Optimal insurance contracts with adverse selection and comonotonic background risk

Alary D.*  Bien F.
TSE (LERNA)  University Paris Dauphine

Abstract

In this note, we consider an adverse selection problem involving an insurance market à la Rothschild-Stiglitz. We assume that part of the loss is uninsurable as in the case with health care or environmental risk. We characterize sufficient conditions such that adverse selection by itself does not distort competitive insurance contracts. A sufficiently large uninsurable loss provides an incentive to high-risk policyholders not to mimic low-risk policyholders without distorting the optimal coverage.

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*Corresponding author: TSE-LERNA, University of Toulouse 1, Manufacture des Tabacs, 21 allée de Brienne, 31000 TOULOUSE, FRANCE.
1 Introduction

The existence of an uninsurable financial risk affects the individual’s insurance choice. Some authors have studied the impact of such background risk on the theory of insurance demand (Doherty and Schlesinger (1983, 1990); Eeckhoudt and Kimball (1992); Hau (1999)).

The purpose of this note is thus to analyse the effects of financial background risk in an adverse selection setting à la Rothschild and Stiglitz (1976) (abbreviated here as RS). To our knowledge, Crocker and Snow (2006) are the only authors to focus on the effect of a background risk on optimal insurance contracts with adverse selection. They assume that the background risk is independent of the insurable loss. This additional risk affects the extent of insurance coverage because it increases the risk aversion of vulnerable agents. Crocker and Snow show that the RS deductible needed to achieve separation is lower than in the case of the standard RS model. Moreover, to achieve a separating equilibrium, the proportion of high-risk agents must be higher than in RS. Otherwise, pooling equilibrium appears. However, uninsurable losses are usually perfectly correlated with financial loss rather than being independent. This is the case when considering environmental risks, human capital or health: part of the losses is unobservable and/or uninsurable.

We study here the incidence of an uninsurable loss comonotonic with the insurable risk in a RS setting. In particular we characterize conditions under which private information does not modify the optimal contracts. Due to the comonotonic uninsurable loss, the high-risk agent balances between a higher coverage (and a higher premium) and a smaller premium (with a smaller coverage). If the agent faces a sufficiently large uninsurable risk, the effect of coverage dominates the effect of premium reduction. The present study has the following structure. The model is presented in section 2 and the private information equilibrium is characterized in section 3. Section 4 reports the conclusions.

2 The model

There are two possible states of nature: the "no loss" state occurs with probability $p_i$ and the "loss" state with probability $1 - p_i$. Consumers share the same Von Neumann-Morgenstern utility function $U(W)$ where $W$ denotes the wealth and $W_0$ the corresponding initial wealth. As in RS, there are two types of agent in the economy ($i \in \{H, L\}$ for high and low risk) differing in their loss probabilities: $p_H > p_L$. We assume that there is a proportion $\lambda$ of high-risk agents in the total population.
In the loss state, their wealth is reduced by an amount \( Z_i = D + \Delta_i \). We assume \( \Delta_H > \Delta_L = 0 \). Moreover we consider that only \( D \) is verifiable.\(^1\) Therefore \( \Delta_i \) is uninsurable and may be considered as a background risk that is comonotonic with \( D \).

Under competition, insurers are constrained to earn zero expected profit. We consider an insurance contract \( \alpha_i, \beta_i \) where \( \alpha_i \) denotes the premium and \( \beta_i \) the indemnity net of the premium.

With public information (both on the probability and the loss \( Z_i \)), the optimal individual contract is given by solving

\[
\max_{\alpha_i, \beta_i} U(W_0 - Z_i + \beta_i) + (1 - p_i) U(W_0 - \alpha_i)
\]

which is subject to zero expected profit on each contract

\[
(1 - p_i) \alpha_i - p_i \beta_i = 0
\]

Since the premium is actuarially fair, it is straightforward to show that the optimal contracts provide full coverage thus implying \( \beta_i + \alpha_i = Z_i \). From the zero expected profit constraint, we obtain \( \alpha_i = p_i Z_i \).

### 3 Private information equilibrium

Under private information, insurers do not observe the individual risk characteristics i.e. the probability \( p_i \) and the uninsurable loss \( \Delta_i \). According to the standard explanation of screening in insurance markets, firms use deductible to sort high and low levels of risks. If this property is valid, then L-risk type agents will accept a large deductible in order to signal their type and pay a lower premium. This is due to the fact that only H-types want to mimic L-types. L-types never choose an H-type contract which can be explained by the single crossing property. This property states that, for any contract \( \alpha, \beta \) the marginal rate of substitution for the H-risk type must be lower than for the L-risk type.

This property is true if the indifference curve has a greater slope for the L-type agents than for the H-type agents. Hence,

\[
\frac{p_L}{1 - p_L} \frac{U'(W_0 - \alpha)}{U'(W_0 - D + \beta)} > \frac{p_H}{1 - p_H} \frac{U'(W_0 - \alpha)}{U'(W_0 - D - \Delta_H + \beta)} \tag{1}
\]

\(^1\)As in Doherty and Jung(1993), we consider that applicants have different losses. Doherty and Jung show that, if losses are observed, adverse selection does not modify the first best contracts. Indeed, insurers may infer risk types from the loss. In our model, we assume that policyholders have the same observable loss. Then, contrary to Doherty and Jung, insurers cannot infer the type of policyholder by observing losses.
As \( p_L < p_H \) and \( \Delta_H > 0 \), this inequality always holds true.

As observed in a standard RS setting, this condition implies that the H-risk type will overvalue a monetary transfer in the bad state compared to the L-risk type agent. The H-risk type is thus willing to purchase more insurance rather than the L-risk agent. Hence, competitive contracts under full information are not incentive compatible. The H-risk agents prefer an L-risk type’s contract because they receive the same indemnity but with a lower premium. Insurers achieve separation by introducing a deductible. Thus the L-risk type agent is offered a contract with limited coverage at a low unit price. Moreover the self selection constraint of L-type agents is never binding. Therefore, we focus here on incentives for H-type agents.

Note that, contrary to RS under perfect information, different contracts exhibit different indemnities. Thus it is not trivial that the self selection constraint of H risk agents is binding.

We can characterize conditions such that first best contracts are optimal under asymmetric information \textit{i.e.} such that the H-type self selection constraint is slack.

The optimal contracts have to satisfy the following program:

\[
\max_{\alpha_H, \beta_H, \alpha_L, \beta_L} \lambda [p_H U(W_0 - D - \Delta_H + \beta_H) + (1 - p_H) U(W_0 - \alpha_H)] \\
+ (1 - \lambda) [p_L U(W_0 - D + \beta_L) + (1 - p_L) U(W_0 - \alpha_L)]
\]

subject to zero expected profit on each contract

\[
(1 - p_L) \alpha_L - p_L \beta_L = 0 \\
(1 - p_H) \alpha_H - p_H \beta_H = 0
\]

and the H-type agent’s self selection constraint

\[
p_H U(W_0 - D - \Delta_H + \beta_H) + (1 - p_H) U(W_0 - \alpha_H) \quad (2_H) \\
\geq p_H U(W_0 - D - \Delta_H + \beta_L) + (1 - p_H) U(W_0 - \alpha_L)
\]

This program is the same as the previous one except that it contains a self selection constraint. Clearly, ignoring this constraint will give rise to the same solution. Then we can characterize situations such that this condition is slack when first best contracts are offered. Therefore, we obtain the following result.

**Proposition 1** If the uninsurable loss is sufficiently large, the equilibrium contracts under private information correspond to the first best contracts.
Note that departing from compatibility constraint is obviously satisfied.

Proof. In order to characterize the equilibrium, first best contracts have to satisfy the incentive-compatibility constraint:

\[ u(w - p_H(D + \Delta_H)) - p_Hu(w - p_LD - \Delta_H) - (1 - p_H)u(w - p_LD) \geq 0 \]

Let us denote \( f(\Delta_H; p_L) \) the left hand side of this constraint:

\[ f(\Delta_H; p_L) = u(w - p_H(D + \Delta_H)) - p_Hu(w - p_LD - \Delta_H) - (1 - p_H)u(w - p_LD) \]

Note that \( f(0; p_L = p_H) = 0 \). If the risk type are the same, the incentive-compatibility constraint is obviously satisfied.

Departing from \((0; p_L = p_H)\) by decreasing \( p_L \) we decrease \( f(\Delta_H; p_L) \) and the incentive-compatibility constraint is no longer satisfied since \( df(\Delta_H; p_L) > 0 \).

However such a decrease may be compensated by a variation of the uninsurable loss \( \Delta_H \) such that the incentive-compatibility constraint remains satisfied. By fully differentiating the function \( f(\Delta_H; p_L) \), we know that

\[ df(\Delta_H; p_L) = p_H[u' (w - p_LD - \Delta_H) - u' (w - p_H(D + \Delta_H))] d\Delta_H + D[p_Hu' (w - p_LD - \Delta_H) + (1 - p_H)u' (w - p_LD)] dp_L \]

Then \( df(\Delta_H; p_L) = 0 \) if and only if

\[ d\Delta_H = -\frac{D[p_Hu' (w - p_LD - \Delta_H) + (1 - p_H)u' (w - p_LD)]}{p_H[u' (w - p_LD - \Delta_H) - u' (w - p_H(D + \Delta_H))]} dp_L \]

However, from \( \Delta_H = 0 \), we can only consider an increase in the uninsurable loss then it must be the case that \( d\Delta_H > 0 \). Since \( dp_L < 0 \), this will be true if and only if

\[ p_H[u' (w - p_LD - \Delta_H) - u' (w - p_H(D + \Delta_H))] < 0 \]

which is equivalent to \( \Delta_H > \frac{(p_H - p_L)(D - p_H)}{1 - p_H} \). Note that \( \Delta_H(p_H) = 0 \) then from \((0; p_L = p_H)\), we know that

\[ f(0; p_L = p_H) = 0 \]

Then, providing that \( \Delta_H > \Delta_H(p_L) \), we can define a function \( \Delta_H^*(p_L) \) such that \( f(\Delta_H^*(p_L); p_L) = 0 \) by using the implicit function theorem. Moreover, for all \( \Delta_H \geq \Delta_H^*(p_L) \) \( f(\Delta_H; p_L) \geq 0 \) meaning that the incentive-compatibility constraint is always satisfied for the first best contracts. ■

The intuition of this result is the following. A H-risk type agent who chooses an L-risk type contract is no longer compensated for the uninsurable
loss.\textsuperscript{2} A higher uninsurable loss decreases the wealth in the unfavourable state but increases the difference between insurance premiums. However if the uninsurable loss is sufficiently large, the ensuing reduction in utility due to it overwhelms the benefit of a lower premium. Therefore the H-type policyholders prefers their own type of contract.s in which their self selection constraint is never binding. Optimal contracts under asymmetric information are those of full information.

4 Conclusion

Crocker and Snow (2006) show that the existence of an unobservable independent background risk may close some insurance markets. However, they show that extending coverage to background risk can have a positive social value. In this paper, we focus on a perfect correlation between risks. In fact we consider that part of the loss is unobservable so it is therefore then uninsurable as in the case of health or human capital. We establish certain conditions whereby private information leads to the absence of inefficiencies in competitive markets. This comonotonic background risk induces policyholders to prefer their own first best contract to any other contract they may choose due to private information.

References


\textsuperscript{2}Note that it is unnecessary to assume $\Delta_L = 0$ is not . Our result is true if $\Delta_H > \Delta_L$. In this case, the H-type agent is partially compensated for the uninsurable loss. Nevertheless, if the remaining uncovered loss is sufficiently large, the same argument applies.
