alpha\) in \(n = 1/13\). Yet, one must have \(\beta \leq \alpha\). Therefore, \(\beta \geq \beta_{ij}(n)\) can be satisfied only if \(n \leq 1/13 < 3/14\). From Corollary 1, we also have \(W_i(B_G) \geq W_i(B_S)\) for any \(n \in [3/41, 1/13]\). If now \(n < 3/41\), then from Lemma 1, \(W_i(B_G) \geq W_i(B_S)\) when \(\beta \leq \tilde{\beta}(n)\).

(ii) From Lemma 2, \(W_i(B_{i,j}) \geq W_i(B_S)\) independently of \(\beta\) when \(n \leq 3/14\). Now using Lemma 1, we have that \(W_i(B_S) \geq W_i(B_G)\) when \(\beta \geq \tilde{\beta}(n)\), which implies \(n \leq 3/41\).

(iii) From (i) just above, \(W_i(B_G) \geq W_i(B_{i,j})\) if \(n \leq 1/13\) and \(\beta \leq \beta_{ij}(n)\) or if \(n \geq 1/13\) since, in

\(^{19}\)For example, Hoel and Schneider (1997) in a reduced-form model show that side payments may substantially reduce the incentives to join an IEA.
this latter case, $\beta \leq \beta_{ij}(n)$ is always satisfied. From Lemma 2, we also have $W_i(B_{(i,j)}) \geq W_i(B_S)$ since $n \leq 3/14$.

### 8.3 Proof of Lemma 4

(i) $W_k(B_{(i,j)}) \geq W_k(B_G)$ whenever $-25\alpha(1-3n) + 2\beta(20-57n) \leq 0$. One have $n \leq 3/14 < 20/57$. Hence $20 - 57n > 0$ and $W_k(B_{(i,j)}) \geq W_k(B_G)$ for any $\beta \leq \beta_k(n) \equiv (25\alpha/2)(1-3n)/(20-57n)$. One must also verify that $\beta_k(n) \geq 0$, which is indeed the case for any $n \leq 3/14$. This is because $\beta_k(n)$ is decreasing in $n$ and is equal to $(125/218)\alpha > 0$ in $n = 3/14$, implying that $\beta_k(n)$ is positive for any $n \leq 3/14$. Now using Lemma 1, when $n_k = 1 - 2n > 4/7 > 1/3$, we have $W_k(B_G) \geq W_k(B_S)$ for $\beta \geq \tilde{\beta}(1-2n)$.

(ii) From Lemma 2, $W_k(B_{(i,j)}) \geq W_k(B_S)$ independently of $\beta$ when $n \leq 3/14$. From (i) just above, $W_k(B_S) \geq W_k(B_G)$ if $\beta \leq \tilde{\beta}(1-2n)$.

(iii) From (i) just above, $W_k(B_G) \geq W_k(B_{(i,j)})$ if $\beta \geq \beta_k(n)$ and, again, from Lemma 2 we also have $W_k(B_{(i,j)}) \geq W_k(B_S)$ since $n \leq 3/14$.

### 8.4 Proof of Proposition 1

From (25), when $n_i + n_j > 3/7 \forall i,j = A,B,C$ and $i \neq j$, we must have at least one country - say country $i$ - for which $W_i(B_{(i,j)}) < W_i(B_S)$. This implies that $B_{(i,j)}$ cannot be an equilibrium coalition structure. Indeed, for $B_{(i,j)}$ to be an equilibrium coalition structure, the strategy profile $s^* = (S^*_i, S^*_j, S^*_k)$ with $S^*_i = S^*_j = \{i,j\}$ must be a CPNE. But when $W_i(B_{(i,j)}) < W_i(B_S)$, country $i$ has a profitable deviation to $S_i = \{i\}$, giving rise to the singleton coalition $B_S$. Therefore, there are only two coalition structures that are equilibrium candidates in the first stage of the game: $B_G$ and $B_S$.

For the grand coalition to be an equilibrium structure, it must be the case that $\hat{s} = (\hat{S}_i, \hat{S}_j, \hat{S}_k)$ with $\hat{S}_i = \hat{S}_j = \hat{S}_k = \{i,j,k\}$ is a CPNE. Therefore, if there is one country - say country $i$ - for which $n_i \leq 3/41$ and $\beta \geq \tilde{\beta}(n_i)$ or $n_i \geq 1/3$ and $\beta \leq \tilde{\beta}(n_i)$, then this country prefers (from Lemma 1) to remain a singleton than to join the grand coalition. This country would then deviate from the strategy profile $\hat{s}$ to $S_i = \{i\}$, thus giving rise to $\Psi(S_i, \hat{S}_j, \hat{S}_k) = B_S$. Such a deviation is immune to further profitable joint deviation - here by countries $j$ and $k$ - because $n_j + n_k > 3/7$ which implies that country $j$ (or $k$) prefers to be a singleton than to form a two-country coalition. Finally, unilateral deviation by country $j$ or country $k$ would have no effect on the resultant coalition structure $B_S$. Therefore, $B_S$ is the unique equilibrium coalition structure. In all other parameter configurations - i.e. $n_i \in [3/41, 1/3]$, or $n_i \leq 3/41$ and $\beta \leq \tilde{\beta}(n_i)$ or $n_i \geq 1/3$ and $\beta \geq \tilde{\beta}(n_i)$ - all countries prefer $B_G$ to $B_S$ and at least one country ($i$ or $j$) is worse off under $B_{(i,j)}$ than under $B_S$. Therefore, $\hat{s}$ being immune to any deviation - unilateral or multilateral - it is the unique CPNE, resulting in the formation of the grand coalition $B_G$.
8.5 Proof of Proposition 2

Consider first that $\beta \leq 2 (1-2n) < \hat{\beta}_k(n)$. Then Lemma 4 implies for country $k$ that $W_k(B_{(i,j)}) \geq W_k(B_S)$, while Lemma 3 implies for countries $i$ and $j$ that $W_i(B_G) \geq W_i(B_{(i,j)}) \geq W_i(B_S)$. In this case any strategy profile $s^* = (S^*_i, S^*_j, S^*_k)$ with $S^*_i = S^*_j = \{i,j\}$ is a CPNE and gives rise the coalition structure $B_{(i,j)}$ regardless of $S^*_k$. Countries $i$ and $j$ prefer the grand coalition $B_G$ to the coalition structure $B_{(i,j)}$, but country $k$ prefers to remain a singleton than to join the grand coalition. Hence, $\hat{s} = (\hat{S}_i, \hat{S}_j, \hat{S}_k)$ with $\hat{S}_i = \hat{S}_j = \{i,j,k\}$ - which is the unique strategy profile giving rise to $B_G$ - is not a CPNE because country $k$ has an incentive to deviate from $\hat{s}$ to $S_k = \{k\}$. Therefore, countries $i$ and $j$ do not have any incentive to deviate from $s^*$ because any deviation - unilateral or multilateral from $S^*_i = S^*_j = \{i,j\}$ - would result in the singleton coalition which is dominated by $B_{(i,j)}$ for countries $i$ and $j$. Finally, a unilateral deviation by country $k$ from $s^*$ has no effect on the resultant coalition structure. Therefore, if $\beta \leq 2 (1-2n)$, the unique equilibrium coalition structure is $B_{(i,j)}$.

Consider now that $2 (1-2n) < \beta \leq \hat{\beta}_k(n)$. Then Lemma 4 implies for country $k$ that $W_k(B_{(i,j)}) \geq W_k(B_G) \geq W_k(B_S)$, while Lemma 3 implies for countries $i$ and $j$ that $W_i(B_G) \geq W_i(B_{(i,j)}) \geq W_i(B_S)$. The coalition structure $B_S$ cannot be supported by a CPNE since countries $i$ and $j$ have a profitable joint deviation to $S^*_i = S^*_j = \{i,j\}$ giving rise to the coalition structure $B_{(i,j)}$, irrespective of country $k$’s choice. In fact, the coalition structure $B_{(i,j)}$ can result from four strategy profiles: $s = (\{i,j\}, \{i,j\}, \{k\})$, $s = (\{i,j\}, \{i,j\}, \{k\})$, $s = (\{i,j\}, \{i,j\}, \{j,k\})$ or $s = (\{i,j\}, \{i,j\}, \{i,j,k\})$. The first three strategy profiles constitute a CPNE. However, the strategy profile $s = (\{i,j\}, \{i,j\}, \{i,j,k\})$ is not a CPNE because countries $i$ and $j$ have a joint profitable deviation to $S_i = S_j = \{i,j,k\}$ so as to perform the grand coalition what gives them a higher welfare than $B_{(i,j)}$. But the resulting strategy profile $\hat{s} = (\{i,j,k\}, \{i,j,k\}, \{i,j,k\})$ is not a CPNE because country $k$ would have a profitable deviation to any other strategy than $S_k = \{i,j,k\}$ so as to induce the coalition structure $B_{(i,j)}$, which is most preferred by country $k$. Indeed, a unilateral deviation from $\hat{s}$ by country $k$ would lead to the singleton coalition $B_S$, which in turn would lead to a further joint deviation by countries $i$ and $j$ to $S^*_i = S^*_j = \{i,j\}$. This is because $S^*_i = S^*_j = \{i,j\}$ leads to the coalition structure $B_{(i,j)}$ that is preferred by both countries $i$ and $j$ to the singleton coalition $B_S$. To summarize, for any $\beta \leq \hat{\beta}_k(n)$, the unique equilibrium coalition structure is $B_{(i,j)}$.

This is also the case when $\beta \geq \hat{\beta}_i(n)$. Indeed, in this case, $B_G$ is the most preferred coalition structure by country $k$ (Lemma 4). However, from Lemma 3, countries $i$ and $j$ prefer the coalition structure $B_{(i,j)}$ to $B_G$ so that they will not accept country $k$ as a coalition partner. Since countries $i$ and $j$ also prefer $B_{(i,j)}$ to the singleton coalition, the unique coalition structure supported by a CPNE is again $B_{(i,j)}$.

To summarize, $B_{(i,j)}$ is the unique equilibrium coalition structure for any $\beta \not\in [\hat{\beta}_k(n), \hat{\beta}_i(n)]$, as stated
in (i) of Proposition 2.

(ii) If \( \hat{\beta}_k(n) \leq \beta \leq \hat{\beta}_{ij}(n) \), then by Lemma 3 and 4, we have that the grand coalition structure \( B_G \) is preferred to both \( B_{(i,j)} \) and \( B_S \) by all three countries. In other words, there is unanimity to form the grand coalition \( B_G \) and hence \( \hat{s} = (\hat{S}_i, \hat{S}_j, \hat{S}_k) \) is the unique CPNE which then leads to \( B_G \), as stated in (ii) of Proposition 2.

References


