Electricity Production with Intermittent Sources of Energy

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Abstract

The paper analyzes the interaction between a reliable source of electricity production and intermittent sources such as wind or solar power. We first characterize the first-best dispatch and investment in the two types of energy. We put the accent on the availability of the intermittent source as a major parameter of optimal capacity investment. We then analyze decentralization through competitive market mechanisms. We show that decentralizing first-best requires to price electricity contingently on wind or solar availability. By contrast, traditional meters impose a second-best uniform pricing, which distorts the optimal mix of energy sources. Decentralizing the second-best requires either cross-subsidy from the intermittent source to the reliable source of energy or structural integration of the two types of technology.

Keywords: renewable resources, wind electricity, solar energy, global warming

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1 Introduction

The substitution of renewable sources of energy such as wind and photovoltaic power for fossil fuel in electricity production is one of the key technological solutions to mitigate global warming. It is currently pushed forward by many scientists and policy makers in the debate on greenhouse gas emission reduction. It has led to environmental policies to support less-carbon intensive renewable sources of energy such as subsidies, feed-in tariffs and mandatory minimal installed capacity. In all developed countries, the generation of electricity from geothermal, wind, solar and other renewables increases by more than 20% a year. Nevertheless there is a large difference between OECD Europe where renewables count for 6% of electricity generation and OECD North America where the ratio is 2.5%.[1] The difference mainly comes from the policy of the European Commission. It has fixed a minimum target of a 20 % share of energy from renewable sources in the overall energy mix for 2020. If all Member States could achieve their national targets fixed in 2001, 21 % of overall electricity consumption in the EU would be produced from renewable energy sources by 2010.[2]

An essential feature of most renewable sources of energy is intermittency. Electricity can be produced from wind turbines only during windy days, from photovoltaic cells during sunny days and certainly not during the night, from waves and swell when the sea is rough. All these intermittent sources of energy rely on an input (wind, sun, waves, tide) whose supply depends on out-of-control conditions. Some of these conditions are perfectly predictable, for example the seasonal duration of day period for sun power or the tide level. Others like wind and sunshine intensity can only be forecasted few days in advance although with some degree of uncertainty.

By contrast, a particular feature of the electricity industry is the commitment

[1] OECD/IEA (2010), Monthly Electric Statistics, November 2009; available at www.iea.org/stats/surveys/mes.pdf

[2] "Renewable Energy Road Map. Renewable energies in the 21st century: building a more sustainable future"; available at europa.eu/scadplus/leg/en/lvb/l27065.htm.

of retailers to supply electricity to consumers at a given price anytime for any level of demand. This business model reflects the consumers' taste for a reliable source of energy viewed as essential, for example for lightning, cooling or heating. Blackouts being very costly, electricity production and supply are designed to match the demand of consumers any time at any location on the grid. Clearly, the variability and unpredictability of intermittent sources of energy conflict with the reliable supply of electricity.

One way to reconcile intermittent supply with permanent demand consists in storing the input, the output or both. To that respect hydropower production is an attractive source of energy. Although it relies on uncertain rainfall and snow, water can be stored in reservoirs to supply peak load with electricity. In particular, in northern countries, water is stored in fall and spring to be used in winter for heating and lightening. By contrast input storage is not possible for the growing sources of renewables, wind and sun power. As regards output storage, it is also very limited. The current storage technologies through batteries are very costly and inefficient so far. An intermediary solution in combination with hydropower is pumped storage.[3]

The introduction of a significative share of intermittent and non storable source of energy is a new challenge for the operators and regulators of the electricity industry. On top of difficulties for the transport and distribution grid, intermittent sources raise problems at the generation stage. In this paper we are mainly interested in three of them. The first one is the efficient mix of intermittent sources (wind, solar,...) and reliable sources such as fossil fuel (coal, oil, natural gas) or nuclear power. The second issue is the compatibility of intermittent sources of energy with market mechanisms. Specifically, do competitive markets allow to decentralize the efficient mix of capacity? The third one is the design of an environmental policy aimed at promoting low carbon technologies by relying on intermittent sources of energy and simultaneously guaranteeing security of supply.

[3] Cheap electricity is used at periods of low demand to restore water resources that can be used to generate electricity at periods of peak demand. See Crampes and Moreaux (2010).

To address those issues, we rely on a stylized model of energy investment and production with two sources of energy: an intermittent and non-storable one, say wind, and a reliable one, say fossil fuel. The two sources differ in cost and in availability. Of course, both sources require installed capacities at a cost. Electricity generation in plants using non intermittent energy costs the price of fossil fuel plus a possible polluting emission tax or price. By contrast, producing electricity from wind is (almost) free once capacity is installed. Nevertheless, it is possible only in the "states of nature" where the input (wind) is available. We characterize the efficient energy mix in installed capacity and production depending on costs. We show that, depending on the cost of operating the reliable source, i.e. the cost of fuel plus pollution permits, wind power is used either as a substitute or a complement with fuel power during windy days. We also show the economic findings remain unchanged when we consider the case of several sources of intermittent energy, *e.g.* wind turbines at two different locations with unequal weather conditions. We determine under which conditions it is efficient to invest in both sources of intermittent energy even if one is more efficient at producing MWh.

Next, we show that decentralizing the efficient energy mix requires to set prices contingent on the availability of the intermittent source of energy, that is on weather conditions. Imposing the same price in all states of nature (either wind turbines are spinning or not) leads to a second-best with under-investment in wind power and over-investment in fossil fuel. The reason is that a uniform price does not reflect energy scarcity in each state of nature. The price is too high during windy days when energy is abundant and too low during windless days when energy is scarce. Wind power production is thus more profitable than fossil power. As a consequence, a regulated electricity monopoly that operates the two technologies under a zero profit condition experiences a deficit on fossil power which is financed by the profit from its wind power division. If, by contrast, electricity is supplied by competing firms owning one of the two technologies, the zero profit condition of the fossil power producers implies strictly positive profits for wind power producers. We then provide policy insights for efficiency improvement and climate change mitigation.

The paper is not the first to address the efficient energy mix with renewable and non-renewable sources of energy, its compatibility with market mechanisms and the related public policies. Notably, Fischer and Newell (2008) assess different environmental policies to mitigate climate change with the two sources of energy including wind power. However, their focus is on innovation and technological improvement of both technologies. They abstract from wind power intermittency by assuming a reliable annual output with wind power. Other papers focus on the storage of energy in reservoirs for hydropower. They examine competition among hydropower plants (Garcia *et al.*, 2001, Ambec and Doucet, 2002) or between a hydropower and a thermal producer (Crampes and Moreaux, 2001). However, all these papers consider deterministic supply of renewable inputs whereas here we focus on input variability.

The economics of intermittent sources of electricity production are still in their infancy. Most papers on the subject are empirical and country specific. For example, Neuhoff *et al.* (2006 and 2007) develop a linear programming model to capture the effects of the regional variation of wind output on investment planning and on dispatching in the UK when transport is constrained. Kennedy (2005) estimates the social benefit of large-scale wind power production (taking into account the environmental benefits) and applies it to the development of this technology in the South of Long Island. Boccard (2008) computes the social cost of wind power as the difference between its actual cost and the cost of replacing the produced energy. He divides the social cost into technological and adequacy components and applies the break-up to Denmark, France, Germany, Ireland, Portugal and Spain. Müsgens and Neuhoff (2006) build an engineering model representing inter-temporal constraints in electricity generation with uncertain wind output. They provide numerical results for the German power system. Coulomb and Neuhoff (2005) focus on the cost of wind turbines in relation with changes in their size using data on German prices. Papers like Butler and Neuhoff (2004) and Menanteau *et al.* (2003) are closer to ours than the former ones. They consider the variety of tools available for public intervention in the development of renewable energy in general, and intermittent sources in particular.[4] Our analysis is upstream the latter papers as it provides a microeconomic framework for the study of optimal investment and dispatching of wind or solar plants. It also allows to determine by how much market mechanisms depart from the outcome of optimal decisions. Garcia and Alzate (2010) compare the performance of two public policies: feed-in tariffs and mandatory portfolio standards. They also examine the efficient energy mix but with an inelastic demand which is nil beyond a maximal price. Their model cannot capture the social cost of rationing demand. By contrast we consider a standard increasing and concave consumer's surplus function (or utility for electricity consumption) which leads to a more general demand decreasing in price. Our framework allows to better assess the social impact of energy production on consumers. Moreover, it is a more realistic assumption to analyze long run decisions concerning investment in generation capacity.

The paper is organized as follows. Section 2 sets up the model with the two sources of energy (reliable and intermittent) and determines the first-best dispatch and generation capacities. Section 3 analyzes the decentralization of first-best with state-contingent prices and second-best with uniform prices. In section 4 we extend the model to two different sources of intermittent energy. Section 5 discusses two policy insights based on our main results, namely the development of smart technologies necessary for the implementation of first best and the structural or financial links between technologies necessary to the implementation of second-best. Section 6 concludes.

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[4] All these papers are devoted to wind power. Borenstein (2008) proposes a deep economic analysis of solar photovoltaic electricity production with a focus on California.

2 First best production

We consider an industry where consumers derive gross utility $S(q)$ from the consumption of q kWh of electricity. This function is unchanged along the period considered.[5] It is a continuous derivable function with $S' > 0$ and $S'' < 0$.

Electricity can be produced by means of two technologies. First a fully controlled technology (e.g. coal, oil, gas, nuclear, hydropower with water storage) allows to produce q_f at unit cost[6] c as long as production does not exceed the installed capacity, K_f. The unit cost of capacity is r_f. This source of electricity will be named the "fossil" source. We assume $S'(0) > c + r_f$; in words, producing electricity from fossil energy is efficient when it is the only source.

The second technology relies on an intermittent source of energy such as solar energy or wind. It allows to produce q_i kWh at 0 cost as long as q_i is smaller than the installed capacity K_i, whose unit cost is r_i and the primary energy is available. We assume two states of nature: “with" and “without" intermittent energy. The state of nature with (respectively without) intermittent energy occurs with probability ν (respectively $1 - \nu$) and is denoted by the superscript w (respectively $\overline{w}$).

For simplicity, we abstract from environmental issues related to electricity production by assuming that c, r_f and r_i include the environmental marginal costs to society. More specifically, burning fossil fuel to produce q_f KWh requires to buy carbon emission permits or to pay a carbon tax. We assume that they are part of the marginal cost c and they reflect the marginal damage due to climate change by one KWh produced from fossil energy once capacity has been installed. Thanks to this assumption it is not unrealistic to consider the case of wind or solar power technologies being more competitive than thermal plants. Therefore, we will take all

[5] The problem we have in mind better corresponds to wind than to solar energy. Indeed, demand is changing along the daily cycle so that there is some positive correlation between demand for electricity and the supply of solar energy. This correlation does not hold as regards wind. And there is no reason for a negative correlation either.

[6] The unit cost includes the tax on polluting emissions or the price of emission permits, if any.

combinations of cost parameters into consideration.

The first-best problem to solve is twofold. First, the central planner determines the capacities K_i, K_f to install. This is the long run commitment of the decision process. Second, it chooses how to dispatch the capacities in each state of nature q_i^w, q_f^w and $q_i^{\overline{w}}$, $q_f^{\overline{w}}$, depending on the availability of the intermittent source. It is a short run decision constrained by the installed capacities. When deciding on the dispatch of plants, the planner knows the state of nature.

Although the problem *a priori* counts six decision variables, three can be easily determined, leaving us with only three unknowns to be determined. Actually,

i. $q_i^{\overline{w}} \equiv 0$: windmills cannot produce if there is no wind and solar batteries cannot produce absent any sun ray;

ii. $q_i^w \equiv K_i$: since the installation of the capacity for producing with the intermittent source is costly, it would be inefficient to install idle capacity.[7]

iii. $q_f^{\overline{w}} = K_f$: without intermittent source of energy, since demand is unchanged and the available capacity is reduced from $K_f + K_i$ to K_f, it would be inefficient to leave idle some production capacity.[8]

For the three remaining decision variables K_i, K_f and q_f^w, the planner's program can be written as follows:[9]

$$(P1) \qquad \max_{K_i, K_f} \nu \left[\max_{q_f^w} S(K_i + q_f^w) - cq_f^w \right] + (1-\nu)[S(K_f) - cK_f] - r_f K_f - r_i K_i$$

$$s.t. \qquad q_f^w \geq 0 \quad , \quad q_f^w \leq K_f \quad , \quad K_i \geq 0$$

As proven in the Appendix, we can establish the following:

[7] We discard the necessary maintenance operations, for example assuming that they can be performed during type $\overline{w}$ periods.

[8] Here again, we discard maintenance operations, for example by assuming that capacity is measured in terms of available plants.

[9] Note that it is not necessary to write explicitly the constraint $K_f \geq 0$ because $K_f > 0$ is granted by the assumption $S'(0) > c + r_f$.

Proposition 1 *First best capacities and outputs are such that*

a) *for* $\frac{r_i}{\nu} > c + r_f$

$$q_f^w = q_f^{\overline{w}} = K_f = S'^{-1}(c + r_f), \qquad q_i^w = K_i = 0$$

b) *for* $c > \frac{r_i}{\nu}$

$$q_f^w = 0 < q_f^{\overline{w}} = K_f = S'^{-1}\left(c + \frac{r_f}{1-\nu}\right) < q_i^w = K_i = S'^{-1}\left(\frac{r_i}{\nu}\right)$$

c) *for* $c + r_f > \frac{r_i}{\nu} > c$

$$q_f^w = q_f^{\overline{w}} = K_f = S'^{-1}\left(\frac{c+r_f-r_i}{1-\nu}\right), \quad q_i^w = K_i = S'^{-1}\left(\frac{r_i}{\nu}\right) - S'^{-1}\left(\frac{c+r_f-r_i}{1-\nu}\right)$$

In case a) the intermittent energy is so scarce (small ν) and/or the technology using this energy is so costly (high r_i) that no plant using intermittent energy should be installed. Then whatever the state of nature, the fossil plant is used at full capacity. The capacity is determined by the equality between the marginal utility of electricity and its long run marginal cost $c + r_f$. In case b) wind is so abundant and wind turbines so cheap that the intermittent energy totally replaces fossil energy in state of nature w. The capacity to install equates the unit cost of capacity r_i discounted by the probability of availability ν to marginal utility. Fossil energy capacity is only used in state of nature $\overline{w}$. Its long run marginal cost is c plus the capacity cost r_f discounted by the probability of using it $1 - \nu$ since it is dispatched only when the intermittent source is not available.

In the intermediary case c), fossil energy is used at full capacity jointly with intermittent energy. This case is illustrated in Figure 1. The merit order in state of nature $\overline{w}$ just consists in dispatching fossil energy up to K_f determined by the equality between marginal utility and long run marginal cost. The latter is equal to the cost of the marginal technology in state $\overline{w}$ that is $\frac{c+r_f}{1-\nu}$ reduced by the saving on the cost of developing the other technology. This is because, at periods w, f is the marginal technology to dispatch (since $c > 0$) but i is the one to develop (since $\frac{r_i}{\nu} < c + r_f$). Then $\frac{r_i}{\nu}$ is the long run marginal cost of the whole system and it determines the total capacity to install $K_i + K_f$ by $S'(K_i + K_f) = \frac{r_i}{\nu}$.

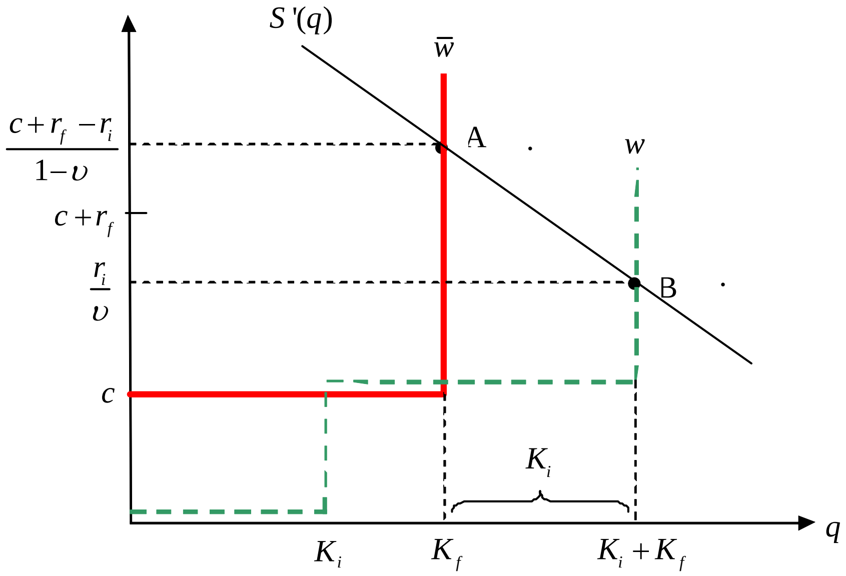

Figure 1: First best when the two technologies are used in state of nature w

Figure 2 allows to understand how the two capacities K_f and K_i must be combined at first best. We have depicted K_i , K_f and the sum $K_i + K_f$ as functions of r_i. The graph clearly shows that when the intermittent technology i becomes profitable (that is when $\frac{r_i}{\nu} \leq c + r_f$) it is not simply substituted for fossil energy f. As r_i decreases, it is true that there occurs some substitution since K_f decreases but the total capacity $K_f + K_i$ increases. Substitution cannot be done on a one-to-one basis since nothing can be produced with technology i in state of nature $\overline{w}$. Nevertheless there is some substitution with the consequence that, as compared with a world without technology i, there is less energy available in state of nature $\overline{w}$ than in state w.

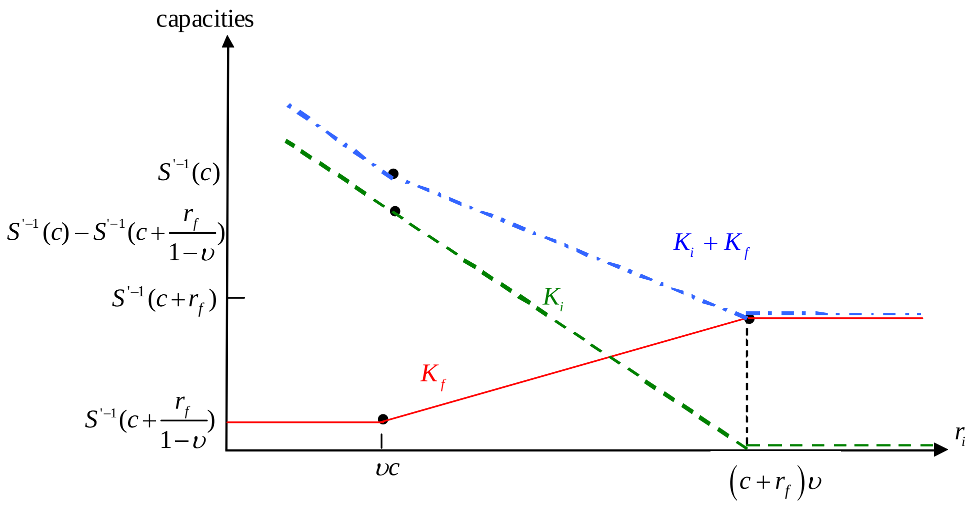

Figure 2: Capacities as functions of the development cost of type-i technology with varying prices

3 Decentralization

Regulation authorities in most developped countries promote simultaneously renewable sources for electricity production and the liberalization of the industry. To assess the consequences of these separate policies, we now consider the decentralization of first best by market mechanisms taking account of the reactivity of consumers to price variations (Section 3.1), then on the contrary their lack of reactivity (Section 3.2).

3.1 Market implementation with reactive consumers

Assume that consumers and firms are price-takers. Suppose also that they are equipped to be price sensitive. It is easy to show that the optimal outcome can be decentralized with prices contingent on states of nature p^w and $p^{\overline{w}}$. In practice, it means that electricity prices should depend on the presence or the absence of the intermittent source of energy.

In each state of nature $s \in \{w, \overline{w}\}$, consumers facing price p^s solve $\max_q S(q^s) - p^s q$. They demand q^s kWh in state s where $S'(q^s) = p^s$ (marginal utility equals price) for $s = w, \overline{w}$.

First consider case a) whereby $\frac{r_i}{\nu} > c + r_f$. The prices that decentralize the optimal outcome are $p^w = p^{\overline{w}} = c + r_f$. Consumers react to that prices by consuming the efficient productions $q^w = q^{\overline{w}} = S'^{-1}(c + r_f)$. Producers owning the intermittent technology i invest nothing since the long term marginal cost r_i of each kWh exceeds the expected unit benefit $p^w \nu$. Producers endowed with the fossil technology f invest up to supply all consumers $K_f = q^w = q^{\overline{w}}$. Since the long run marginal cost of each kWh $c + r_f$ equals the market price in both states of nature $p^w = p^{\overline{w}}$, they make zero profit. Clearly, the prices that decentralize first-best are unique. With lower prices, fossil electricity producers would not recoup their investment and thus would invest nothing. Symmetrically, with higher prices, more fossil fuel capacities would be installed and competitive entry would reduce prices to the long term marginal cost.

Second, in case b) where $c > \frac{r_i}{\nu}$, the prices that decentralize first best are $p^w = \frac{r_i}{\nu}$ and $p^{\overline{w}} = c + \frac{r_f}{1-\nu}$ per kWh. As before, consumers react to those prices by consuming $q^w = S'^{-1}(\frac{r_i}{\nu})$ in state of nature w and $q^{\overline{w}} = S'^{-1}(c + \frac{r_f}{1-\nu})$ in state $\overline{w}$. In state w, firms producing energy from fossil sources cannot compete with those producing from intermittent sources. They therefore specialize in producing only during state of nature $\overline{w}$. Their expected return on each unit of capacity is thus $(1 - \nu)(p^{\overline{w}} - c) = r_f$. Since it exactly balances the marginal cost of capacities, the plants using fossil source have a zero expected profit. Similarly, firms with intermittent technology obtain an expected return $\nu p^w = r_i$ per unit of investment and thus zero profit on average. In other words, under those prices, each type of producer recoups exactly its long term marginal cost, taking into account the probability of using capacities.

Third, in case c) whereby $c + r_f > \frac{r_i}{\nu} > c$, with prices $p^w = \frac{r_i}{\nu}$ and $p^{\overline{w}} = \frac{c+r_f-r_i}{1-\nu}$ the market quantities also are at first-best levels. Consumers' demand is $q_f^{\overline{w}} = S'^{-1}(\frac{c+r_f-r_i}{1-\nu})$ when the wind is not blowing and $q_f^w + q_i^w = S'^{-1}(\frac{r_i}{\nu})$ when it is. In state w, competing producers are ordered on the basis of their bids which are equal to short run marginal cost under perfect competition, that is 0 for i-producers and c for f-producers. The investment in capacity depends on expected returns and long run marginal costs. Fossil electricity firms produce in both states of nature. The return per unit of capacity is thus $\nu p^w + (1-\nu)p^{\overline{w}} - c$ which matches exactly the capacity unit cost r_f. Thus f-producers make zero profit. On the other hand, i producers get in expectation νp^w per unit of investment which also matches exactly the cost r_i. They therefore make zero profit as well which is the equilibrium under free entry condition. Therefore, we have established the following.

Proposition 2 *State contingent prices p^w and $p^{\bar{w}}$ with $p^{\bar{w}} \geq p^w$ and free entry allow market mechanisms to reach first best. When it is efficient to install intermittent sources of energy, $p^{\bar{w}} > p^w$.*

3.2 Market implementation with non-reactive consumers

The decentralization process described in the former section faces a serious hurdle. The first best dispatch and investment can be driven with state contingent prices only if consumers have smart meters signaling scarcity values and if they are able to adapt to price signals. Actually, most consumers, particularly among households, are equipped with traditional meters. Consequently they are billed at a price independent of the state of nature.

To assess the consequences of a uniform pricing constraint, we determine the efficient production and investment levels constrained by uniform delivery. Actually, with a stationary surplus function as assumed here, consumers react to uniform prices by consuming the same amount of electricity in both states of nature. Formally, non state contingent pricing implies the constraint $q_i^w + q_f^w = q_f^{\overline{w}}$. Yet, since the intermittent (resp. reliable) technology is used under full capacity in state w (resp. $\overline{w}$) the later constraint leads to $K_i + q_f^w = K_f$. To distinguish it from first-best, we denote the solution of the (second-best) uniform pricing constrained program by ($\tilde{q}_i^w$, $\tilde{q}_f^w$, $\tilde{K}_i$ $\tilde{K}_f$).

As shown in the Appendix, the main consequence of this restriction is that the intermittent source of energy will never be used in complement to fossil energy in state w. More precisely, case c) of Proposition 1 where both technologies are operated in state w (namely for $c < \frac{r_i}{\nu} < c + r_f$) disappears. This is because the constraint of uniform provision $K_i + q_f^w = K_f$ makes the two technologies perfect substitutes in state w. It results in a bang-bang solution. If $c < \frac{r_i}{\nu}$ only technology f is installed and $S'(K_f) = c + r_f = \tilde{p}^w = \tilde{p}^{\overline{w}}$. The uniform price just matches the long term marginal cost of the f technology. On the other hand, if $c > \frac{r_i}{\nu}$, both technologies are installed but only technology i is used when possible, *i.e.* in state w with $S'(K_f) = S'(K_i) = (1-\nu)c + r_f + r_i = \tilde{p}^w = \tilde{p}^{\overline{w}}$. The uniform price equals the long term marginal cost of each kWh, namely $(1-\nu)c + r_f + r_i$, taking into account that the two technologies are developed to insure uniform delivery and c is incurred only in state $\overline{w}$ which arises with probability $1-\nu$. We therefore can assert the following.

Proposition 3 *When prices cannot be state-contingent, second best capacities and outputs are such that:*

a) for $\frac{r_i}{\nu} > c$

$$\tilde{q}_f^w = \tilde{q}_f^{\overline{w}} = \tilde{K}_f = S'^{-1}(c + r_f), \quad \tilde{q}_i^w = \tilde{K}_i = 0$$

b) for $\frac{r_i}{\nu} < c$,

$$\tilde{q}_f^w = 0 < \tilde{q}_f^{\overline{w}} = \tilde{K}_f = \tilde{q}_i^w = \tilde{K}_i = S'^{-1}((1-\nu)c + r_f + r_i)$$

The disappearance of the possibility to jointly operate the two technologies in state w can be illustrated using Figure 1. The fact that consumers are weakly price-sensitive can be viewed as if their marginal surplus curve $S'(q)$ were more vertical.[10] Consequently the horizontal difference between points A and B is smaller and smaller, which means that K_i converges to zero in this interval of cost.

[10] Notice that consumers are not inelastic to price since their demand (marginal utility) function has a finite negative slope. They would change their consumption if they could receive state-contingent price signals. But they cannot react since they receive a uniform price signal.

Figure 3 shows how the uniform-pricing constraint transforms the capacities as functions of the cost of renewables r_i. In the left part of the graph where it is socially profitable to invest in technology i, the two technologies become strict complements, contrary to what we have observed in Figure 2. The consequence is that, except if $\nu = 1$ where $\tilde{K}_f = 0$, the smaller r_i the larger $\tilde{K}_f = \tilde{K}_i$. In words, even if the renewable source has a very high probability of availability but cannot be totally guaranteed, when prices are not state-contigent the whole capacity must be duplicated. Actually, the problem is the same as for reserve capacities that must be available to replace failing plants or to supply unexpected demand, except that in state $\bar{w}$ the whole type-i capacity is failing and must be replaced.

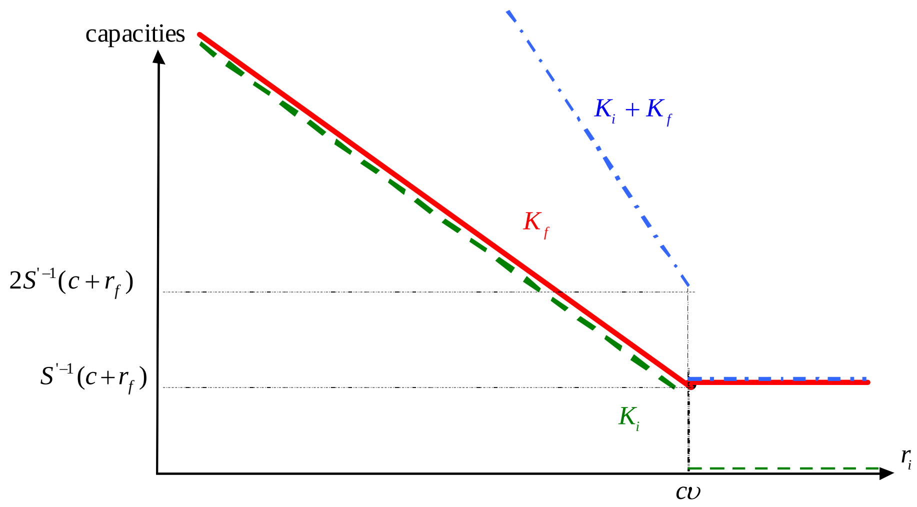

Figure 3: Capacities as functions of the development cost of type-i technology with constant prices

An important drawback of the second-best solution with a mix of the two technologies (that is when $c > \frac{r_i}{\nu}$) is that it requires some form of subsidy from technology i to technology f to secure non-negative profits. Absent any external financial transfer, second-best can be decentralized only under certain conditions, for example a regulated electricity monopoly or competitive firms owning the two technologies.

To prove this surprising result, we first observe that the expected unit profit of a firm using the two technologies is nil since

$$\nu\tilde{p}^{w} - r_i + (1-\nu)(\tilde{p}^{\overline{w}} - c) - r_f = 0$$

Thus the division operating technology i obtains positive cash flows

$$\nu\tilde{p}^{w} - r_i = \nu\left[(1-\nu)(c - \frac{r_i}{\nu}) + r_f\right] > 0$$

whereas the fossil energy division incurs financial losses $(1-\nu)(\tilde{p}^{\overline{w}} - c) - r_f < 0$. These losses are obviously larger and larger when ν increases since $i)$ the price decreases, $ii)$ the type-f technology is less often called into operation and $iii)$ the capacity to install increases. This can result in huge financial resources so that transfers from division i towards division f are necessary to sustain second-best.

What occurs when the two technologies are owned by separate operators and transfers are not allowed? In a competitive industry with free-entry, the fossil energy-based electricity producers will exit the market under the second-best electricity price. This will reduce the supply of energy in state $\overline{w}$ and, therefore, increase the price of electricity in both states of nature above the second best level. The free entry equilibrium price in a competitive industry with a unique price in the two states of nature is such that firms with fossil technology make zero profit. It thus matches the fossil energy producer's long term marginal cost $c + r_f$. The firms with intermittent energy technology i enjoy strictly positive profits. They free-ride on the uniform price constraint.

Finally, note that since $p^{\overline{w}} = c + \frac{r_f}{1-\nu} > \tilde{p}^{w} = \tilde{p}^{\overline{w}} > p^{w} = \frac{r_i}{\nu}$, and prices signal investment opportunities, the capacity of intermittent energy installed under uniform price is smaller than at first-best whereas the opposite stands for fossil energy, i.e. $\tilde{K}_i < K_i$ and $\tilde{K}_f > K_f$. This is true when $\tilde{K}_i = \tilde{K}_f$, that is when $\frac{r_i}{\nu} < c$, but it is also obviously true when $c < \frac{r_i}{\nu} < c + r_f$ since $\tilde{K}_i = 0 < K_i$ and $\tilde{K}_f < K_f + K_i$.

We therefore can assert the following:

Proposition 4 *When the price of electricity cannot be state contingent, second best is implementable only if the two sources of energy are owned by the same financial* *entity or if the government transfers revenues from intermittent sources to reliable sources. Otherwise, free entry with uniform price leads to (i) over-investment and zero profits in fossil fuel electricity production; (ii) under-investment and strictly positive profits in the intermittent source industry.*

To summarize, this section the decentralization of first-best production levels of electricity calls for a lower price when intermittent sources of energy are available. If not feasible for technological or institutional reasons, the second-best production levels under the uniform pricing constraint distort first-best prices by increasing the price of intermittent energy p^w and reducing the price of fossil energy when intermittent energy in not available $p^{\overline{w}}$. Intermittent energy is therefore overvalued and fossil-fuel electricity undervalued compared to first-best. It thus leads to under-investment into intermittent energy and over-investment into fossil fuel electricity. In a nutshell, since a uniform price does not reflect state-of-nature marginal costs, consumers tend to over-consume electricity when it is costly to produce (in state $\overline{w}$) and under-consume it when it is cheap (in state w). Compared to first-best, this increases demand for fossil energy and reduces it for intermittent energy. Long run supply through investment in capacities is adapted accordingly.

4 Two sources of intermittent energy

The former results can be easily generalized to cases where several sources of intermittent energy are available. Assume there are two sources, labelled 1 and 2. The two sources can be of different kind, e.g. wind and solar. They also can be of the same kind but at different locations e.g. turbines facing different wind conditions. As a consequence, the two sources differ potentially both in their occurrence and in the energy produced when available. For instance, they might face different dominant winds (north versus south), one being stronger on average than the other.

The results of the former sections can be extended to the multiplicity of sources by increasing the number of states of nature. For example, with two turbines located at different places, we have four states of nature: in state 1 only the intermittent source of energy 1 is available, in state 2 only the intermittent source of energy 2 is available, in state 12 both are available and, as before, in state $\overline{w}$ none is available (and therefore electricity can only be produced from fossil energy). These states of nature occur with probabilities ν_1, ν_2, ν_{12} and $1-\nu$ respectively where $\nu = \nu_1 + \nu_2 + \nu_{12}$. Let us denote by K_i the investment into intermittent source of energy i for $i = 1, 2$. The unit cost of capacity of source i is denoted $r_i > 0$ for $i = 1, 2$ where $r_2 > r_1$. For instance, if wind turbines are at different locations and the mean wind is stronger[11] at location 1 than at 2 when it is windy, then with a smaller number of wind turbines at location 1 one can produce the same amount of electricity at location 1 and at location 2. Yet, the occurrence of the two sources of intermittent energy might make location 2 attractive.

The planner must determine the capacity of the two intermittent sources of energy K_1 and K_2 in addition to the fossil source K_f and the production levels q_f^s, q_1^s and q_2^s in states $s = \overline{w}, 1, 2$ and 12. Using notations similar to the former section's, we can easily determine that for $i = 1, 2$, $q_i^{\overline{w}} \equiv 0$, $q_i^s = K_i$ in states $s = 1, 2, 12$ and $q_f^{\overline{w}} = K_f$. The remaining decision variables K_1, K_2, K_f, q_f^1, q_f^2 and q_f^{12} are determined by solving program $(P2)$ below:

$$
\begin{aligned}
(P2) \max_{K_1, K_2, K_f} \quad & \nu_1 \max_{q_f^1} \left[S(K_1 + q_f^1) - c q_f^1 \right] + \nu_2 \max_{q_f^2} \left[S(K_2 + q_f^2) - c q_f^2 \right] \\
& + \nu_{12} \max_{q_f^{12}} \left[S(K_1 + K_2 + q_f^{12}) - c q_f^{12} \right] + (1-\nu)[S(K_f) - cK_f] \\
& - r_f K_f - r_1 K_1 - r_2 K_2
\end{aligned}
$$

subject to

$$0 \leq q_f^s \leq K_f \text{ for } s = 1, 2, 12; \; K_i \geq 0 \text{ for } i = 1, 2.$$

Depending on the cost parameters and the probability of each state of nature we can obtain a large spectrum of results, some with only one intermittent source of

[11] Nevertheless the wind should not be "too strong" because windmills could not resist.

energy to operate, others combining the two sources. In each case we can derive the capacity to install and the dispatch that maximize net social welfare. Using the proof in the appendix, we just establish the following proposition.

Proposition 5 *First best capacities in the intermittent sources of energy* 1 *and* 2 *are such that*

- *a) For* $\nu_1 > 0$ *and* $\nu_2 > 0$*,* $K_1 = K_2 = 0$ *if and only if* $c + r_f < \frac{r_i}{\nu_i + \nu_{12}}$ *for* $i = 1, 2$.
- *b) For* $c + r_f > \frac{r_i}{\nu_i + \nu_{12}}$
  - *1)* $K_1 > 0$ *and* $K_2 = 0$ *if* $\nu_1 = \nu_2 = 0$ *and* $\nu_{12} > 0$*,*
    (perfect positive correlation),
  - *2)* $K_1 > 0$ *and* $K_2 > 0$ *if* $\nu_1 > 0$*,* $\nu_2 > 0$ *and* $\nu_{12} = 0$
    (perfect negative correlation).

As in investment portfolios, the decision to invest in various intermittent technologies does not only depend on the return on investment but also on the risk associated to each return. According to *a)* in Proposition 5, a necessary and sufficient condition for investing in an intermittent source of energy is $\frac{r_i}{\nu_i + \nu_{12}} < c + r_f$ for one $i \in \{1, 2\}$ at least: the long run marginal cost of electricity produced from source i discounted by the probability of its availability $\nu_i + \nu_{12}$ must be lower than the long run marginal cost of electricity produced from fossil energy. Depending on the value of the parameters, in some cases, the two sources of intermittent energy are installed and, in other cases, only one is installed. For instance, consider the extreme cases *b.1)* and *b.2)* of perfectly positive and negative correlations respectively. If sources 1 and 2 are always available only at the same time (perfect positive correlation), we have $\nu_1 = \nu_2 = 0$. Then only the more efficient source of intermittent energy should be installed. Even if $\frac{r_i}{\nu_{12}} < c + r_f$ for $i = 1, 2$ so that the two sources of intermittent energy have lower discounted marginal cost than fossil energy, only source 1 is installed since we have assumed $r_1 < r_2$. On the contrary, if sources 1 and 2 are never available at the same time (perfect negative correlation), which translates formally into $\nu_{12} = 0$, then as long as $\frac{r_i}{\nu_i} < c + r_f$ for $i = 1, 2$ both sources of intermittent energy are installed. In particular, source 2 is installed even if it is more costly ($r_2 > r_1$) and/or less frequent ($\nu_2 < \nu_1$). Concretely, if wind turbines can be developed at two different locations, one being superior in terms of wind speed and frequency, it is efficient to install turbines at both locations to exploit the complementarity of the two sources of energy as long as their discounted long run marginal costs are lower than the fossil energy cost. Like in all portfolio management problems, negative correlation allows some form of insurance. But as long as $\nu_1 + \nu_2 + \nu_{12} < 1$, it is necessary to install reliable capacity to replace the intermittent technologies in "bad" states of nature. This cost should be internalized by the builders and operators of the plants using intermittent sources. Notice that the multiplicity of locations make more costly the transmission of any signal about which source of energy is currently generating electricity. Consequently, our former developments about the difficulty to implement first and second best are made even more relevant.

5 Policy insights

The model developed in the above sections analyzes cases where intermittent technologies can compete against fossil fuel technologies. This may be the case in the future after a technological break or some drastic learning effect, or due to more stringent climate change mitigation policies (higher carbon taxes or fewer emission permit) leading to a higher marginal cost for fossil combustion.[12] Meanwhile, intermittent technologies are sustained by public aids (e.g. certificates, feed-in tariffs) or purchase requirement that represent a financial burden for society. These costs are well known. They are so high that at the beginning of 2010, some governments

[12] According to the Royal Swedish Academy of Sciences (2010), the full cost for electricity generated by wind power, excluding the costs for expanding the electricity power line network and the back-up power, is currently 6-10c€/kWh. This is slightly larger than the generation cost in coal power plants but twice the cost in nuclear plants.

(in particular France and Germany) have decided to step back from a blind policy of support to photovoltaic energy. By contrast the back-up costs we have identified with our model are less emphasized and still entail important policy implications as regards the future of the energy industry. We first focus on the cost of adapting the consumption electrical appliances and the network to decentralize the first best. We then consider the structural or institutional arrangement required to decentralize the second-best with uniform pricing.

5.1 Smart equipment and smart network

Our first-best analysis suggests that intermittent technologies should be promoted in parallel to smart meters and/or smart boxes. These intelligent devices can make electricity consumption dependent on the state of nature that prevails at the location of production plants. By controlling in real time some programmed electric equipments such as boilers and heaters, disconnecting them when the intermittent source of energy is not available, smart meters and boxes renders electricity consumption sensitive to the energy scarcity across time and space. They are likely be more receptive and reactive than consumers exposed to messages such as "the wind turbines you are connected to are currently running; therefore the price of electricity is low" (or the opposite). The smart meters or boxes that would dispatch automatically consumption across time need to be connected with information technologies to be installed in the shadow of the energy network. More generally, the growth of intermittent energy calls for further investment in network, increasing both connection and information processing. Indeed, compared to thermal power plants, wind and solar power plants are more likely to be scattered on a given territory. This has two consequences. First, connection requires large investment in small scale lines, transformers and meters. This obviously makes coordination necessary between producers, transmitters and system and market operators. Second, random local injections radically modify the business model of distributors since they now have to balance the flows on the grid under their responsibility and maybe to install new lines to guarantee the reliability of the local system under the constraint to accept all injections by authorized generators. The adaptation of networks to the development of intermittent sources has been underestimated so far. In most developed countries making the grid smart is now a priority, which means huge investments for embedding Information and Communication Technologies (ICT) into the grid.[13]

5.2 Structural arrangements

It may appear that the huge cost of installing smart appliances at consumption nodes coupled with ICT devices all along the grid is too high compared to the difference between first best and second best welfare levels. If so, consumers will continue to face a single price whenever wind turbines are producing or not. Compared to the first-best with state-contingent prices, they demand too little energy when the intermittent source is available and too much when it is not. Were the resulting equilibrium price an average value of the marginal costs of production in the different types of generation plants, the generators using fossil energy would lose money. Then they would exit the industry. We therefore have to consider several structural and legal solutions to implement the second-best outcome. Under free entry and exit, in order to keep generators using fossil fuel in the market, the price should be equal to the long run marginal cost of their MWh. It is as if consumers had to pay for a guarantee of service. The drawback of this solution is that the owners of plants using intermittent energy pocket a benefit equal to the difference between the long run marginal cost of electricity from fossil fuel and the long run marginal cost of electricity from intermittent energy. Consumers pay for an insurance and the money thay pay is seized by those who create randomness. Two public policies can reduce the rent assigned to intermittent energy producers. A first one consists in taxing windmills to subsidize thermal plants in order to balance the budget of all producers. A second one is mandatory technological mix or insurance. Each producer should

[13]See for example the website www.smartgrids.eu.

either control the two technologies or buy an insurance contract that guarantees energy supply any time. These legal arrangements would force any new entrant to insure production in all states of nature. Both policies have their drawbacks. The first one, more “market-based”, is at a cost of levying and redistributing public funds. The second one, more “command-and-control", restricts the firms’ flexibility in their technological choices.

6 Conclusion

The development of intermittent sources of energy to produce electricity creates a series of difficulties as regards the adaptation of behavior, structures and institutions to the characteristics of these sources. Satisfying the demand for non contingent electricity at a non contingent price clearly requires an installed capacity of non intermittent sources equal to the capacity of intermittent source, whatever the availability duration of the intermittent source. Actually, because availability periods are not known with certainty and fossil fuel plants cannot be dispatched instantaneously when it is necessary to replace intermittent sources, the back-up capacity must even be larger. In Ireland for example, *"incorporating 30GW of additional renewable capacity into the grid, to meet EU’s 2020 target, will require a further 14-19GW of new fossil fuel and nuclear capacity to replace plants due to close and to meet new demand (almost doubling the total new installed electricity generating capacity required by 2020, compared to a scenario where renewable generation was not expanded)."*[14] In our model we have analyzed the basic parameters that should be considered to determine the capacity of intermittent and non-intermittent production plants anticipating their efficient dispatch. Nevertheless these first-best decisions are not implementable because they necessitate prices varying with the state of nature and consumers reacting accordingly. We also have shown that second best constrained by fixed delivery is not financially feasible. The conclusion is that an electricity industry with a large

[14] Northern Ireland Assembly (2009).

share of intermittent sources is not sustainable without an obligation of integration in production, either structural or financial. An alternative solution that we did not consider is random supply, but we can be sure that it is not politically sustainable.

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A First best production and capacity

The Lagrange function corresponding to problem (P1) in the text is[15]

$$\begin{aligned}\mathcal{L} &= \nu\left[S(K_i+q_f^w)-cq_f^w+\xi_f^w q_f^w+\eta_f^w(K_f-q_f^w)+\xi_i K_i\right] \\ &\quad +(1-\nu)\left[S(K_f)-cK_f\right]-r_f K_f-r_i K_i\end{aligned}$$

Given the linearity of technologies and the concavity of the surplus function, the following first order conditions are sufficient to determine the first best allocation:

$$\frac{\partial\mathcal{L}}{\partial q_f^w}=\nu\left[S'(K_i+q_f^w)-c+\xi_f^w-\eta_f^w\right]=0 \tag{A1}$$

$$\frac{\partial\mathcal{L}}{\partial K_f}=\nu\eta_f^w+(1-\nu)\left[S'(K_f)-c\right]-r_f=0 \tag{A2}$$

$$\frac{\partial\mathcal{L}}{\partial K_i}=\nu\left[S'(K_i+q_f^w)+\xi_i\right]-r_i=0 \tag{A3}$$

plus the complementary slackness conditions derived from the three constraint of (P1).

We first identify conditions for $K_i>0$.

>From (A3), if $K_i>0$, $S'(K_i+q_f^w)=\frac{r_i}{\nu}$ and we can write from (A1) $\frac{r_i}{\nu}-c=\eta_f^w-\xi_f^w$. Then, we face two possibilities:

- if $\frac{r_i}{\nu}>c$, $\eta_f^w>0$ so that $q_f^w=K_f>0$ and $\xi_f^w=0$.

[15] $\xi_f^w\geq 0, \eta_f^w\geq 0$ and $\xi_i\geq 0$ are the multipliers respectively associated to $q_f^w\geq 0, q_f^w\leq K_f$ and $K_i\geq 0$.

Plugging $\eta_f^w = S'(K_i + K_f) - c$ into (A2) we obtain

$$\nu S'(K_i + K_f) + (1 - \nu)S'(K_f) = c + r_f$$

$>$From $K_i > 0$ and $S'' < 0$, we have $S'(K_i + K_f) < S'(K_f)$ so that

$$S'(K_i + K_f) = \frac{r_i}{\nu} < c + r_f$$

- in the second possibility, $\frac{r_i}{\nu} < c$, the condition $\frac{r_i}{\nu} < c + r_f$ is obviously satisfied.

We conclude that $\frac{r_i}{\nu} > c + r_f$ is sufficient for $K_i = 0$.

We can therefore partition the set of parameters as follows:

a) for $\frac{r_i}{\nu} > c + r_f$, $K_i = 0$. As regards the output of the reliable technology in state of nature w, we have $q_f^w = K_f$. Indeed, assume $q_f^w < K_f$. Then $\eta_f^w = 0$ and from (A1) $S'(q_f^w) - c = -\xi_f^w \leq 0$. Similarly from (A2) $S'(K_f) - c = \frac{r_f}{1-\nu} > 0$. But since $S'' < 0$ these two inequalities are not compatible. It results that $q_f^w = q_f^{\overline{w}} = K_f$ and combining (A1) and (A2) , $S'(K_f) = c + r_f$.

b) for $c > \frac{r_i}{\nu}$, since from (A1) $\frac{r_i}{\nu} - c = \eta_f^w - \xi_f^w$, we have $\xi_f^w > 0$ so that $q_f^w = 0$. Knowing that $K_f > 0$, this implies $\eta_f^w = 0$. Then, from (A2) we have

$$S'(K_f) = c + \frac{r_f}{1 - \nu}$$

and from (A3) and $K_i > 0$

$$S'(K_i) = \frac{r_i}{\nu}.$$

c) for the intermediary case $c + r_f > \frac{r_i}{\nu} > c$, we saw formerly that $K_i > 0$ and $q_f^w = K_f$. From equation (A3).

$$S'(K_i + K_f) = \frac{r_i}{\nu}$$

and combining (A1) and (A2)

$$\nu S'(K_i + K_f) + (1-\nu)S'(K_f) = c + r_f$$

Plugging the first equation into the second,

$$S'(K_f) = \frac{c + r_f - r_i}{1-\nu}.$$

B Second best under uniform provision of electricity

Adding the constraint $K_i + q_f^w = K_f$ and the multiplier γ to the Lagrange function of first best, the first order conditions become

$$\frac{\partial \mathcal{L}}{\partial q_f^w} = \nu[S'(K_i + q_f^w) - c + \xi_f^w - \eta_f^w + \gamma] = 0 \tag{B1}$$

$$\frac{\partial \mathcal{L}}{\partial K_f} = \nu(\eta_f^w - \gamma) + (1-\nu)\left[S'(K_f) - c\right] - r_f = 0 \tag{B2}$$

$$\frac{\partial \mathcal{L}}{\partial K_i} = \nu\left[S'(K_i + q_f^w) + \xi_i + \gamma\right] - r_i = 0 \tag{B3}$$

We already know that $K_i = 0$ when $\frac{r_i}{\nu} > c + r_f$. We then focus on $\frac{r_i}{\nu} < c + r_f$. Combining (B1) and (B3) we have that

$$\frac{r_i}{\nu} - c = \eta_f^w + \xi_i - \xi_f^w$$

- If $\frac{r_i}{\nu} < c$, $\xi_f^w > 0$ so that $q_f^w = 0$ and $K_i = K_f$. Because $K_f > q_f^w = 0$, $\eta_f^w = 0$ and $\xi_i = 0$. Consequently we can combine (B2) and (B3) to get

$$S'(K_i = K_f) = (1-\nu)c + r_f + r_i.$$

- If $\frac{r_i}{\nu} > c$, $\eta_f^w > 0$ so that $q_f^w = K_f$ and $\xi_i > 0$ so that $K_i = 0$. In effect, we cannot have $K_i > 0$ because, if so, $\xi_i = 0$ and $\eta_f^w > 0$ so that $q_f^w = K_f$. The uniform delivery constraint becomes $K_i + K_f = K_f$ which cannot be true for $K_i > 0$. Then, second best commands $S'(K_f) = c + r_f$ like for $\frac{r_i}{\nu} > c + r_f$.

C Two sources of intermittent energy

In our modelling, there is a one-to-one relationship between states of nature and the technology using the energy available in this state of nature. This allows to simplify notations by dropping the index naming states of nature s. Using the same notation as before for the multipliers, the first-order conditions yield:

$$\frac{\partial \mathcal{L}}{\partial q_f^i} = \nu_i \left[S'(K_i + q_f^i) - c + \xi_f^i - \eta_f^i\right] = 0 \text{ for } i = 1, 2$$

$$\frac{\partial \mathcal{L}}{\partial q_f^{12}} = \nu_{12} \left[S'(K_1 + K_2 + q_f^{12}) - c + \xi_f^{12} - \eta_f^{12}\right] = 0$$

$$\frac{\partial \mathcal{L}}{\partial K_f} = (1-\nu)\left[S'(K_f) - c\right] + \nu_1 \eta_f^1 + \nu_2 \eta_f^2 + \nu_{12} \eta_f^{12} - r_f = 0$$

$$\frac{\partial \mathcal{L}}{\partial K_i} = \nu_i \left[S'(K_i + q_f^i) + \xi_i\right] + \nu_{12}\left[S'(K_1 + K_2 + q_f^{12}) + \xi_i\right] - r_i = 0 \quad \text{for} \quad i = 1, 2$$

plus the complementary slackness conditions derived from the constraints of (P2).

Since in states of nature with intermittent energy, one can always use the fossil fuel equipment, production cannot be lower: $K_1 + K_2 + q_{12}^f \geq K_i + q_i^f \geq K_f$ for $i = 1, 2$. Since S' is decreasing, these inequalities imply $S'(K_1 + K_2 + q_{12}^f) \leq S'(K_i + q_i^f) \leq S'(K_f)$ for $i = 1, 2$.

Proof of *a)*. We show that $K_i = 0$ if and only if $c + r_f < \dfrac{r_i}{\nu_i + \nu_{12}}$ for $i = 1, 2$. Suppose that $c + r_f < \dfrac{r_i}{\nu_i + \nu_{12}}$ for $i = 1, 2$. By $(C5-6)$,

$$\frac{r_i}{\nu_i + \nu_{12}} = \frac{\nu_i S'(K_i + q_i^f) + \nu_{12} S'(K_1 + K_2 + q_f^{12}}{\nu_i + \nu_{12}} + \xi_i \qquad (1)$$

Moreover, by $(C4)$,

$$c + r_f = (1 - \nu)S'(K_f) + \nu c + \nu_1 \eta_f^1 + \nu_2 \eta_f^2 + \nu_{12} \eta_f^{12}$$

Using $(C1 - 3)$, we substitute for η_f^j $(j = 1, 2, 12)$ to obtain:

$$c + r_f = E[S'(K_i + q_i^f)] + \nu_1 \xi_f^1 + \nu_2 \xi_f^2 + \nu_{12} \xi_f^{12} \tag{2}$$

where $E[S'(K_i + q_i^f)] \equiv \nu_1 S'(K_1 + q_1^f) + \nu_2 S'(K_2 + q_2^f) + \nu_{12} S'(K_1 + K_2 + q_f^{12}) + (1 - \nu)S'(K_f)$ is the expected marginal surplus. The assumption $c + r_f < \frac{r_i}{\nu_i + \nu_{12}}$ for $i = 1, 2$ combined with (2), (1) and the non-negativity of ξ_f^j for $j = 1, 2, 12$ leads to

$$E[S'(K_i + q_i^f)] < \frac{\nu_i S'(K_i + q_i^f) + \nu_{12} S'(K_1 + K_2 + q_f^{12})}{\nu_i + \nu_{12}} + \xi_i, \tag{3}$$

for $i = 1, 2$. Suppose first that $K_1 + q_1^f \leq K_2 + q_2^f$ then $S'(K_1 + K_2 + q_{12}^f) \leq S'(K_2 + q_2^f) \leq S'(K_1 + q_1^f) \leq S'(K_f)$ which implies:

$$E[S'(K_i + q_i^f)] \geq \frac{\nu_2 S'(K_2 + q_2^f) + \nu_{12} S'(K_1 + K_2 + q_f^{12})}{\nu_2 + \nu_{12}},$$

for $i = 1, 2$. For the last inequality to be consistent with (3) for $i = 2$, it must be that $\xi_2 > 0$ which implies $K_2 = 0$. Since by assumption $K_1 + q_1^f \leq K_2 + q_2^f = q_2^f = K_f$ and then we must have $K_1 = 0$ and $q_1^f = K_f$.

Suppose now that $K_i = 0$ for $i = 1, 2$ then $\xi_i > 0$ for $i = 1, 2$ in (C5-6) which leads to

$$(\nu_i + \nu_{12})S'(K_f) < r_i \tag{4}$$

for $i = 1, 2$. Moreover by (C1-3), $\eta_f^j = S'(K_f) - c$ for $j = 1, 2, 12$ which combined with (C4) leads to $S'(K_f) = c + r_f$. The last equality joint with (4) leads to $c + r_f < \frac{r_i}{\nu_i + \nu_{12}}$ for $i = 1, 2$.

Proof of *b.1)*. Suppose $\nu_1 = \nu_2 = 0$, $\nu_{12} > 0$ and $c + r_f > \frac{r_i}{\nu_{12}}$ for $i = 1, 2$. The first-order conditions simplify to:

$$S'(K_1 + K_2 + q_f^{12}) = c - \xi_f^{12} + \eta_f^{12} \tag{C'3}$$

$$(1-\nu)\left[S'(K_f)-c\right]=r_f-\nu_{12}\eta_f^{12} \tag{C'4}$$

$$S'(K_1+K_2+q_f^{12})=\frac{r_i}{\nu_{12}}-\xi_i \;\text{ for }\; i=1,2 \tag{C'5-6}$$

Conditions (C'5-6) lead to $\xi_2-\xi_1=\frac{r_2}{\nu_{12}}-\frac{r_1}{\nu_{12}}>0$ where the last inequality is due to the assumption $r_2>r_1$. Therefore $\xi_2>0$ which implies $K_2=0$. Since there are only two states of nature with only one source of intermittent energy in one state like in Section 2, Proposition 1 holds. In particular, with our notation we have $K_1>0$ for $c+r_f>\frac{r_1}{\nu_{12}}$.

Proof of *b.2)*. Suppose that $\nu_1>0$, $\nu_2>0$ and $\nu_{12}=0$ and $c+r_f>\frac{r_i}{\nu_i}$ for $i=1,2$. The first-order conditions simplify to:

$$S'(K_i+q_f^i)=c-\xi_f^i+\eta_f^i \text{ for } i=1,2 \tag{C"1-2}$$

$$(1-\nu)\left[S'(K_f)-c\right]=r_f-\nu_1\eta_f^1-\nu_2\eta_f^2 \tag{C"4}$$

$$S'(K_i+q_f^i)=\frac{r_i}{\nu_i}-\xi_i \;\text{ for }\; i=1,2 \tag{C"5-6}$$

Case 1: $\frac{r_i}{\nu_i}<c$ for one $i\in\{1,2\}$. The conditions (C"1-2) and (C"5-6) imply $c-\frac{r_i}{\nu_i}=\xi_f^i-\eta_f^i-\xi_i>0$ which implies $\xi_f^i>0$ and therefore $q_f^i=0$, i.e. no fossil power in state i. As long as $\nu_i>0$ and $S'(0)=+\infty$, $q_f^i=0$ is optimal only if $K_i>0$.

Case 2: $\frac{r_i}{\nu_i}>c$ for $i=1,2$. Suppose first that $K_1=K_2=0$. Then $K_i+q_i^f=K_f$ for $i=1,2,12$ (use of fossil power under full capacity in all states of nature). Moreover, $K_i=0$ implies $\xi_i>0$ and therefore $S'(K_f)<\frac{r_i}{\nu_i}$ by (C"5-6). The first-order conditions (C"1-2) and (C"4) imply $S'(K_f)=c+r_f$ which combined with the last inequality contradicts the assumption $c+r_f>\frac{r_i}{\nu_i}$. Suppose now that $K_1>0$ and $K_2=0$ which implies $K_2+q_2^f=K_f$ and $\xi_2>0$. The first-order conditions (C"4) and (C"5-6) imply respectively $(1-\nu_1)\left[S'(K_f)-c\right]=r_f-\nu_1\eta_f^1$ and $S'(K_f)<\frac{r_2}{\nu_2}$. The two last relations lead to $\nu_1\eta_f^1>r_f+c-\frac{r_2}{\nu_2}+\nu_1\left[\frac{r_2}{\nu_2}-c\right]$. Since by assumption $r_f+c>\frac{r_2}{\nu_2}>c$, $\eta_f^1>0$ and therefore $q_1^f=K_f$. The first-order conditions imply $E[S'(K_i+q_i^f)]=r_f+c$. Since $S'(K_1+K_f)<E[S'(K_i+q_i^f)]<S'(K_f)$, the last equality combined with $S'(K_f)<\frac{r_2}{\nu_2}$ contradicts our starting assumption $c+r_f>\frac{r_2}{\nu_2}$.