Optimal policies to preserve tropical forests Preliminary draft

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Abstract

We develop a North/South growth model to investigate the normative question of international transfers to preserve tropical forests. The South converts forest lands into agricultural lands to produce a final consumption good which is consumed by the North. We consider two ways of incorporating the externality coming from deforestation in the utility of the North: i) through an amenity value, which reflects a continuous willingness to preserve forests, or ii) through a minimum stock of tropical forests to be preserved. First, we solve the optimal program and we characterize the dynamic properties of the optimal deforestation regime. Second, we derive the decentralized equilibrium outcome in which the externality can be corrected by a transfer scheme from the North to the South. This transfer function depends both on the stock of forest (positively) and on the deforestation rate (negatively). Third, we characterize the dynamic implementation rule which must be satisfied by this transfer to restore the optimum. Finally, we analyze the specific cases of a pure forest subsidy policy and a pure deforestation taxation policy and we illustrate these findings with an analytical specified model.

Keywords: REDD mechanism, deforestation, dynamics, optimal regulation, tax, subsidy.

JEL classifications: Q23, Q27, Q28.

1 Introduction

Reducing emissions from deforestation and forest degradation (REDD) in developing countries has been recently at the center of negotiations on a renewed climate regime. Deforestation in tropical countries is responsible for about a quarter of total world carbon emissions

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and represent the main source of emissions in some developing countries. Reducing deforestation could offer both environmental gains as a mitigation option and an opportunity to integrate developing countries into the international negotiations on climate change. An official agreement has been recently achieved on the REDD (or REDD+) mechanism during the UNFCCC conference of the Parties in Cancun, in December 2010, as regards its scope, eligible activities, safeguards, and methodological issues.[1] Agreeing on safeguards represent a major achievement due to complex governance and institutional issues, that includes respect for the rights of Indigenous Peoples and local communities, forest governance, and actions that are consistent with conservation of natural forests and biodiversity. To be part of the REDD mechanism, countries need to undertake activities according to a phased approach. In the first phase called readiness, countries develop national strategies, assess national reference emission levels, and implement a robust and transparent national forest monitoring systems. In the second phase, countries implement their national policies aiming at preserving tropical forests. In the third phase, countries can receive performance-based incentive payments, that is, payments for verified emissions reductions. Designing the performance-based incentive scheme is one of the important challenges of the REDD mechanism, but it also offers better prospects for realizing forest preservation outcomes than most previous forest policy interventions (Pfaff et al., 2010).

Since REDD projects are mostly in their infancy, result-based incentive schemes have hardly been implemented, apart from the large bilateral programs of Norway in Brazil, Indonesia, Guyana and Tanzania. As illustrated by the Amazon Fund, donations (USD 1 billion) are linked directly to results, i.e. to emission trends.[2] More precisely, payments in a particular year will depend on the difference between emissions from deforestation in the previous year and a reference level, which is the average for the current ten-year calculation period, and which is updated every five years. If emissions in a particular year are higher than the reference level, no payment will be made to the Amazon fund in the subsequent year. Hence, this scheme rests on ex-post evaluation of the emissions due to deforestation, and respects national sovereignty since the fund is managed by the Brazilian Development Bank (BNDES). However, the incentive scheme that performs well for Brazil

[1] See www.un-redd.org. REDD stands for Reducing emissions from deforestation and degradation, whereas REDD+, in addition to REDD, includes enhancing forest carbon stocks through activities such as forest conservation, forest restoration and sustainable forest management (Angelsen et al., 2009; Kanowski et al., 2010).

[2] See Norwegian initiative on http://www.regjeringen.no/en/dep/md/Selected-topics/climate/the-government-of-norways-international-/norway-amazon-fund.html; and more details on the Amazon Fun on http://www.amazonfund.gov.br/FundoAmazonia/fam/site_en/

is unlikely to suit Guyana, where deforestation hardly occurs despite the large tropical forest. In this context, Norway has offered up to USD 250 millions for preserving the stock of forest in Guyana. The design of the incentive scheme is crucial to avoid emissions from deforestation, and it needs to take into consideration both the stock of forest and the rate of deforestation depending on the characteristics of the recipient countries.

To address the question of the design of the REDD mechanism, we develop a growth model with deforestation in a forest abundant economy that will receive a transfer from the international community. Assuming that deforestation generates a global externality that affects the welfare of the international community, we analyze the optimal policies that restore the social optimum in a decentralized setting where the recipient economy does not immediately suffer from the externality. We adopt several simplifying assumptions. First, the analysis is held in partial equilibrium since we are only considering one productive sector in the recipient economy, and the sector is export-oriented since all products are consumed by the North. We make this assumption with the case of Brazilian exports of soybeans, Indonesian exports of palm oil, and exports of tropical wood and biofuels from tropical countries in mind. These products are often linked to deforestation patterns in tropical countries. This leads us to our second assumption: we assume that agriculture is the main driver for deforestation, that is, forested lands are cleared for productive purpose, and agricultural expansion is fueling growth in tropical countries. In this context, reducing deforestation may have detrimental impacts on growth trends and requires that the country will be compensated for the opportunity costs of not clearing land for agriculture. The third assumption is that only the international community (rich industrialized countries) or the North is willing to preserve tropical forests. It reflects the fact that tropical countries are developing countries, hence their priority is development even though they face a trade-off between environmental preservation and development. We consider two ways of incorporating the externality in the utility of the North: through an amenity value made explicit in consumers' preferences, or through an environmental threshold. Northern consumers are affected by a loss in amenity value when the stock of tropical forests is reduced. This reflects a continuous willingness to preserve tropical forests. Since forests are sequestrating carbon and contribute to climate stability, we can also relate the atmospheric carbon cap that climate scientists recommend not to overshoot to avoid high climate disruptions with a minimum stock of tropical forests to be preserved. The intuition is that, if the stock of tropical forest is reduced beyond this threshold, the loss of carbon sequestration sinks and the carbon released into the atmosphere by deforestation would induce some catastrophic damages. Interestingly, the North faces a trade-off between preserving tropical forests and consuming goods that are produced in deforesting countries. Finally, the fourth assumption determines how to implement the REDD mechanism so that the tropical country would reduce its deforestation trends. Since there is no consensus on the most appropriate transfer scheme in the literature, we consider that the transfer can depend both on the stock of forest or on the rate of deforestation. We obtain that...

To the best of our knowledge, the literature on payments for preserving tropical forests has not proposed an optimal incentive scheme in the context of a growth model with deforestation. It can serve as a benchmark for the negotiations between Northern donors (e.g. Norway) and the recipient tropical countries. Using a renewable resource model, Stähler (1996) compares two transfer schemes aiming at preserving a forest stock, and concludes that the scheme which offers a constant price per hectare of forest is more efficient than the scheme which offers a compensation price that rises with forest scarcity, when forest-abundant countries adopt strategic behaviors. van Soest and Lensink (2000) use a transfer scheme that depends on both the stock of forest and the rate of deforestation to show that penalizing the rise in the rate of deforestation can be efficient for limiting and postponing carbon emissions from deforesting. The more recent debate on the evaluation of the opportunity costs of REDD projects reveals that the willingness to finance forest preservation depends crucially on the expected benefits in terms of carbon emission reductions (Myers, 2007; Eliasch, 2008). Many drawbacks limit the expected carbon emission reductions: carbon sequestration in forest is not permanent; there is a risk that deforestation will move toward non participating countries, leading to carbon leakage; and the governance issue is often problematic in tropical developing countries, implying a high risk that the transfer will be diverted to corruption and rent-seeking (Myers, 2007). We abstract from these issues by considering only one recipient economy characterized with neither market nor institutional failures.

The remainder of the paper is organized as follows. Section 2 presents the simple model of growth with deforestation. Section 3 presents the optimum, that is, what would be the optimal deforestation trends for the consumers who are consuming the good that is responsible for the externality. Section 4 presents the optimal behaviors of the producing South and of the consuming North, and section 5 describes the incentive mechanism that is required to restore the optimum in this context. Section 6 illustrates with a specified model, and section 7 provides concluding remarks.

2 The model

Consider a forest abundant economy with an infinitely lived representative agent. The economy is composed of one sector, whose product is consumed. Only one factor of production, land, is required, but we distinguish between old agricultural land and newly deforested land, the latter being more productive than the former. And deforesting generates an externality suffered by the representative consumer.

In our economy, deforestation is entirely driven by agricultural expansion. We focus on the issue of land allocation between a productive land use and an "idle" land use, hence we neglect sources of revenue from forest such as timber production van Soest and Lensink (2000). We assume that deforestation is irreversible, that is, once a hectare of forest has been cleared, it is used for agriculture and thus forest cannot regrow. Following Hartwick et al. (2001), the clearing process is represented as a sticky adjustment of endowments, where the stock of "unproductive" forest F_t decreases while the stock of agricultural land A_t rises, since the economy has initially "too much" forested land (large F_0). The dynamics is

$$\dot{F}_t = -h_t = -\dot{A}_t, \tag{1}$$

where h_t denotes the rate of deforestation or the amount of newly cleared land at period t, and F_0 and A_0 are given. Given the total amount of land $\bar{L}$ in the economy, we have for any t:

$$\bar{L} = F_t + A_t. \tag{2}$$

The production function of the agricultural sector, whose product x_t is homogeneous, takes the following form:

$$x_t = f(A_t, h_t) \tag{3}$$

where both the stock of agricultural land, A_t, and the newly deforested land, h_t, are factors of production. We assume that their productivity differ since the clearing and burning of biomass create a boost in productivity in newly deforested land, due to the release of nutrients from ashes. The fertility gap only lasts one period, and after one period, the land falls into the stock of agricultural land whose productivity is normalized to one. The partial derivatives of the production function with respect to agricultural land and to newly cleared land are denoted by f_A and f_h, respectively. We assume that technology $f(.)$ exhibits decreasing returns to scale, since there is only one input considered (land). The use of inputs is not costly, since the resource is in open access and once deforested the land is owned by the producer.

The entire production of the homogeneous good is consumed by the representative household. Since his preferences exhibit amenity values, his instantaneous utility function $U(.)$ depends both on his consumption x_t and on the stock of tropical forest F_t. Denote by W the intertemporal utility function, which is defined by:

$$W = \int_0^{\infty} U(x_t, F_t) e^{-\rho t} dt \tag{4}$$

where ρ is the social discount rate, which is assumed to be constant. The partial derivatives of the instantaneous utility function with respect to consumption and to forest preservation are denoted by U_x and U_F, and they are both positive. We also assume that the function $U(.)$ is strictly concave and satisfies the Inada conditions. We do not need to make any assumption on the sign of the cross derivative U_{xF}.[3]

To avoid any environmental catastrophe, we assume that the stock of forest, F_t, needs to stay above a minimum $\underline{F}$ or, equivalently, that the stock of deforested land A_t needs to remain inferior to a threshold $\bar{A} = \bar{L} - \underline{F}$. Hence, we have

$$F_t \geq \underline{F} \;\Leftrightarrow\; A_t \leq \bar{A},\ \forall t \geq 0, \tag{5}$$

with $\underline{F} < F_0$ and $\bar{A} > A_0$. Assuming $0 < \underline{F} < \bar{L}$ implies that $0 < \bar{A} < \bar{L}$. Combining this constraint with the amenity value of the forest implies that the following discontinuous welfare function $V(x_t, F_t)$ captures the twofold representation of the externality from deforesting:

$$\forall x_t \geq 0, \quad V(x_t, F_t) = \begin{cases} U(x_t, F_t), & \text{if } F_t \geq \underline{F} \\ 0, & \text{if } F_t < \underline{F} \end{cases} \tag{6}$$

3 The optimum

The social planner chooses the deforestation path h_t that maximizes the intertemporal utility W subject to (3)-(2), (5) and to the non-negativity constraint $h_t \geq 0$. The extended Hamiltonian (in current value) of this one-state-variable problem is:

$$H_t = U(x_t, F_t) + \lambda_t h_t + \mu_t h_t + \eta_t \left(\bar{A} - A\right) \tag{7}$$

where λ_t is the co-state variable associated with agricultural land accumulation, and μ_t and η_t are the Lagrangian multipliers associated with the non-negativity constraint on the

[3]The term U_{xF} measures the degree of substitution or complementarity between consumption and natural capital preservation in welfare. In the Michel and Rotillon (1996) terminology, the utility function exhibits a "*compensation effect*" if $U_{xF} < 0$ (i.e. a decrease in natural capital implies an increase in the marginal utility of consumption that leads society to consume more) and a "*disgust effect*" if $U_{xF} > 0$.

deforestation rate and with the forest threshold constraint, respectively. The static and dynamic first-order conditions are:

$$\begin{aligned} U_x f_h &= -\lambda_t - \mu_t \quad (8) \\ \dot{\lambda}_t &= \rho\lambda_t - U_x f_A + U_F + \eta_t, \quad (9) \end{aligned}$$

and the transversality condition is given by

$$\lim_{t\to\infty} e^{-\rho t}\lambda_t A_t = 0, \quad (10)$$

and the complementary slackness conditions are

$$\begin{aligned} \mu_t h_t &= 0, \quad \mu_t \geq 0, \quad h_t \geq 0 \quad (11) \\ \eta_t(\bar{A} - A) &= 0, \quad \eta_t \geq 0, \quad A_t \leq \bar{A}. \quad (12) \end{aligned}$$

Condition (8) implies that the marginal gain of deforestation in terms of utility, which is related to an increase in consumption, must be equal to the social marginal cost of deforestation, $-\lambda_t$, as long as $h_t > 0$. Denote by $\phi(F_t, h_t) \equiv U_x f_h$ the marginal gain of deforestation. Integrating the differential equation (9) from 0 up to t yields:

$$\lambda_t = \left[\lambda_0 + \int_0^t (U_F + \eta_s - U_x f_A)e^{-\rho s} ds\right] e^{\rho t} \quad (13)$$

From the transversality condition (10), it implies that $\lambda_0 = -\int_0^\infty (U_F + \eta_s - U_x f_A)e^{-\rho s} ds$, which leads to

$$-\lambda_t = \int_t^\infty (U_F + \eta_s - U_x f_A)e^{-\rho(s-t)} ds. \quad (14)$$

At any period t along the optimal trajectory, the social marginal cost of deforestation must be equal to the discounted sum from t up to ∞ of the flow of the marginal amenity value of preserving the forest plus the marginal social value of the threshold constraint, minus the marginal utility gain from an increase in agricultural land, $U_x f_A$. Reducing the stock of forest by one unit provides a direct marginal decrease in utility, U_F, and an indirect marginal loss since we are approaching the threshold by one more unit, which is captured by η_t. We denote by $\psi(F_t, h_t) \equiv U_F + \eta_t - U_x f_A$ the net marginal gain of preserving the forest.

Proposition 1 *Characterization of the optimal solution:*

1. *Interior solution: at each time t, an optimal deforestation path is characterized by the following equivalent equations:*

$$\begin{aligned} \phi(F_t, h_t) &= \int_t^\infty \psi(F_s, h_s)e^{-\rho(s-t)} ds \quad (15) \\ \rho &= \frac{\dot{\phi}(F_t, h_t)}{\phi(F_t, h_t)} + \frac{\psi(F_t, h_t)}{\phi(F_t, h_t)} \quad (16) \end{aligned}$$

2. If the economy converges toward a steady-state, it must be characterized by:

$$\rho = \frac{1}{f_h}\left(\frac{U_F + \eta_\infty}{U_x} - f_A\right). \tag{17}$$

Proof: Equation (15) results from combining (8) and (14), and (16) is obtained by differentiating (15) with respect to time. Equation (17) is derived by setting $\dot{\phi}$ equal to zero in (16). η_∞ corresponds to the value of η_t at steady state.

In our economy, no deforestation occurs at the steady-state, hence the stock of forest remains constant (if positive). Condition (17) can be interpreted as a modified Clark and Munroe Golden Rule (Swallow, 1990) that equalizes the social discount rate to the social marginal rate of return of undeforested land. Deforestation ends when the marginal gain of deforestation multiplied by the social discount rate equals the social net marginal gain of preserving the forest. Denoting by A^* the optimal stationary stock of agricultural land, we observe two cases: if $A^* = \bar{A}$, then $\eta_\infty \geq 0$, the threshold constraint is binding, which impedes more deforestation; whereas if $A^* < \bar{A}$, then $\eta_\infty = 0$, and the threshold plays no role in the ending of deforestation. Given that $\bar{A} < \bar{L}$, some forest will remain at steady state, even for negligible amenity values.

4 The decentralized economy

Now, consider that there is no international social planner who balances the benefits and the damages from deforestation. Instead, we represent a simplified world economy where South is the producer of a single exported good and the landowner of tropical forests, and where North consumes the good and suffers from the externality of deforestation. The price of the exported good is normalized to one. Since the environmental externality is not considered by South, North must offer to South a conditional transfer to avoid that most of the forest is cut down. Denote by $S(F_t, h_t)$ the functional class of transfers that depends on both the remaining stock of forest F_t and the instantaneous deforestation rate h_t. Intuitively, an efficient transfer scheme should be designed such that $S_F \geq 0$ and $S_h \leq 0$.

4.1 Deforesting South

We assume that forest is an open access resource in South, hence there is no market for land. Clearing the land is sufficient to obtain the property rights. Assuming that deforestation is irreversible implies that South will take into account the shadow cost of deforestation.

However, only an international transfer from North can compel South to consider the social opportunity cost of not preserving the forest.

The program of South is:

$$\max_{\{h_t, t\geq 0\}} \int_0^\infty [f(A_t, h_t) + S(F_t, h_t)] e^{-\int_0^t r_s ds} dt \tag{18}$$

subject to (1), (2) and to $h_t \geq 0$. The extended Hamiltonian of this program is:

$$H_t^S = f(A_t, h_t) + S(F_t, h_t) + \lambda_t^S h_t + \mu_t^S h_t \tag{19}$$

where λ_t^S is the co-state variable associated with the accumulation of the stock of agricultural lands, and where μ_t^S is the Lagrangian multiplier associated with the non-negativity constraint on the deforestation rate. The static and dynamic first-order conditions, the transversality condition, and the complementary slackness condition are, respectively:

$$f_h + S_h = -\lambda_t^S - \mu_t^S \tag{20}$$

$$\dot{\lambda}_t^S = r_t \lambda_t^S + S_F - f_A \tag{21}$$

$$\lim_{t\to\infty} e^{-\int_0^t r_s ds} \lambda_t^S A_t = 0 \tag{22}$$

$$\mu_t^S h_t = 0, \quad \mu_t^S \geq 0, \quad h_t \geq 0 \tag{23}$$

Using (21) and (22), we can express λ_t^S as follows:

$$-\lambda_t^S = \int_t^\infty (S_F - f_A) e^{-\int_t^s r_u du} ds \tag{24}$$

As long as $h_t > 0$, we have $\mu_t^S = 0$, hence given (20) and (21), the interior solution respects the following arbitrage condition:

$$r_t = \frac{\dot{\phi}^S(F_t, h_t)}{\phi^S(F_t, h_t)} + \frac{\psi^S(F_t, h_t)}{\phi^S(F_t, h_t)} \tag{25}$$

where $\phi^S(F_t, h_t) \equiv f_h + S_h$ is the net marginal profit from deforesting and $\psi^S(F_t, h_t) \equiv S_F - f_A$ is the net marginal profit of preserving one unit of forest. Comparing (25) with (16) shows that the instantaneous profit flows are now discounted at the interest rate r_t, instead of ρ, and that the gains or losses from deforesting are not evaluated in terms of utility.

4.2 Environmentally-friendly North

North is the only consumer of the good whose production generates deforestation and is willing to preserve tropical forest due to its amenity value. The program of the donor is:

$$\max_{\{x_t, t\geq 0\}} \int_0^\infty U(x_t, F_t) e^{-\rho t} dt \tag{26}$$

subject to

$$\dot{B}_t = r_t B_t - x_t - S(F_t, h_t), \tag{27}$$

where B_t denotes Northern wealth, which can be used for consumption or for investment at the interest rate r_t. It can be easily shown that solving the Northern consumer's program leads to the standard Keynes-Ramsey condition:

$$\rho - \frac{\dot{U}_x}{U_x} = r_t \tag{28}$$

4.3 Characterization of the decentralized equilibrium

A particular equilibrium, associated with a particular transfer function $S(F_t, h_t)$, is a vector of trajectories $\{h_t, A_t; r_t\}_t^\infty$ such that: i) Southern producer maximizes its intertemporal profit function; ii) Northern consumer maximizes its utility; iii) the market of the final good is perfectly competitive and cleared: $\forall t \geq 0$, $x_t = f(A_t, h_t)$; and iv) South receives a transfer $S(F_t, h_t)$ from North. From the previous analysis of individual behaviors, we obtain

Proposition 2 *For a given class of transfer functions $S(F_t, h_t)$, the decentralized equilibrium is characterized as follows:*

1. *During any strictly positive deforestation regime, the two following equivalent equation must hold at the equilibrium:*

$$\phi^S(F_t, h_t) = \frac{1}{U_{x,t}} \int_t^\infty \psi^S(F_s, h_s) U_{x,s} e^{-\rho(s-t)} ds \tag{29}$$

$$\rho - \frac{\dot{U}_x}{U_x} = \frac{\dot{\phi}^S(F_t, h_t)}{\phi^S(F_t, h_t)} + \frac{\psi^S(F_t, h_t)}{\phi^S(F_t, h_t)} \tag{30}$$

2. *If the economy converges toward a steady-state equilibrium, it must be such that:*

$$\rho = \frac{\psi^S(F, h)}{\phi^S(F, h)} = \frac{S_F - f_A}{S_h + f_h} \tag{31}$$

Proof: Integration of (28) from t up to s yields: $e^{-\int_t^s r_u du} = (U_{x,s}/U_{x,t})\, e^{-\rho(s-t)}$. Combining this last result with (20) and (24) implies (29). (30) is a direct implication of (25) and (28).

5 Optimal transfer analysis

5.1 First-best optimum implementation rule

The optimal transfer function is such that the characterizing conditions of the optimum in Proposition 1 coincide with the characterizing conditions of the decentralized equilibrium in Proposition 2. We obtain

Proposition 3 *During any strictly positive deforestation regime at the decentralized equilibrium, the transfer function $S(F_t, h_t)$ must satisfy the two following equivalent conditions in order to restore the first-best optimality:*

$$S_h U_x = \int_t^{\infty} (S_F U_x - U_F - \eta_s) e^{-\rho(s-t)} ds \;\Leftrightarrow\; \frac{U_F + \eta_t}{U_x} \;=\; S_F + \dot{S}_h - S_h \left(\rho - \frac{\dot{U}_x}{U_x} \right) \quad (32)$$

The condition states that when the threshold $\underline{F}$ is not reached, the marginal rate of substitution between consumption and forest preservation is equal to the marginal transfer per unit of forest, plus the change of the marginal transfer per unit of deforestation, minus the marginal transfer per unit of deforestation multiplied by the rate of interest. A priori, there exists an infinity of transfer functions among the functional class $S(F_t, h_t)$ that satisfy the condition (32). In the next subsection, we will restrict the analysis to a set of standard political tools, namely taxes and subsidies.

5.2 Example of transfers

5.2.1 Subsidy on the remaining stock of forest

Consider that the transfer takes the form of a unitary subsidy on the remaining stock of forest, i.e. $S(F_t, h_t) = \sigma_t F_t$, with $\sigma_t \geq 0$ for any $t \geq 0$. From (32), the optimal subsidy rate must be equal to:

$$\sigma_t^* = \frac{U_F + \eta_t}{U_x} \quad (33)$$

When the threshold $\underline{F}$ has not been reached ($\eta_t = 0$), the optimal subsidy rate must be equal to the marginal rate of substitution between consumption and preservation of the forest. Once the threshold is reached, the optimal subsidy rate is larger since $\eta_t \geq 0$. Its dynamic properties cannot be characterized without a deeper investigation on the properties of the utility function $U(x_t, F_t)$. An analytic example will be examined later.

If the damage due to deforestation was only captured by the threshold constraint, the optimal subsidy rate must be equal to zero as long as the threshold of forest is not reached and to the marginal social cost of the constraint in monetary terms (i.e. divided by the marginal utility) thereafter. North would then perceive a subsidy only once his forest stock has been driven down to the allowed minimal level, to stop further deforestation.

5.2.2 Tax on the deforestation flow

Consider that the transfer takes the form of a tax on the deforestation flow, hence $S(F_t, h_t) = -\tau_t h_t$, with $\tau_t \geq 0$ for any $t \geq 0$. From (32), we have

$$\tau_t^* = \frac{1}{U_x} \int_t^{\infty} (U_F + \eta_s) e^{-\rho(s-t)} ds \tag{34}$$

At each period t, the optimal tax rate must be equal to the discounted sum of the instantaneous flow of social marginal gain from preserving the forest, divided by the marginal utility of consumption to obtain the expression in monetary terms (given that the consumption good is the numeraire). Since $\eta_t = 0$ for any $t < T$, where T denotes the date at which the stock of forest reaches the minimal threshold, the expression of τ_t^* can be rewritten as:

$$\tau_t^* = \begin{cases} \frac{e^{\rho t}}{U_x} \left[\int_t^T U_F e^{-\rho s} ds + \int_T^{\infty} (U_F + \eta_s) e^{-\rho s} ds \right], & t \in [0, T) \\ \frac{e^{\rho t}}{U_x} \int_t^{\infty} (U_F + \eta_s) e^{-\rho s} ds, & t \in [T, +\infty) \end{cases} \tag{35}$$

If the deforestation externality was captured only by the minimal threshold, the optimal tax would be strictly positive even when $t < T$. This is in contrast with the subsidy on the stock of forest in the case where forest has no amenity value.

Log-differentiating (34) gives the growth rate of the optimal tax on deforestation at each time t:

$$\frac{\dot{\tau}_t}{\tau_t} = \rho - \frac{\dot{U}_x}{U_x} - \frac{U_F + \eta_t}{\int_t^{\infty} (U_F + \eta_s) e^{-\rho(s-t)} ds} \tag{36}$$

The dynamics of the optimal tax results from the difference of two components. The first one, $\rho - \dot{U}_x / U_x$, is simply the interest rate, i.e. the rate at which future gains or losses are discounted, assumed here to be positive.[4] The second term is the ratio of the instantaneous social marginal utility gain of preserving the forest over the discounted sum of the same marginal gain, which is also positive. We then have two opposite effects that drive the dynamics of the tax, which implies that the tax can either rise or fall over time.

5.2.3 Policy-mix

Since Southern participation into the transfer program is voluntary, the tax scheme is unlikely to be accepted. Hence, the participation constraint requires that $S(F_t, h_t) \geq 0$.

[4] Note that, since social preferences exhibit a conservation motive for tropical forest, this discount rate can be expanded as $r_t = \rho + \epsilon(x_t) g_{x,t} + h_t U_{xF} / U_x$, where $\epsilon(x_t)$ is the inverse of the elasticity of intertemporal substitution of consumption and $g_{x,t}$ is the instantaneous growth rate of consumption. As compared with the standard decomposition of the discount rate, we must take into account the amenity effect captured by $h_t U_{xF} / U_x$, whose sign is not specified a priori. An extensive discussion on this point can be found in Schubert (2006).

To respect this condition, the scheme needs to combine a subsidy for forest preservation at rate σ_t and a tax on the deforestation process at rate τ_t: $S(F_t, h_t) = \sigma_t F_t - \tau_t h_t$. If one of these two policy instruments is set equal to its first-best optimal level defined above, then obviously, the second one must be equal to zero. Since the model considers a single source of externality, a unique policy instrument is required to restore optimality. However, the enforcement of the first-best solution through one instrument may not be achievable due to budgetary, socioeconomic or political additional constraints, hence the use of both instruments may be required. To illustrate this, consider that North has not set the subsidy rate at its optimal level, hence $\tilde{\sigma}_t < \sigma_t^*$, $\forall t$. In this particular case, North could also implement a second-best optimal tax τ_t^{sb} to restore the optimum, such as

$$\tau_t^{sb} = \frac{1}{U_x} \int_t^{\infty} U_x(\sigma_s^* - \tilde{\sigma}_s) e^{-\rho(s-t)} ds \, > \, 0 \tag{37}$$

Since the donor was not able to offer the optimal subsidy rate, the incentive for preserving forest is not sufficiently high, hence a tax is required to limit deforestation. However, we need to have $\tilde{\sigma}_t F_t \geq \tau^{sb} h_t$ for all t to ensure Southern participation.

Conversely, consider that the tax rate $\tilde{\tau}_t$ effectively levied by North is such that $\tilde{\tau}_t < \tau_t^*$, $\forall t$. From (32), the second-best subsidy rate σ_t^{sb}, which is necessary to restore the optimum, is given by:

$$\int_t^{\infty} U_x \sigma_s^{sb} e^{-\rho(s-t)} ds = U_x(\tau_t^* - \tilde{\tau}_t) \, \Rightarrow \, \sigma_t^{sb} = \left(\rho - \frac{\dot{U}_x}{U_x} \right) (\tau_t^* - \tilde{\tau}_t) - (\dot{\tau}_t^* - \dot{\tilde{\tau}}_t) \, > \, 0 \tag{38}$$

Since North has not set the tax at its optimal level, South is tempted to deforest more, hence a subsidy on the forest stock is necessary to curb deforestation. However, we need to have $\sigma_t^{sb} F_t \geq \tilde{\tau}_t h_t$ to allow South to participate.

6 Illustration

6.1 Analytical specifications and identification of the possible cases

In order to further develop the analysis, we adopt the following specifications. First, assume a separable instantaneous utility function:

$$U(x_t, F_t) = \frac{x_t^{1-\epsilon}}{1-\epsilon} + \gamma F_t, \quad \text{with} \;\; \epsilon > 0, \; \gamma > 0 \tag{39}$$

where $1/\epsilon$ is the constant intertemporal elasticity of substitution in consumption. γ is positive to ensure that the higher the stock of preserved tropical forest, the higher the utility. Second, consider a production function with decreasing returns to scale, where two types of land are perfect substitutes:

$$x_t = f(A_t, h_t) = (A_t + \nu h_t)^{\beta}, \quad \text{with} \quad 0 < \beta < 1, \ \nu > 1, \tag{40}$$

where ν is the fertility boost of the newly deforested land, h_t, compared to the old agricultural land, A_t.

We first look at the optimal stationary regime. There are two possible stationary stocks of agricultural lands: the "natural" steady-state level A^* towards which the economy would tend in the absence of the maximal threshold constraint on A_t and the imposed steady-state level $\bar{A}$. In both cases, the deforestation rate h equals zero and the resulting consumption level simply becomes $x = x^* = (A^*)^{\beta}$ or $x = \bar{x} = \bar{A}^{\beta}$, depending on the stationary amount of agricultural land. Assume that level $\bar{A}$ is high enough so that $A^* < \bar{A} \leq \bar{L}$. In this case, the economy converges towards a steady-state which is characterized by (17) with $\eta_{\infty} = 0$, starting from $A_0 < A^*$. Using the specified analytical forms (39) and (40), $U_x f_h = \nu U_x f_A = \nu \beta x^{1-\epsilon-1/\beta}$ so that (17) implies:

$$A^* = \left[\frac{\beta(\rho\nu + 1)}{\gamma} \right]^{\frac{1}{1-\beta(1-\epsilon)}} \tag{41}$$

where $[1 - \beta(1-\epsilon)] > 0$ for any $0 < \beta < 1$ and $\epsilon > 0$.

Conversely, assume that $\bar{A}$ is low enough so that the threshold constraint becomes binding at some date $T > 0$. Hence, from (12), we have $\eta_t = 0$ for any $t < T$ and $\eta_t \geq 0$ for any $t \geq T$. Equation (17) implies $\eta_t = \beta(\rho\nu + 1)\bar{A}^{\beta(1-\epsilon)-1} - \gamma$ for any $t \geq T$. Replacing γ by $\beta(\rho\nu + 1)(A^*)^{\beta(1-\epsilon)-1}$ in this last expression, using (41), we obtain:

$$\eta_t = \begin{cases} 0, & t \in [0, T) \\ \beta(\rho\nu + 1)\Psi, & t \in [T, +\infty) \end{cases} \tag{42}$$

where Ψ is defined as:

$$\Psi = \bar{A}^{\beta(1-\epsilon)-1} - (A^*)^{\beta(1-\epsilon)-1}. \tag{43}$$

with $\beta(1-\epsilon) - 1 < 0$ for any $\beta \in (0, 1)$ and $\epsilon > 0$. The multiplier η_t is non-negative if and only if $\Psi \geq 0$, that is, if $\bar{A} \leq A^*$. Consequently, we have two cases: either $\bar{A} > A^*$ and the optimal stationary regime is unconstrained, or $\bar{A} \leq A^*$ and the stationary regime is driven by the threshold constraint. We investigate these two cases in the following subsections.

6.2 The unconstrained case: $A^* < \bar{A}$

6.2.1 Optimal trajectories

If the threshold constraint is not binding, $\eta_t = 0$ for any t, and the optimal trajectories are given by the following proposition:

Proposition 4 *If $A^* < \bar{A}$, the optimal trajectories are (the upperscript u refers to optimality in the unconstrained case), for any $t \geq 0$:*

$$A_t^u = A^* - (A^* - A_0)e^{-\frac{t}{\nu}} = \bar{L} - F_t^u \tag{44}$$

$$h_t^u = \left(\frac{A^* - A_0}{\nu}\right)e^{-\frac{t}{\nu}} \tag{45}$$

$$x_t^u = x^* = (A^*)^\beta. \tag{46}$$

Proof: See the proof in appendix.

Given that the utility function is additively separable in x and F, and is linear in F, the economy instantaneously jumps from the beginning of the planning horizon to a stationary consumption regime. However, the optimal deforestation rate decreases exponentially over time from its initial level $h_0 = (A^* - A_0)/\nu$, and tends asymptotically to zero. The stock of agricultural lands continuously increases over time, from A_0 at the initial period to its asymptotic stationary value A^*. Observe that a strictly positive stock of forest F^* will remain in the steady state, where $F^* = \bar{L} - A^*$, due to the amenity value of forest. In the absence of such a conservation motive, the optimal deforestation path could exhaust entirely the forest stock in finite time[5]. These optimal trajectories are depicted in Figure 1.

Table 1 shows the sign of the partial derivative of the variables of interest with respect to the main parameters. We find notably that the optimal stationary levels of agricultural land and consumption are increasing in the pure rate of time preference ρ, as well as in the new deforested land productivity boost ν, and are decreasing in the marginal amenity value γ.

Table 1: Comparative dynamics analysis – The unconstrained case

X	$\partial X/\partial \rho$	$\partial X/\partial \gamma$	$\partial X/\partial \nu$
A_t^u	$+$	$-$	depends on $A_0, \beta, \epsilon, \rho, \nu$
h_t^u	$+$	$-$	depends on $A_0, \beta, \epsilon, \rho, \nu$
x_t^u	$+$	$-$	$+$
A^*	$+$	$-$	$+$
F^*	$-$	$+$	$-$

[5] Proof available upon request.

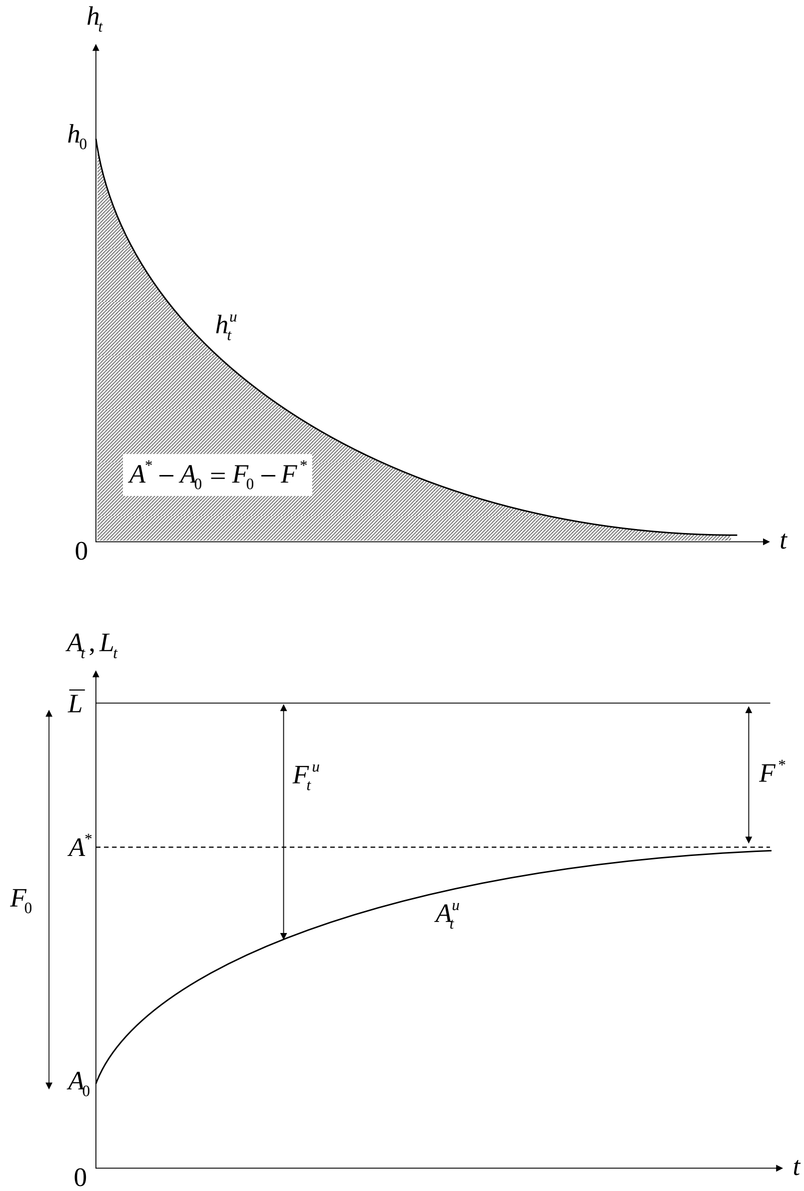

Figure 1: Optimal trajectories of h_t, A_t and F_t – The unconstrained case

6.2.2 Optimal policies

In the unconstrained case, consider first a pure subsidy on the remaining stock of forest: $S(F_t, h_t) = \sigma_t F_t$. From (33), the optimal subsidy rate is $\sigma_t^u = \gamma(A^*)^{\epsilon\beta}$, $\forall t \geq 0$. Since the marginal rate of substitution between consumption and preservation of the forest is constant through time in our specified framework, the optimal rate of subsidy must also be constant. Obviously, it is increasing in the marginal amenity value, γ. It is also increasing in ρ and ν, since a rise in these two parameters results in a larger stationary level of agricultural land.

Second, consider a pure taxation policy: $S(F_t, h_t) = -\tau_t h_t$. From (34), it implies: $\tau_t^u = \gamma(A^*)^{\epsilon\beta}/\rho$, $\forall t \geq 0$. The optimal deforestation tax rate corresponds to the discounted sum from t up to ∞ of the flows of the optimal marginal rate of substitution (MRS) between x and F. Given that the optimal MRS is constant over time in our specified example, the unitary tax simplifies to MRS$/\rho$. The optimal tax rate is constant through time, increasing in γ and ν and decreasing in ρ since short run consumption, and deforestation, are more valuable when ρ is high.[6]

Imagine that North is not willing to offer more that half of the optimal subsidy: we have $\tilde{\sigma}_t^u = \sigma_t^u/2$. To restore the optimum, the transfer scheme must combine the subsidy with a second-best tax, which is set as follows: $\tau_t^{sb} = \tau_t^u/2$. To ensure Southern participation, the scheme must respect the following condition:

$$\bar{L} - A^* \geq (A^* - A_0)e^{-t/\nu}[1/(\nu\rho) - 1]. \tag{47}$$

If $\rho\nu \geq 1$, the inequality is satisfied for all t, whereas, if $\rho\nu < 1$, the condition is satisfied if $\rho\nu$ is high, and if the stationary stock of forest is large compared to the rise in agricultural land from its initial level, given that $e^{-t/\nu} < 1$.

6.3 The constrained case: $\bar{A} \leq A^*$

6.3.1 Optimal trajectories

Consider now that the threshold constraint becomes binding at some date T. In this case, the Lagrange multiplier associated with the threshold constraint is given by (42).

[6] A simple calculus leads to:

$$\frac{\partial \tau^u}{\partial \rho} = \frac{\beta^{\frac{\epsilon\beta}{1-\beta(1-\epsilon)}}}{\rho^2}\left(\frac{\rho\nu+1}{\gamma}\right)^{\frac{\beta-1}{1-\beta(1-\epsilon)}}\left[\frac{(\beta-1)(\rho\nu+1)-\epsilon\beta}{1-\beta(1-\epsilon)}\right],$$

which is negative for any $\epsilon \geq 0$ and $0 \leq \beta \leq 1$.

Proposition 5 *If $\bar{A} \leq A^*$ then, the optimal trajectories are (the upperscript c refers to optimality in the constrained case), for any $t \geq 0$:*

$$A_t^c = \begin{cases} \left[A_0 + \frac{1}{\nu}\int_0^t (z_s^c)^{\frac{1}{\beta(1-\epsilon)-1}} e^{\frac{s}{\nu}} ds\right] e^{-\frac{t}{\nu}}, & t \in [0, T) \\ \bar{A}, & t \in [T, +\infty) \end{cases} \tag{48}$$

$$F_t^c = \bar{L} - A_t^c \tag{49}$$

$$h_t^c = \frac{1}{\nu}\left[(z_t^c)^{\frac{1}{\beta(1-\epsilon)-1}} - A_t^c\right] \tag{50}$$

$$x_t^c = (z_t^c)^{\frac{\beta}{\beta(1-\epsilon)-1}}, \tag{51}$$

where z_t^c is defined by:

$$z_t^c = \begin{cases} \bar{A}^{\beta(1-\epsilon)-1} + \Psi\left[e^{-\left(\frac{\rho\nu+1}{\nu}\right)(T-t)} - 1\right], & t \in [0, T) \\ \bar{A}^{\beta(1-\epsilon)-1}, & t \in [T, +\infty) \end{cases} \tag{52}$$

Proof: See the proof in appendix.

Let us analyze the dynamic properties of these optimal trajectories. Using (52), the trajectory of z_t^c is increasing and convex for any $t \in [0, T)$ and constant for any $t \geq T$. It is depicted by the North East quadrant of Figure 2. By construction, x_t^c is obtained as a decreasing and convex transformation of z_t^c as shown in the South East quadrant of Figure 2. The optimal consumption path is declining through time as long as the deforestation process goes on and it reaches the imposed stabilization level $\bar{x} = \bar{A}^\beta$ at time T. Consumption decreases because production is restricted, due to the constraint on agricultural land.

The trajectory of A_t^c must be increasing since $\dot{A}_t^c = h_t^c$ is non-negative by assumption. Using (50) and (51), $\ddot{A}_t^c = \dot{h}_t^c = (1/\nu)[\dot{x}_t^c(x_t^c)^{1/\beta-1}/\beta - \dot{A}_t^c]$ which is clearly non-positive since $\dot{x}_t^c \leq 0$ and $\dot{A}_t^c \geq 0$. Finally, differentiating again this last expression with respect to time leads to $\ddot{h}_t^c \geq 0$ since $\ddot{x}_t^c \geq 0$ and $\ddot{A}_t^c \leq 0$. We obtain an increasing and concave optimal trajectory of agricultural land accumulation and a decreasing and convex optimal deforestation path for any $t \in [0, T)$ as represented in Figure 3.

The optimal date T at which the stock of deforested land reaches the imposed maximal threshold $\bar{A}$ is endogenously determined by the continuity condition on the optimal trajectory A_t^c. It is thus solution of the following equation:

$$\frac{1}{\nu}\int_0^T (z_s^c)^{\frac{1}{\beta(1-\epsilon)-1}} e^{\frac{s}{\nu}} ds = \bar{A}e^{\frac{T}{\nu}} - A_0. \tag{53}$$

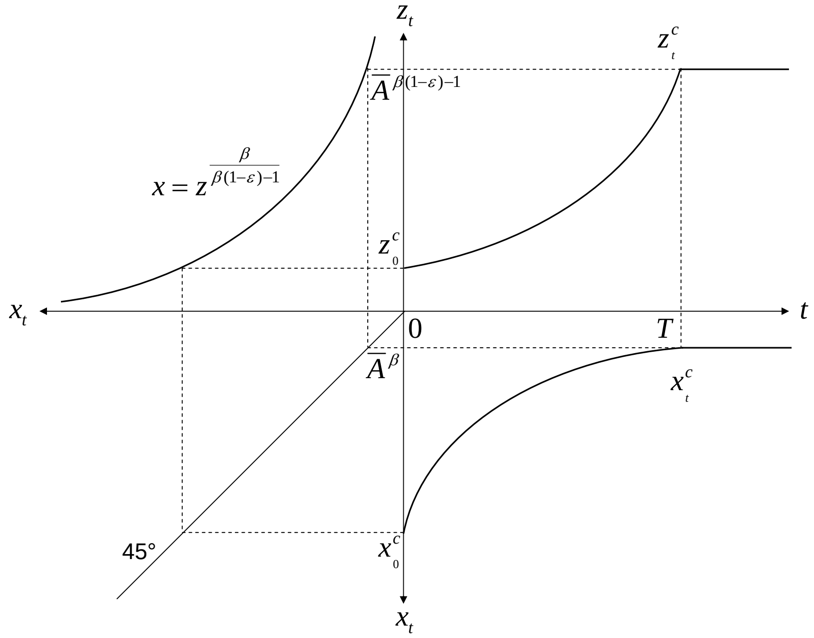

Figure 2: Optimal trajectory of x_t – The constrained case

Performing a comparative dynamics analysis on the optimal trajectories amounts to performing a comparative statics analysis on T. We undertake such an exercise by studying the impact of the pure rate of time preference and of the marginal amenity value on T. Partially differentiating (53) with respect to ρ leads to:

$$\frac{1}{\nu}\left[\frac{\partial T}{\partial \rho}\left(z_T^c\right)^{\frac{1}{\beta(1-\epsilon)-1}} e^{\frac{T}{\nu}} + \int_0^T \frac{\partial z_s^c}{\partial \rho}\left(\frac{1}{\beta(1-\epsilon)-1}\right)\left(z_s^c\right)^{\frac{1}{\beta(1-\epsilon)-1}-1} e^{\frac{s}{\nu}} ds\right] = \frac{\partial T}{\partial \rho}\frac{\bar{A}e^{\frac{T}{\nu}}}{\nu} \quad (54)$$

Given $z_T^c = \bar{A}^{\beta(1-\epsilon)-1}$, this expression simply reduces to $\partial z_t^c/\partial\rho = 0$, for any $t \in [0, T)$. From (41), (43) and (52), we obtain after simplifications:

$$\left.\frac{\partial z_t^c}{\partial \rho}\right|_{t\in[0,T)} = 0 \Leftrightarrow \Psi\left[\left(\frac{\rho\nu+1}{\nu}\right)\frac{\partial T}{\partial \rho} + (T-t)\right] = \frac{\gamma\nu}{\beta(\rho\nu+1)^2}\left[1 - e^{\left(\frac{\rho\nu+1}{\nu}\right)(T-t)}\right] \quad (55)$$

For any $t \in [0, T)$, the RHS of this equality is negative. Since Ψ is positive, a necessary condition for getting a negative LHS is $\partial T/\partial\rho \leq 0$. Intuitively, the optimal date at which the stock of forest reaches its threshold level occurs earlier when ρ increases. This result implies that the initial deforestation level h_0^c must increase, since the area below the trajectory of h_t^c between 0 and T is constant: $\int_0^T h_t^c dt = \bar{A} - A_0$. Then, an increase in ρ results in faster deforestation: F_t^c is reduced and A_t^c is augmented. The effect of an increase in ρ on the optimal trajectories of the deforestation rate and the stock of agricultural land is represented in Figure 4 by a shift from the solid line to the dashed line.

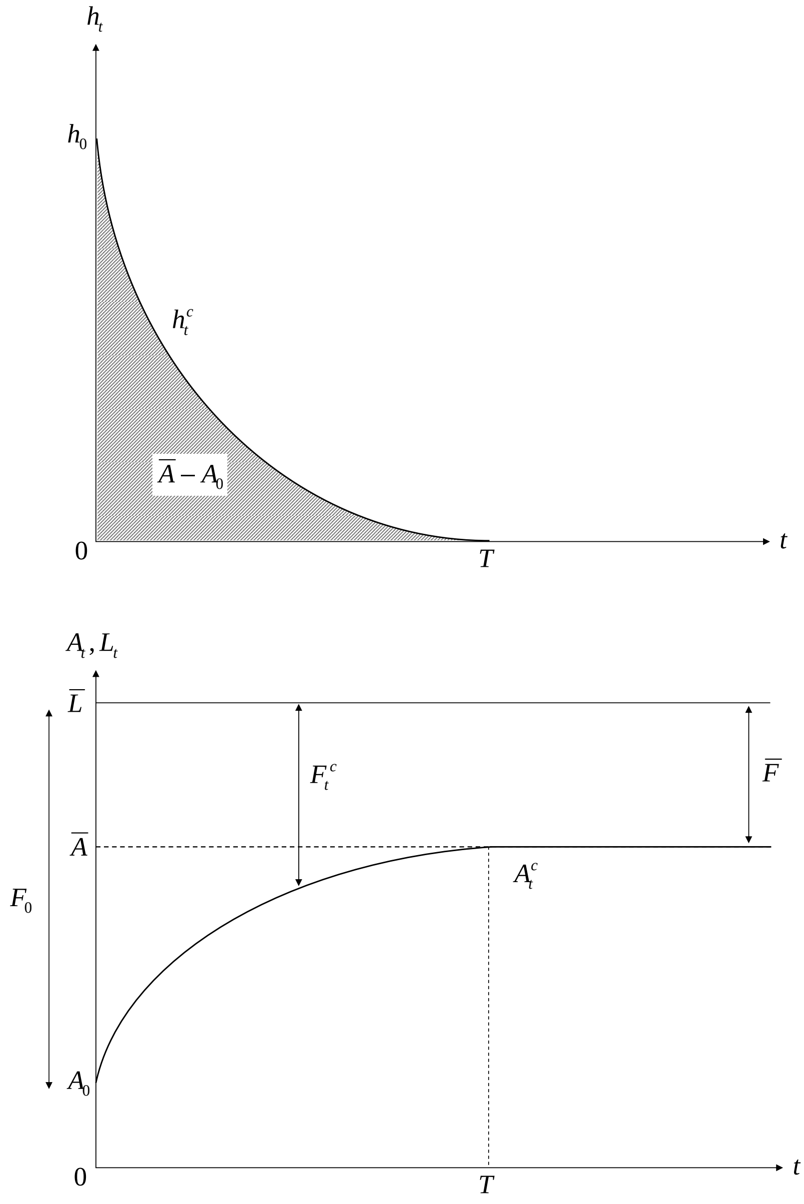

Figure 3: Optimal trajectories of h_t, A_t and F_t – The constrained case

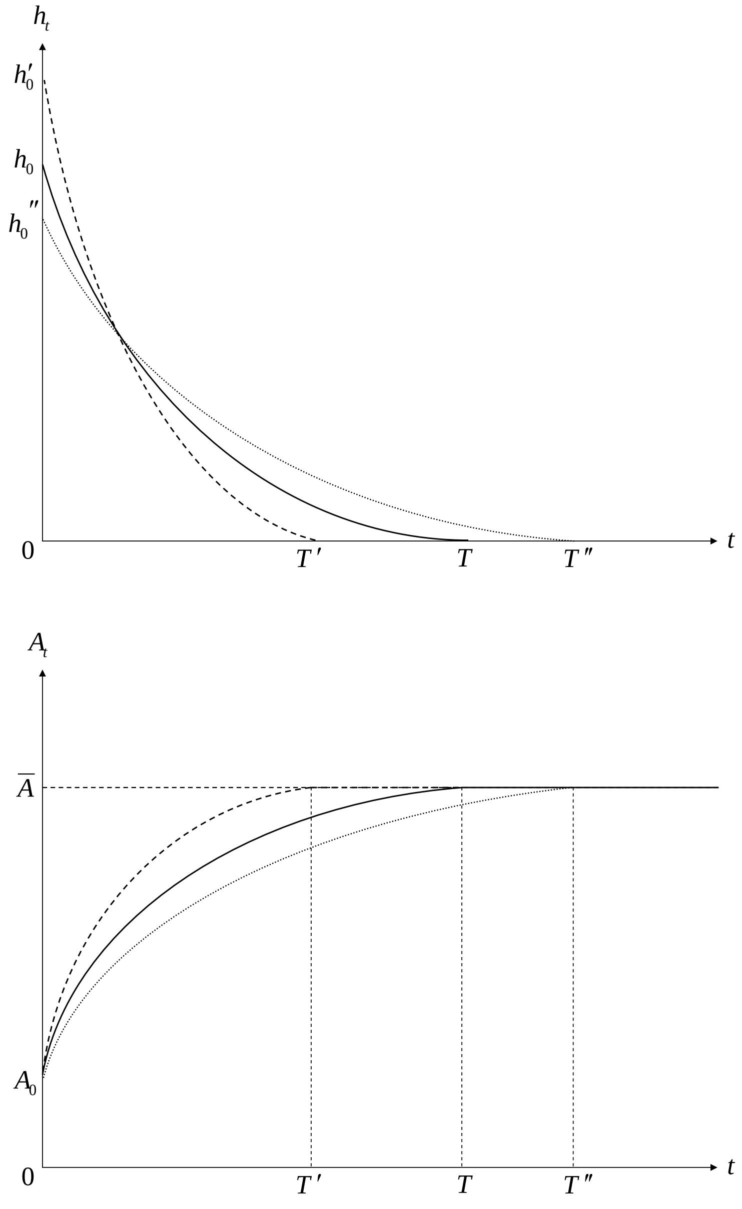

Figure 4: Impact of an increase in ρ and γ on h_t^c and A_t^c

Similarly, the partial derivative of each side of (53) with respect to γ implies: $\partial z_t^c/\partial\gamma = 0$, $\forall t \in [0, T)$, which can be rewritten as:

$$\left.\frac{\partial z_t^c}{\partial \gamma}\right|_{t\in[0,T)} = 0 \;\Leftrightarrow\; \frac{-1}{\beta(\rho\nu+1)}\left[1 - e^{\left(\frac{\rho\nu+1}{\nu}\right)(T-t)}\right] = \Psi\left(\frac{\rho\nu+1}{\nu}\right)\frac{\partial T}{\partial \gamma} \tag{56}$$

For any $t \in [0, T)$, the LHS of (56) is unambiguously positive. Hence, $\partial T/\partial\gamma$ in the RHS must be positive. An increase in the marginal amenity value results in a later optimal date at which the forest stock attains its threshold. The initial level of deforestation is reduced, and deforestation proceeds more slowly along the time frame $[0, T)$. The effect of an increase in γ on h_t^c and A_t^c is represented in Figure 4 by a shift from the solid line to the doted line. The findings of this comparative dynamics analysis are summarized in Table 2.

Table 2: Comparative dynamics analysis – The constrained case

X	$\partial X/\partial\rho$	$\partial X/\partial\gamma$	$\partial X/\partial\bar{A}$
T	$-$	$+$	
h_0^c	$+$	$-$	
$\dot{h}_t^c$	$-$	$+$	
A_t^c	$+$	$-$	
F_t^c	$-$	$+$	
x_t^c			

6.3.2 Optimal policies

Consider a pure subsidy policy on the stock of forest: $S(F_t, h_t) = \sigma_t F_t$. From (33), we obtain $\sigma_t^c = (\gamma+\eta_t)(x_t^c)^\epsilon$. Using (42), (43), (51) and (52), the expended formulation of the optimal subsidy rate is:

$$\sigma_t^c = \begin{cases} \gamma\left\{\bar{A}^{\beta(1-\epsilon)-1} + \Psi\left[e^{-\left(\frac{\rho\nu+1}{\nu}\right)(T-t)} - 1\right]\right\}^{\frac{\epsilon\beta}{\beta(1-\epsilon)-1}}, & t \in [0, T) \\ [\gamma + \beta(\rho\nu+1)\Psi]\,\bar{A}^{\epsilon\beta} = \beta(\rho\nu+1)\bar{A}^{\beta-1}, & t \in [T, +\infty) \end{cases} \tag{57}$$

It can be shown that the trajectory of σ_t^c is decreasing and convex from $t = 0$ up to $t = T$ and constant thereafter. It is surprising that North offers a decreasing subsidy while the remaining forest stock decreases. Given $\bar{A} < A^*$ in the constrained case, it follows that $\sigma_{T^-}^c = \gamma\bar{A}^{\epsilon\beta} < [\gamma+\beta(\rho\nu+1)\Psi]\bar{A}^{\epsilon\beta}$, meaning that the optimal subsidy rate is discontinuous through time and makes an upper jump at the instant the stock of forest reaches its minimal threshold, as depicted in Figure 5.

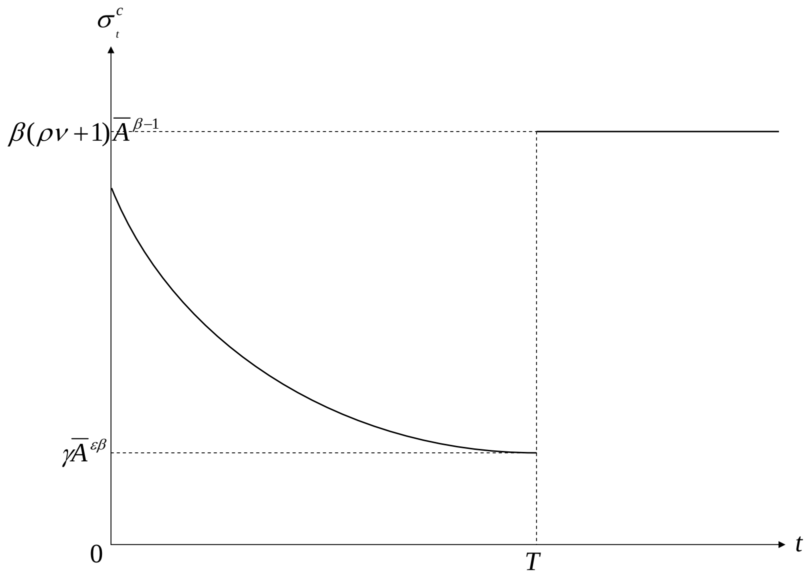

Figure 5: Optimal subsidy rate on the stock of forest – The constrained case

In the deforestation taxation context, i.e. when $S(F_t, h_t) = -\tau_t h_t$, we obtain from (35), (42), (51) and (52):

$$\tau_t^c = \begin{cases} \frac{(x_t^c)^\epsilon}{\rho}\left[\gamma + \beta(\rho\nu+1)\Psi e^{-\rho(T-t)}\right], & t \in [0, T) \\ \frac{\beta(\rho\nu+1)}{\rho}\bar{A}^{\beta-1}, & t \in [T, +\infty) \end{cases} \tag{58}$$

Contrary to the optimal subsidy trajectory in the pure subsidy policy case, the optimal tax trajectory is continuous at time T. Moreover, from (36), its instantaneous growth rate is given by:

$$\frac{\dot{\tau}_t^c}{\tau_t^c} = \rho + \epsilon\frac{\dot{x}_t^c}{x_t^c} - \frac{\gamma(x_t^c)^\epsilon}{\tau_t^c} \tag{59}$$

$$= \rho + \left(\frac{\epsilon\beta}{\beta(1-\epsilon)-1}\right)\frac{\dot{z}_t^c}{z_t^c} - \left[\frac{\gamma\rho}{\gamma + \beta(\rho\nu+1)\Psi e^{-\rho(T-t)}}\right] \tag{60}$$

whose sign is, a priori, undetermined.

7 Conclusion

Appendix

A. Proof of Proposition 4

Replacing ϕ_t by $\nu\beta x_t^{[\beta(1-\epsilon)-1]/\beta}$ and ψ_t by $\left(\gamma - \beta x_t^{[\beta(1-\epsilon)-1]/\beta}\right)$ into (16), it comes:

$$\left[\frac{\beta(1-\epsilon)-1}{\beta}\right]\frac{\dot{x}_t}{x_t} = \left(\frac{\rho\nu+1}{\nu}\right) - \frac{\gamma}{\beta\nu}x_t^{\frac{1-\beta(1-\epsilon)}{\beta}}. \tag{61}$$

Adopting the variable change $z_t = x_t^{[\beta(1-\epsilon)-1]/\beta}$, the previous differential equation is reduced to the following non-homogeneous ordinary differential equation:

$$\dot{z}_t = \left(\frac{\rho\nu+1}{\nu}\right) z_t - \frac{\gamma}{\beta\nu} \tag{62}$$

whose solution is given by:

$$z_t = \frac{\gamma}{\beta(\rho\nu+1)} + Ke^{\left(\frac{\rho\nu+1}{\nu}\right)t} \tag{63}$$

and where K is a constant of integration to be determined. We know that variable z_t must reach, at some date, its steady-state value z^* which is equal to $(x^*)^{[\beta(1-\epsilon)-1]/\beta}$ or, equivalently, to $\gamma/[\beta(\rho\nu+1)]$ from (41). Since the exponential term in (63) is positive we must have $K = 0$, which implies $z_t = z^*$ for any $t \geq 0$. Consequently, the instantaneous production level x_t is also constant over time: $x_t = x^* = (A^*)^\beta$ for any $t \geq 0$. Hence, from (1) and (40), we obtain the following simplified dynamics of land accumulation:

$$A_t + \nu\dot{A}_t = A^*, \quad A_0 \text{ given,} \tag{64}$$

whose solution is given by (44). Expression (45) is simply obtained by differentiating (44) with respect to time. To fully describe the optimal solution in the unconstrained case, we use (14) and (8) to characterize λ_t and μ_t. We find: $\lambda_t = -\gamma\nu/(\rho\nu+1)$ and $\mu_t = 0$ for any $t \geq 0$.

B. Proof of Proposition 5

As long as $h_t > 0$ and $A_t < \bar{A}$, i.e. for $t \in [0, T)$, we have $\mu_t = \eta_t = 0$ and (16) reduces to the same differential equation than (62), after having adopted the variable change $z_t = x_t^{[\beta(1-\epsilon)-1]/\beta}$. The solution is then also given by (63). Continuity of the optimal consumption trajectory implies that $x_T = \bar{A}^\beta$, and then $z_T = \bar{A}^{\beta(1-\epsilon)-1}$. The constant of integration K must now satisfy:

$$\frac{\gamma}{\beta(\rho\nu+1)} + Ke^{\left(\frac{\rho\nu+1}{\nu}\right)T} = \bar{A}^{\beta(1-\epsilon)-1} \Rightarrow K = \Psi e^{-\left(\frac{\rho\nu+1}{\nu}\right)T} \tag{65}$$

Replacing K by this last expression into (63) gives (52) as stated in Proposition 5. Next, from (1) and (40), we can write $x_t = (A_t + \nu\dot{A}_t)^\beta = z_t^{\beta/[\beta(1-\epsilon)-1]}$, which implies:

$$\dot{A}_t = \frac{1}{\nu}\left[z_t^{\frac{1}{\beta(1-\epsilon)-1}} - A_t\right]. \tag{66}$$

(48) is obtained by solving this non-homogeneous ordinary differential equation and (50), by setting $h_t = \dot{A}_t$.

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