Persuasive Subsidies in a Clean Environment

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Abstract

This paper examines a signaling explanation for environmental subsidies: they attract attention to environmental friendly goods in a credible way. When some households overestimate the level of environmental damage, a perfectly informed government can use subsidies to reveal that the environment is clean. Subsidies are the efficient means of signaling because the consequent consumption distortions are most damaging in a dirty environment where, at the same time, the consumption good generates polluting emissions and households

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value it less than if it were environmental friendly. The more informed households, the lower the level at which a subsidy is an efficient signal.

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1 Introduction

There is a widespread mistrust of subsidies among economists, which contributes in particular to question their efficiency in addressing environmental problems. Although subsidies are, in this respect, the mirror image of taxes, economists rely more on taxes to provide correct incentives for protecting or enhancing environmental quality.

According to Baumol and Oates (1988), the effects of subsidies and taxes on the reduction of the environmental damage are far to be equivalent. For instance, subsidies, unlike taxes, may be suspected to induce excessive entry of firms with consequent resource misallocations (see chapter 14 of Baumol and Oates (1988) for a comparison between the effects of subsidies and taxes on the environment). More recently, Stavins (2000) observes that, “in practice, many subsidies promote economically inefficient and environmentally unsound practices”. He quotes the US Forest Service’s “below-cost timber sales” as an example of market distortion. When Jaffe and Stavins (1995) produce econometric evidence that energy-efficiency technology adoption subsidies may be more effective in some cases than energy taxes, they present this finding as “unanticipated” and “at odds with economic think-
ing”. Even when a subsidy is recognized the desirable role of correcting a market imperfection such as the monopolist’s tendency to underproduce, it seems weird in an environmental context to advise that pollution be subsidized.

This paper examines a novel motive for environmental subsidies, based on their wasteful nature. A subsidy may be a “money burning” message intended to signal that the environment is clean when some uninformed households overestimate the true environmental damage. A formal model is developed to explain how environmental subsidies attract attention to environmental friendly goods in a credible way.

Subsidies will be thought of as tax cuts and credits that serve to encourage the consumption of environmental friendly goods and services. These subsidized goods can be exemplified by “green electricity”, that is, electricity generated with renewable sources of energy.

The model investigates an economy characterized by households having differential information about the state of the environment. This is consistent with recent research by Mullainathan and Shleifer (2005) who give some per-

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1In the US Energy Policy Act of 1992, for instance, electricity produced from wind and biomass fuels have received a credit, and solar and geothermal investments have received up to a 10% tax credit. In many European countries like Britain, France, Germany or Norway, governments also back the adoption of low-carbon-emitting alternatives with generous subsidies. The French government offers various kinds of tax cuts to encourage the residential use of renewable energies and help households pay back the cost of expensive new equipment like solar cells. In Germany, consumers who sell renewable energy like solar power to the central electricity grid, are offered highly attractive tariffs.
suasive arguments in favor of the assumption that people are likely to form various beliefs from the news delivered by media. As noticed by Schneider and Volkert (1999), environmental problems are susceptible to substantial information asymmetries kept up, on one hand, by the publications of industry and business associations, and on the other hand, by the spectacular actions of environmental groups (for example, Greenpeace vs. Shell in Fall 1995). Differential information might indeed come from difficulties in communicating accurate information about pollution effects\(^2\) as well as about the feasibility of alternative technologies, public disclosure of labeling programs, the mass media\(^3\), or from the views of politicians, business associations or environmental groups households trust. Boyer and Laffont (1999) argue that the government generally has superior knowledge of the environmental damage because she is endowed with superior data from confidential reports of the public service bureaucracy. A second reason also mentioned by Boyer and Laffont (1999) is that the government’s scientific information on nonverifiable environmental variables may be costly to communicate.

In the present framework, some households – called “uninformed” – are

\(^2\) As noted by Pearce (1991), scientific opinion on this topic is changing rapidly as new information comes to light. This author observes that “the scale of the threat from chlorofluorocarbons has been revised upwards several times” during the experience of the Montreal Protocol.

\(^3\) According to Lomborg (2001), the environmental message delivered on television and in the newspapers is generally characterized by a tendency to overemphasize pessimistic viewpoints and confronts the public with a lopsided version of reality giving the impression that the global environment is in the worst shape.
less informed than others – called “informed” – about the environmental consequences of private consumption. The government is assumed to be perfectly informed about the state of the environment. All households are willing to pay more for environmental friendly goods. Since they have the opportunity to observe the environmental action chosen by the government, uninformed households can infer the true state of the environment. This imposes additional constraints in the standard welfare maximization program, which give the government correct incentives to reveal information. The subsidy emerges in equilibrium as the only “efficient” way of conveying all the information in a clean environment where the environmental damage is overestimated by some uninformed households. This is all the more striking as there would be no reason to subsidize the clean good under complete information. The subsidy is efficient in the sense that it minimizes the losses in welfare which are accepted by the government to transmit information in a credible way. In other words, the subsidy has the same “persuasive” role as that acknowledged for advertising in the industrial organization literature: it raises the valuation of the subsidized good by changing households’ beliefs (see Bagwell (2005)). Furthermore, with constant relative risk aversion utility functions, it turns out that the subsidy is an efficient device for signaling the clean environment in a poorly informed economy, as long as the level of environmental damage is not too high and households have a sufficiently large willingness-to-pay for an environmental friendly product.
When the environment is dirty, the Pigovian tax plays the same signaling role as that played by the subsidy in a clean environment. However, it fully reveals that environment is dirty with no distortion relative to what would be optimal under complete information. Hence, it correctly internalizes the environmental externalities while signaling the true damage.

The present analysis is somewhat related to two strands of the economics literature. The first one deals with pollution control under imperfect competition and the other one analyzes fiscal action as a signaling device for the government.

Barnett (1980) originates the first strand in providing theoretical foundations for second-best Pigovian taxes in a monopoly context. Building on his seminal work, Requate (1993) and Simpson (1995) have recently shown that the government must subsidize the output of a monopoly or a symmetric oligopoly in addition to the Pigovian tax so as to achieve a first-best outcome. All these works abstract from informational considerations and emphasize the essential role of subsidy in solving the firms tendency to underproduce due to their excessive market power.

The present analysis also links up with the literature on fiscal actions as signals in assuming that the government has an informational advantage over economic agents. However, most of the works in this range of literature focus on taxes – environmental taxes when the issue at stake is market-based instruments for environmental policy – and fail to provide direct insight
into the informational role of subsidies. In Rogoff (1990), citizens infer the government’s administrative competence from tax and expenditure policies distortions. This author argues that the political budget cycles may be a socially efficient mechanism for transmitting up-to-date information about the government’s performance. Our approach differs not only in that it abstracts from electoral pressures, but also we do not need to postulate that the government in office receives ego rents to explain signaling distortions. The assumption that the politician’s interests may be entirely disjointed from those of the citizens is also central in the analysis of Brett and Keen (2000). They assume that incumbent policymakers have relevant private information about the true environmental costs and benefits of alternative policies. They investigate the role of earmarking as a way of signaling both the type of incumbent policymaker and the level of environmental damage. They characterize equilibria for which earmarking is informative but the choice of environmental tax is not. The analysis of Barigozzi and Villeneuve (2004) is similar to ours in that the objective of a privately informed government is to maximize welfare. In their model, households, unlike the government, cannot ascertain the negative external effects of individual consumption. As a result, the government must distort downward the Ramsey-Sandmo tax in order to fully reveal information to households. The emergence of signaling distortions crucially hinges on the fact that raising public funds is costly for the government. In contrast, we deliberately leave aside such costs to start
with the benchmark of an economy with no distortionary tax under symmetric information. Finally, all the papers that have previously explored the strategic behavior of a privately informed government, like this one, draw on the seminal articles in the literature of industrial organization by Milgrom and Roberts (1982 and 1986), in which a firm uses prices, among other variables, to signal its private information either on production costs or on product quality.

The paper is organized as follows. Section 2 derives a reduced form signaling framework from primitives on the economy, states some useful properties and presents the benchmark case of complete information. Section 3 investigates the equilibria and testes their robustness to the intuitive criterion in order to enlighten the informative role of subsidies and taxes. Section 4 offers conclusions and proposes some extensions of the model.

2 The model

The economy is made up by \( N \) households who have identical preferences and work to consume a good. This private consumption may have detrimental effects on the environment. Households do not have the same information about the state of the environment. Let \( \varepsilon^j \) be an index of the environmental damage and assume, for simplicity, that the environment is either clean \((j = c)\) or dirty \((j = d)\). The government and \( I \) households perfectly know the
state of the environment, whereas \( N - I \) households cannot ascertain it. The latter will be called “uninformed”. Uninformed households hold prior beliefs on the environmental damage represented as follows: they believe the environment is clean with a commonly known probability \( \mu \in (0, 1) \).

The household’s utility depends on one public good: environmental quality, and two private goods: consumption of the polluting good \( x \) and labor \( l \) which is taken as the numeraire. Given a state \( \varepsilon^j \) of the environment, the informed household \( i \)'s utility function \( U(x_i, l_i, \varepsilon^j) \) is assumed to take the quasi-linear form:

\[
U(x_i, l_i, \varepsilon^j) = Q(\varepsilon^j) u(x_i) - l_i - \varepsilon^j \sum_{i=1}^{N} x_i,
\]

where \( u(x_i) \) is twice continuously differentiable, increasing and satisfies the usual strict concavity and Inada conditions. The last term in the right hand side of (1) represents the environmental damage (or environmental benefit from pollution reduction) which is directly related to the total consumption of the polluting good. The per-unit damage \( \varepsilon^j \) is normalized to \( \varepsilon^c = 0 \) when the environment is clean, and so \( \varepsilon^d = \varepsilon > 0 \) when the environment is dirty. Households are assumed to be environmentally aware in the sense that they are willing to pay more for a cleaner consumption good, hence we have \( Q'(\varepsilon) < 0 \). The assumption of the externality non-separability is consistent

\[\text{As usual, primes denote derivatives.}\]
with recent works in environmental economics that recognize the existence of feedbacks between economic activity and environmental externalities (see, for instance, Carbone and Smith (2007)).

Let \( \tilde{\varepsilon}(\mu) \equiv (1 - \mu) \varepsilon \) be the uninformed households’ perception of the environmental damage. The uninformed household \( i \)'s utility depends on his perception of environmental quality \( Q(\tilde{\varepsilon}(\mu)) \) and is given by:

\[
U(x_i, l_i, \tilde{\varepsilon}(\mu)) = Q(\tilde{\varepsilon}(\mu)) u(x_i) - l_i - \tilde{\varepsilon}(\mu) \sum_{i=1}^{N} x_i.
\]

Note that the uninformed households’ willingness-to-pay for the product is larger when the prior beliefs attach a higher probability to the environment being clean: denoting \( \tilde{Q}(\mu) \equiv Q(\tilde{\varepsilon}(\mu)) \), we have \( \tilde{Q}'(\mu) > 0 \).

We shall denote the price of the private good by \( p \). The government chooses a tax or a subsidy \( t \) (the value of \( t \) will be positive for a tax and negative for a subsidy) on polluting consumption. Let \( T \equiv (-p, +\infty) \) be the set of possible taxes or subsidies. It will be assumed, as in Diamond (1973), that any tax revenue is returned to the people via lump-sum transfers \( T \) which will be negative in case of subsidies. The budget constraint for household \( i \) amounts to \( (p + t) x_i = l_i + T/N \).

Uninformed households maximize the expected value of utility and, when they decide on the consumption of polluting good, they neglect the adverse effect of their personal polluting consumption on environmental quality.
Household $i$ chooses $x_i$ to maximize

$$\hat{Q}(\mu)u(x_i) - (p + t) x_i + T/N. \quad (3)$$

The first-order condition for utility maximization is

$$\hat{Q}(\mu)u'(x_i) = p + t. \quad (4)$$

The equivalent equation is $Q(\varepsilon^j)u'(x_i) = p + t$ when households are informed and the the actual state of the environment is $\varepsilon^j$. By solving equation (4), we can write the demand for the polluting good by an uninformed household as a function of the environmental tax and the beliefs about the environment. Let $X(t, \mu)$ and $X(t, \varepsilon^j)$ be the per capita demands of respectively the uninformed and informed households.

Differentiating (4) yields the following partial derivatives for all $t \in \mathcal{T}$ and $\mu \in [0, 1]$

$$X_t(t, \mu) = \frac{1}{\hat{Q}(\mu)u''},$$

$$X_{\mu}(t, \mu) = \frac{\varepsilon Q' u'}{\hat{Q}(\mu)u''}, \quad (5)$$

where subscripts denote partial derivatives. Some instructive properties emerge in our economy made of environmental friendly households. Clearly, the uninformed households’ individual demand is both strictly decreasing
with tax and strictly increasing with the probability that the environment is clean. The latter property reflects that “green” households are less willing to consume the polluting good when the environment is perceived to be dirty. Similar calculations for the informed households yield

\[ X(t, \varepsilon) = \frac{1}{Q(\varepsilon)} u_0 < 0 \]

and

\[ X(t, \varepsilon) = -\frac{Q_u'}{Q(\varepsilon)u'} < 0, \]

hence informed households consume more when the environmental tax is lower (the subsidy is higher) and/or the environment is cleaner.

Market equilibrium for the polluting good is such that total output must equal the sum of individual consumptions. Treating all individuals identically, we omit subscript \( i \) and denote by \( L(\mu) \) (resp. \( L(\varepsilon) \)) the labor force of an uninformed (resp. informed) household. Market equilibrium for each state of the environment can be written as:

\[ IL(\varepsilon) + (N - I) L(\mu) = IP(t, \varepsilon) + (N - I) pX(t, \mu), j = c, d. \quad (6) \]

Let us consider the first best outcome when the state of the environment is \( \varepsilon \). Then, the government aims to maximize welfare subject to the decentralized optimizing behavior of households. Following Atkinson and Stiglitz (1980, chap 16, p. 493), the government’s budget constraint can be obtained by summing the individual budget constraints and subtracting the market
clearing condition (6):

\[ tIX(t, \varepsilon^j) + t(N - I)X(t, \mu) = T. \quad (7) \]

It follows that the social welfare when the environmental state is \( \varepsilon^j \) and is perceived by the uninformed households to be clean with probability \( \mu \) upon seeing \( t \), can be written in the following reduced form function \( W(t, \varepsilon^j, \mu) : \mathcal{T} \times \{\varepsilon^e, \varepsilon^d\} \times [0,1] \to [0, +\infty) \):

\[
W(t, \varepsilon^j, \mu) \equiv I\left[Q(\varepsilon^j)u(X(t, \varepsilon^j)) - pX(t, \varepsilon^j)\right] + (N - I)\left[\hat{Q}(\mu)u(X(t, \mu)) - pX(t, \mu)\right] \\
- \varepsilon^jN\left[IX(t, \varepsilon^j) + (N - I)X(t, \mu)\right]
\]

The expression given in (8) shows that social welfare has three components: first, private welfare of informed households which depends on the true state of the environment, second, private welfare of the uninformed households which depends on their perception of the environmental state, and third, environmental welfare which depends on both the true \( \varepsilon^j \) and the uninformed households’ beliefs. By underestimating the environmental damage, an uninformed household consumes more than an informed household, and so raises the social cost of pollution in a dirty environment. The wrong perception of the dirty environment strengthens the negative external effect of the uninformed households’ consumption on the environment. This is no
longer true when the environment is clean. Then, an uninformed household consumes less than an informed household because he attaches some probability to the environment being dirty. However, this has no external effect on the environment. In some sense, when the environment is dirty, the government has to address the problem of the informational externality exerted by uninformed households, in addition to the environmental externality problem.

Benchmark case of complete information.

Let us now define $t^j(\mu)$ as the maximizer of $W(t, \varepsilon^j, \mu)$ with respect to $t$, for $j = c, d$. It can be found in Appendix 1 that $t^j(\mu) = N\varepsilon^j$, hence it does not depend on the uninformed households’ perception of the environment. Consider the benchmark cases where all households are informed. Then, the optimal discretionary tax is simply the Pigovian level $N\varepsilon^j$ that fully internalizes the marginal environmental damage. Under complete information, the government has no reason to tax or subsidize the good when the environmental damage is nil. When the environment is dirty, the government must levy the Pigovian tax $N\varepsilon$. Hence, there is no rationale for subsidies here, under complete information.

The model has two essential properties which will prove useful to investigate the reduced form signaling model in the subsequent analysis.

Single-crossing property

The model is strongly structured by a single-crossing property which takes
the following form:

\[ W_{t\varepsilon}(t, \varepsilon^j, \mu) > 0. \] (9)

In the present context, the single-crossing property (9) is quite intuitive. It means that the government is more inclined to give subsidies when the environment is clean than when it is dirty. The main reason is that, by boosting consumption, subsidies increase the environmental damage whenever it exists, that is, in the dirty environment and not in the clean one. Moreover, it turns out that the effects of subsidies on welfare in a clean environment are less detrimental either when households are more willing to pay for environmental friendly goods or when households are more informed about the true environmental damage (see equation (20) in Appendix 1). It can be shown that the single-crossing property always holds for subsidies. To ensure that condition (9) is also met for high levels of taxes, it suffices to consider that the households’ valuation of the consumption good does not change too much from one state of the environment to the other, i.e., \(|Q'|\) is sufficiently small in the sense defined in Appendix 1.

**Stochastic dominance property**

It is shown in Appendix 2 that the following property of stochastic dominance is satisfied.

For all \(\varepsilon^j \in \{\varepsilon^d, \varepsilon^c\}, \mu \in [0, 1]\), and \(t \in T\), \(W_\mu(t, \varepsilon^j, \mu) > 0\).

This property states that, for a wide range of taxes and subsidies, social
welfare is higher when the uninformed households attach a higher probability to the environment being clean. Hence, $\mu = 0$ is the least favorable belief for the government, whatever the state of the environment. The stochastic dominance is an essential property of standard signaling games\(^5\). Figure 1 depicts possible shapes for the social welfare functions $W(t, \varepsilon^j, \mu)$, for $j = c, d$ and $\mu = 0, 1$.

An increase in $\mu$ makes the consumption good more attractive because environmental friendliness as perceived by households is considered as a vertical attribute of the good. This raises the uninformed demand for the good, thereby lifting up their private component of social welfare (see (8)). Consequently, the government prefers uninformed households to think that the environment is clean regardless of the true state of the environment. When the environment is clean, the stochastic dominance property provides the government with an incentive to convey information on the true state of the environment. Nevertheless, when the environment is dirty, the government may be reluctant to modify the uninformed households' perception of the environmental damage. Instead of transmitting all the information through her choice of tax, the government can also mimic the environmental policy she would choose were the environment clean, thereby providing no information to the uninformed. Without any further information on the state of the

\(^5\)This property is implied by assumptions A1', A2 and A3 taken together page 392 in Cho and Sobel (1990) or directly assumed page 255 in Mailath, Okuno-Fujiwara and Postlewaite (1993.)
environment, the uninformed households consume more than if they were fully informed and attain a high level of private well-being which is likely to compensate the corresponding degradation of environmental quality.

3 The Signaling Role of the Government’s Behavior

3.1 The Game and the Definition of Equilibrium

The government’s choice of environmental policy can be observed by uninformed households before they make their consumption decision. From the level of tax or subsidy, uninformed consumers may be able to infer the true state of the environment. This gives the model a structure of signaling game, for which strategies must form a perfect Bayesian equilibrium. As suggested by Harsanyi (1967-8), such a game of incomplete information can be replaced by a game of complete but imperfect information which unfolds in three stages. First, “Nature” draws a state $\varepsilon_j$ of the environment from the set $\{\varepsilon^c, \varepsilon^d\}$ according to the probability distribution $\mu_0$. Second, the government learns the state of the environment and chooses a tax or a subsidy on the consumption good. After observing this choice, households in the third stage rely on their inferences upon the true value of $\varepsilon_j$ to make their consumption decision. Let $\mu(t) : (0, 1) \times T \to [0, 1]$ denote the uninformed households’ pos-
terior belief that the state of the environment is $\varepsilon^c$, which updates the prior $\mu_0$ when the tax (eventually negative) is $t$. The government, in turn, must take into account how her choice of $t$ influences the uninformed households’ inferences. Restricting attention to pure strategies, a perfect Bayesian equilibrium of this game is a set of strategies $\{(t^j)_{j=c,d}, (X^*_i(t, \tilde{\mu}^*(t)))_{i=1,2,...,N}\}$ and a probability distribution $\mu^*(t)$ such that, at any stage of the game, strategies must be optimal given beliefs:

**Condition 1:** optimality condition for the government.

For $j = c, d, t^j \in \arg\max_t W(t, \varepsilon^j, \tilde{\mu}^*(t))$.

**Condition 2:** perfection condition for households.

$X^*_i(t, \tilde{\mu}^*(t))) \in \arg\max_{x_i} \left\{ \tilde{Q}(\mu^*(t))u(x_i) - (p + t)x_i + T/N \right\}$ for the uninformed households

$X^*_i(t, \varepsilon^j) \in \arg\max_{x_i} \left\{ Q(\varepsilon^j)u(x_i) - (p + t)x_i + T/N \right\}$ for the informed households.

**Condition 3:** Bayes’ consistency of beliefs on the equilibrium path.

If $t^c \neq t^b$, then $\mu^*(t^b) = 0$ and $\mu^*(t^c) = 1$;

If $t^c = t^b$, then $\mu^*(t^b) = \mu^*(t^c) = \mu_0$. 
Condition 1 demands that the government’s choice maximizes social welfare given that households respond optimally. Condition 2 states that households’ consumption of the good should maximize their utility given, in the case of uniformed households, their beliefs induced by the government’s behavior. Finally, condition 3 requires the uniformed households’ posterior beliefs about $\varepsilon^j$ to be formed from their prior beliefs by using Bayes’ rule for the governments’ equilibrium strategies. As usual, the equilibrium concept places no restriction on beliefs off the equilibrium path. To tackle the problem of equilibria multiplicity, we will impose the additional restrictions on off-the-equilibrium-path beliefs required by the “intuitive criterion” (see Cho and Kreps (1987)). Formally, consider an equilibrium in which the level of social welfare is $W^j$ when the state of the environment is $\varepsilon^j$. Then the equilibrium fails to survive the intuitive criterion if there exists $t'$ such that:

\[
W^c < W(t', \varepsilon^c, 1), \quad (10)
\]

\[
W(t', \varepsilon^d, 1) < W^d. \quad (11)
\]

### 3.2 The Analysis of Equilibrium

Our interest now is not really a characterization of all perfect Bayesian equilibria in the model but rather a characterization of the set of perfect Bayesian equilibrium taxes and subsidies. We can without loss of generality let the
uninformed households’ beliefs be \( \mu(t) = 0 \) for all \( t \not\in \{t^c, t^d\} \). We know from the property of stochastic dominance that such out-of-equilibrium beliefs are always the least favorable for the government. Such beliefs are the strongest too in that, if a government does not have an incentive to set \( t \) when \( \mu(t) \neq 0 \), then she will not have an incentive when \( \mu(t) = 0 \), since social welfare is lower, whatever the state of the environment. Therefore setting \( \mu(t) = 0 \) will generate all of the possible perfect Bayesian equilibrium paths.

Let \( t^c \) and \( t^d \) denote the separating equilibrium levels of tax or subsidy when, respectively, the environment is clean and dirty. The next lemma states that the best choice for the government is to set \( t^d \) equal to the Pigovian level when the environment is dirty. In this case, the Pigovian tax is serving two functions at the same time: it conveys all the information from the government to uninformed households, while internalizing the pollution externalities in a conventional way.

**Lemma 2:** If \( \mu(t) = 0 \) for all \( t \not\in \{t^d, t^c\} \), then in any separating equilibrium \( t^d = N\varepsilon \).

**Proof:** (see Appendix 3)

Consider now situations where the uninformed households believe they know the true state of the environment. If the uninformed households assign probability 1 to the clean environment while the environment is in fact dirty,
uninformed demand for the polluting good after observing some tax $t$ is $X(t,1)$ and social welfare is given by $W(t,\varepsilon^d,1)$. On the other hand, if the uninformed households are wrongly convinced that the environment is dirty, demand and social welfare are respectively $IX(t,\varepsilon^c) + (N - I)X(t,0)$ and $W(t,\varepsilon^c,0)$. Setting a zero tax in such a context would yield a welfare of $W(0,\varepsilon^c,0)$, which is the lowest level that can be attained with no tax in the presence of uninformed households. The welfare $W(0,\varepsilon^c,0)$ can be interpreted as the opportunity cost for the government to fully transmit information on the clean environment.

To achieve separation when the environment is clean, the government must choose an environmental tax $t^c$ that satisfies the two following conditions:

$$W(t^c,\varepsilon^d,1) \leq W(t^d,\varepsilon^d,0) \quad \text{(12)}$$

$$W(t^c,\varepsilon^c,1) \geq W(0,\varepsilon^c,0). \quad \text{(13)}$$

Condition (12) ensures that uninformed households would not mistake the dirty state of the environment for the clean state if they were observing $t^c$ in a dirty environment. We have previously seen that the government could conceal information when the environment is dirty in an attempt to enhance the uninformed households’ willingness-to-pay for the polluting good and lift
up their private welfare. Condition (12) rules out such a pooling strategy. The government must choose the complete information Pigovian tax and fully transmit information on the true state of the environment rather than trick uninformed households into believing that the environment is clean by setting \( t^c \). As can be seen in Figure 1, constraint (12) defines a set \( \mathcal{T}_d \) of possible taxes or subsidies \( t^c \) that satisfy (12). The equality version of (12) admits an upper and lower root which will be denoted by \( t_d \) and \( t_c \) respectively.

Condition (13) provides the government with an incentive for revealing information when the environment is clean. From (13), the government would rather choose \( t^c \) and transmit all information than let the uniformed households wrongly perceive the environment as dirty and optimize accordingly. Let \( \mathcal{T}_b \) denote the interval of taxes or subsidies for which condition (13) is met (see Figure 1) and define \( t_c \) and \( t_d \) as, respectively, the upper and lower root of equation \( W(t, \varepsilon^c, 1) = W(0, \varepsilon^c, 0) \).

In order to fully reveal that the environment is clean, the government must choose \( t^c \) in the interval \( \mathcal{T}_d \cap \mathcal{T}_c \) provided that the latter is non empty. This interval is depicted in Figure 1. Proposition 1 establishes necessary conditions for the existence of separating equilibria in which manifold taxes or subsidies may signal the clean environment.

**Proposition 1:** If \( \mu(t) = 0 \) for all \( t \notin \{ t_d, t^c \} \), then any pair \( (t^d, t^c) \in \mathcal{T} \times \mathcal{T} \) such that \( t^d = N \varepsilon \) and \( t^c \in [t_c, t_d] \) is part of a separating equilibrium.
Proof: (see Appendix 4)

One surprising insight is the emergence of a tax or a subsidy when the environmental damage is nil. Indeed, the analysis in Appendix 4 identifies two possible cases for $t^c$ depending on whether $t_d$ is lower or higher than 0 (or, equivalently, $W(0, \varepsilon^d, 1)$ is higher or lower than $W(t^d, \varepsilon^d, 0)$). First, if $t_d < 0$, then separation is achieved with a subsidy $t^c < 0$ when the environment is clean. Second, if $t_d > 0$, then the separating equilibrium involves a tax $t^c > 0$. Hence, signaling a clean environment can take the the form of a subsidy as well as a tax. What really matters here is that either instrument generates a social cost which would be unbearable were the environment dirty.

By examining an usual motive for taxes, the model emphasizes a novel rationale for subsidies. Not only are taxes being used here to internalize the pollution externalities when the environment is dirty, but they convey information in either state of the environment. A subsidy can only play the signaling role. Subsidizing the good is socially costly in both states of the environment because it distorts the households’ consumption from the first-best level. Nevertheless, it is relatively less costly for the government to subsidize the consumption of an environmental friendly good which causes no environmental damage. Consequently, there may exist a subsidy inside $T_d \cap T_c$ when the environment is clean, too high to be duplicated when the environment is dirty, at which all information is conveyed from the government to uninformed households. Equilibrium taxes and subsidies have the
common feature in the clean environment that they entail signaling costs that must be paid to achieve separation.

Note that if we were to consider the case in which the per-unit damage $\varepsilon$ is a random variable described by a cumulative distribution function and a density with continuum support $[\varepsilon^c, \varepsilon^d]$, the reduced form signaling framework derived from primitives on the economy satisfies the conditions stated by Mailath (1987) which guarantee the existence of separating equilibria.

The potential existence of separating equilibria does not dismiss pooling equilibria. Let $t^*$ denote the uninformative tax or subsidy that gives a pooling equilibrium. As the government provides no information at equilibrium, the uninformed households’ beliefs remain unchanged after observing $t^*$. To be part of a pooling equilibrium, $t^*$ must satisfy the two following conditions:

$$W(t^*, \varepsilon^d, \mu_0) \geq W(t^d, \varepsilon^d, 0)$$
$$W(t^*, \varepsilon^c, \mu_0) \geq W(0, \varepsilon^c, 0).$$

The right-hand side of the inequalities above reflects, in each state of the environment, the welfare levels that can be attained at best when uninformed households hold the least favorable beliefs. Given the prior beliefs about the true state of the environment, the government should at least reach these levels to successfully conceal information in equilibrium with $t^*$.

Consider the equality version of (14) and define $\bar{t}_d(\mu_0)$ and $\underline{t}_d(\mu_0)$ as,
respectively, the upper and lower root of this equation in \( t \). Define \( \tilde{t}_c(\mu_0) \) and \( t_c(\mu_0) \) in the same way with \( W(t, \epsilon^c, \mu_0) = W(0, \epsilon^c, 0) \). An argument similar to that given in Appendix 4 yields that \( t_d(\mu_0) < \tilde{t}_c(\mu_0) \). The set of pooling equilibrium taxes and subsidies is characterized in the following lemma.

**Lemma 3:** Any tax \( t^* \in [t_d(\mu_0), \tilde{t}_c(\mu_0)] \) can be supported as a pooling equilibrium by beliefs \( \mu^*(t) = 0 \) for all \( t \neq t^* \).

Any deviation \( t \) from the pooling tax \( t^* \) is such that \( W(t, \epsilon^d, 0) \leq W(t^d, \epsilon^d, 0) \leq W(t^*, \epsilon^d, \mu_0) \) and \( W(t, \epsilon^c, 0) \leq W(0, \epsilon^c, 0) \leq W(t^*, \epsilon^c, \mu_0) \). Therefore, out-of-equilibrium beliefs involving \( \mu^*(t) = 0 \) are likely to support \( t^* \) as a pooling equilibrium.

The next proposition establishes that restrictions on off-the-equilibrium path beliefs required by (10) and (11) eliminate all the pooling equilibria and single out the so-called “least-cost separating equilibrium” which has received much emphasis in the work of Spence (1974), Riley (1979) and Cho-Kreps (1987), among others.

The two cases mentioned in the analysis of separating equilibria yield two different “intuitive” equilibrium outcomes. Their emergence depends on the value of the differential \( \epsilon = \epsilon^d - \epsilon^c \). For small values of \( \epsilon \), we have \( W(0, \epsilon^d, 1) > W(t^d, \epsilon^d, 0) \). This may also be the case if, for a given \( \epsilon \), the economy is poorly informed about the environment, i. e., \( I \) is close to zero.
Then, the government faced with a dirty environment can be said to “envy” the clean environment policy that achieves the high level of consumption $X(0, 1) = X(0, \varepsilon^c)$ with zero tax. In the opposite case, such an envy vanishes since it is socially too costly for the government faced with a dirty environment to deviate from the Pigovian tax and set zero tax, even if doing so would convince uninformed households that the environment is clean and so boost their consumption. The latter case arises only if the two possible values $\varepsilon^d$ and $\varepsilon^c$ are sufficiently different. Note that such a case could not be captured by considering that $j^d$ is continuous.

**Proposition 2:** The unique equilibrium robust to the intuitive criterion is the least-costly separating one characterized by:

- the Pigovian tax $t^d = \frac{N}{\varepsilon}$ when the environment is dirty, and the subsidy $t^c = t_d < 0$ when the environment is clean, if $W(0, \varepsilon^d, 1) \geq W(t^d, \varepsilon^d, 0)$;

- the Pigovian tax $t^d = \frac{N}{\varepsilon}$ when the environment is dirty, and $t^c = 0$ when the environment is clean, if $W(0, \varepsilon^d, 1) < W(t^d, \varepsilon^d, 0)$.

**Proof:** (see Appendix 5)

When the government faced with a dirty environment is envious of the clean environment policy, i.e., $W(0, \varepsilon^d, 1) \geq W(t^d, \varepsilon^d, 0)$, the subsidy $t_d$ is the efficient means to reveal to uninformed households that the environment...
is clean. This is the only case where signaling entails a welfare sacrifice since the subsidy raises households’ consumption above the first-best level. The subsidy can work as a signal because the government has less to lose from getting households to consume more when the environment is clean than when it is dirty. In fact, consumption distortions have negative polluting effects in a dirty environment, not in a clean one. Furthermore, any pooling equilibrium is removed by attractive deviations in a range of subsidies susceptible to convince uninformed consumers that the environment is clean. It is important to stress that, in this case, no information could be revealed in the absence of subsidies. Suppose that the government has neither the authority not the inclination to subsidize the environmental friendly good, then only pooling equilibrium taxes can emerge in equilibrium. Moreover, such taxes would be robust to the intuitive criterion since it would be forbidden to offer subsidies in the range mentioned in the Appendix, where deviations might be attractive. Relative to the least-costly separating equilibrium with subsidy, households may or may not be better off in a pooling equilibrium, depending on the costs of the lost information versus the saving in signaling.

When the clean environment policy is no longer a temptation for the government faced with a dirty environment, i.e., $W(0, \varepsilon^d, 1) < W(t^d, \varepsilon^d, 0)$, the equilibrium selection yields that the government should adopt the same policy as that under complete information. The Pigovian tax turns to be an efficient instrument for signaling the true state of the environment, that
is, zero tax when the environment is clean. Although, by Proposition 1, a positive tax may signal the clean environment in equilibrium, it is not robust to the intuitive criterion. Indeed, with zero tax, the government can also convince uninformed households that the environment is clean, and at a lower social cost. Intuitively, the government will choose the least-costly way of fully revealing information. Unlike subsidies in the previous case, taxes in this case entail no social cost and households become informed for free.

As the role of subsidies is purely informative in the clean environment, one expects subsidies to decline as more households become informed about the state of the environment. This has already been suggested while examining the single-crossing property: a larger number of informed households was shown to mitigate the detrimental effects of subsidies on welfare in a clean environment. Intuitively, the burden of the signaling cost should be reduced in a more informed economy, thereby decreasing the level of consumption distortion necessary to fully convey information. The result in the next corollary formally captures this intuition.

**Corollary 1:** The least-costly separating equilibrium subsidy decreases with $I$: $\lim_{I \to 0} \frac{dt}{dI} > 0$.

**Proof:** (see Appendix 6)

Clearly from this result, the informed households generate a positive externality favorable to the uninformed households. Let us now state the nec-
essary and sufficient condition for a separating equilibrium subsidy to exist when the government faced with a dirty environment is envious of the clean environment policy and no household is informed.

**Corollary 2:** The least-costly separating equilibrium subsidy exists when $I = 0$ if and only if

\[
\frac{Q(0)u(X(0, \varepsilon^c)) - Q(\varepsilon)u(X(t^d, \varepsilon^d))}{X(0, \varepsilon^c) - X(t^d, \varepsilon^d)} > p + \varepsilon N. \quad (16)
\]

**Proof:** (see Appendix 7)

Whether condition (16) is met depends on a number of factor. To get more intuition, it is useful to examine this for a particular case. Consider for instance the constant relative risk aversion (CRRA) utility functions $u(x) = \frac{x^{1-\alpha}}{1-\alpha}$, where $0 < \alpha \leq 1$, so that $u(x) = \ln x$ as $\alpha \to 1$. Moreover, normalize as follows: $N = 1, Q(\varepsilon) = 1$, and denote $Q(0) \equiv q$. For this case, the informed demands are $X(t, \varepsilon^c) = \left(\frac{q}{p+t}\right)^\frac{1}{\alpha}$ and $X(t, \varepsilon^d) = (p + t)^{-\frac{1}{\alpha}}$, which can be substituted into (16) to yield

\[
q^\frac{1}{\alpha} (\alpha p + \varepsilon (\alpha - 1)) (p + \varepsilon)^{\frac{1-\alpha}{\alpha}} > \alpha p^\frac{1}{\alpha}. \quad (17)
\]

It turns out that the least-costly separating equilibrium exists when $I = 0$ unless $\alpha p + \varepsilon (\alpha - 1) < 0$. As the single-crossing condition always holds
when \( t \) is a subsidy (see Appendix 1), the separating equilibrium subsidy \( L_d \) will actually exist when \( I = 0 \) if \( \varepsilon \) is sufficiently small and \( q \) is sufficiently large. In other words, the subsidy is an efficient device for signaling the clean environment in a poorly informed economy, as long as the level of environmental damage is not too high and the incremental willingness-to-pay for an environmental friendly product is sufficiently large.

4 Conclusion

This paper develops a signaling rationale for subsidies in an economy where the government has better information about the environmental damage than some households do. When these uninformed households overestimate the level of environmental damage, the government can use subsidies to fully reveal information. Like money burning expenditures for advertising, subsidies are persuasive rather than directly informative in that they lift up the uninformed households’ willingness-to-pay for a clean good.

Subsidies entail losses in welfare that the government must accept to convey her superior information in a credible manner. Subsidies turn out to be optimal means for signaling the clean environment because the government has less to lose from getting households to consume more a good that protects the environment than a good that deteriorates it. Furthermore, compared to a clean environment, the government faced with a dirty environment increases
less private welfare with subsidies since informed households are less eager
to consume the private good. Understanding that the society in a dirty
environment will suffer from further degradation of environmental quality,
whereas it will not in a clean environment, uninformed households rationally
expect subsidies to mean clean environment.

As a result, the intuitive criterion singles out the equilibrium in which
the subsidy is the minimum waste necessary to signal the clean environment.
This is shown to arise only when the government faced with a dirty environ-
ment is envious of the clean environment policy. In this case, it would be
socially detrimental to forbid subsidies. Taxes, either positive or null, would
then emerge as the only existing pooling equilibria. However, the cost of con-
cealing information, even with zero tax, may be greater than the signaling
cost of subsidies when the environment is clean, depending on the level of
prior beliefs.

Two further observations are in order. First, the government can effi-
ciently signal the clean environment with a lower subsidy as more households
are informed about the true environmental damage. Second, two factors
make separating equilibrium subsidies more likely to appear in a poorly in-
formed economy: a low level of environmental damage in a dirty environment
and a high incremental willingness-to-pay for a clean good.

It would be worthwhile to examine how robust the conclusions of the
paper would be in a dynamic setting. In this line of research, one possible
extension borrowed from Bagwell and Riordan (1991) would be to consider that the number of informed households grows as time passes. Since it would become easier for the government to signal the clean environment, subsidies might decline over time. Another line of research is to seek empirical support for the theoretical finding that subsidies are correlated with higher environmental quality and to test whether this relationship depends on the amount of households’ information.
5 Appendix

5.1 Appendix 1: Complete information and single-crossing property

Differentiating $W(t, \varepsilon^i, \mu)$ with respect to $t$ yields

$$W_t(t, \varepsilon^i, \mu) = I \left[ Q(\varepsilon^i) u'(X(t, \varepsilon^j)) - p \right] X_t(t, \varepsilon^i) + (N - I) \left[ \bar{Q}(\mu) u'(X(t, \mu)) - p \right] X_t(t, \mu)$$
$$\quad - \varepsilon^i N \left[ I X_t(t, \varepsilon^i) + (N - I) X_t(t, \mu) \right].$$

Using the first-order conditions for utility maximization, that is, $Q(\varepsilon^i) u'(x_i) = p + t$ for informed households and (4) for uninformed households, we get

$$W_t(t, \varepsilon^i, \mu) = (t - \varepsilon^i N) \left[ I X_t(t, \varepsilon^i) + (N - I) X_t(t, \mu) \right]. \quad (19)$$

Thus, for all $\mu \in [0, 1]$, $t^i(\mu) = N \varepsilon^i$. The Inada conditions $u''(0) = +\infty$ and $u'(+\infty) = 0$ ensure that the maximum is interior. The second-order condition for welfare maximization when evaluated at the optimum yields $W_{tt}(t, \varepsilon^i, \mu) = I X_t(t, \varepsilon^i) + (N - I) X_t(t, \mu)$ which is negative due to the strict concavity of $u(x)$.

Moreover, the derivative of $W_t(t, \varepsilon^i, \mu)$ with respect to $\varepsilon^i$ is well defined
and gives

\[ W_{t\varepsilon^j}(t, \varepsilon^j, \mu) = \left( t - N\varepsilon^j \right) IX_{t\varepsilon^j}(t, \varepsilon^j) - N \left[ IX_t(t, \varepsilon^j) + (N - I) X_t(t, \mu) \right]. \]

(20)

From the equation above, when \( I \) is close to 0, \( W_{t\varepsilon^j}(t, \varepsilon^j, \mu) \) tends to \(-N^2X_t(t, \mu) > 0\). Thus, there exists some \( \tilde{I} > 0 \) such that, for all \( t \in T \), \( I < \tilde{I} \) and \( \mu \in [0, 1] \), we have \( W_{t\varepsilon^j}(t, \varepsilon^j, \mu) > 0 \).

As \( X_t(t, \varepsilon^j) = \frac{1}{Q(\varepsilon^j)u''} \), we have \( X_{t\varepsilon^j}(t, \varepsilon^j) = -\frac{Q'}{Q^2u''} < 0 \). Note that, for all \( t \leq N\varepsilon^j \), the condition \( W_{t\varepsilon^j}(t, \varepsilon^j, \mu) > 0 \) is met. In particular, the single-crossing condition always holds when \( t \) is a subsidy. In such a case, the first term in the right-hand side of (20) increases with \( I \) and \( |Q'| \). Hence, the single-crossing property is more likely to hold either when households are more willing to pay for environmental friendly goods or when households are more informed about the true environmental damage.

When \( t > N\varepsilon^j \), the first term in the right hand side of (20) is negative, hence the single-crossing property may be reversed for sufficiently high values of \( t \). More precisely, let \( \tilde{t}(\varepsilon^j, \mu) \) denote the value of \( t \) that implicitly solves \( W_{t\varepsilon^j}(t, \varepsilon^j, \mu) = 0 \):

\[ \tilde{t}(\varepsilon^j, \mu) \equiv N\varepsilon^j + \frac{N \left[ IX_t(t, \varepsilon^j) + (N - I) X_t(t, \mu) \right]}{IX_{t\varepsilon^j}(t, \varepsilon^j)} = N\varepsilon^j - \frac{N \left[ I/Q(\varepsilon^j) + (N - I)/\tilde{Q}(\mu) \right]}{IQ'} Q^2(\varepsilon^j). \]

(21)

Note first that the lower is \( |Q'| \), the higher is \( \tilde{t}(\varepsilon^j, \mu) \). As for all \( \mu \in [0, 1] \),
\( \hat{t}(\varepsilon_j, 1) < \hat{t}(\varepsilon_j, \mu) \), we have that, for all \( t < \hat{t}(\varepsilon_j, 1) \), condition \( W_{t, \varepsilon_j}(t, \varepsilon_j, \mu) > 0 \) is met whatever \( \mu \). For expositional convenience, we will assume henceforth that \( |Q'| \) is sufficiently small to fulfill the two following conditions:

\[
W(\hat{t}(\varepsilon_j, 1), \varepsilon_j, 1) < W(\varepsilon_j N, \varepsilon_j, 0), j = c, d. \quad (22)
\]

This will ensure that the single-crossing holds for every tax lower than \( \hat{t}(\varepsilon_j, 1) \).

### 5.2 Appendix 2: Stochastic dominance property

From the expression given in (8), differentiating \( W(t, \varepsilon_j, \mu) \) with respect to \( \mu \) yields:

\[
W_\mu(t, \varepsilon_j, \mu) = (N - I) \left[ \hat{Q}'(\mu)u(X(t, \mu)) + \left( \hat{Q}(\mu)u'(X(t, \mu)) - p - N\varepsilon_j \right) X_\mu(t, \mu) \right]. \quad (23)
\]

Using (4), (23) can be rewritten

\[
W_\mu(t, \varepsilon_j, \mu) = (N - I) \left[ \hat{Q}'(\mu)u(X(t, \mu)) + (t - N\varepsilon_j) X_\mu(t, \mu) \right]. \quad (24)
\]

For all \( t \geq N\varepsilon_j \), \( W_\mu(t, \varepsilon_j, \mu) > 0 \), and so the latter inequality is met for taxes higher than the Pigovian level. If now \( t < N\varepsilon_j \), consider first that \( t = -p \). Recall from (5) that \( X_\mu(t, \mu) = -\frac{\hat{Q}(\mu)}{\hat{Q}(\mu)\mu'} \). Substituting this
expression into (24) yields for $t = -p$:

$$W_{\mu}(-p, \varepsilon^j, \mu) = (N - 1) \hat{Q}'(\mu) \left[ u(X(-p, \mu)) + (p + N\varepsilon^j) \frac{u'(X(-p, \mu))}{\hat{Q}(\mu)u''(X(-p, \mu))} \right],$$

where $u'(X(-p, \mu)) = 0$. Thus, we have $W_{\mu}(-p, \varepsilon^j, \mu) > 0$. Furthermore, by (19), $W_{\mu}(t, \varepsilon^j, \mu) = (t - \varepsilon^j N) (N - 1) X_{\mu}(t, \mu)$ where $X_{\mu}(t, \mu) > 0$. Hence, $W_{\mu}(t, \varepsilon^j, \mu)$ is strictly monotonic for $t < N\varepsilon^j$. It follows that inequality $W_{\mu}(t, \varepsilon^j, \mu) > 0$ holds for all $t \in T$.

5.3 Appendix 3: Proof of lemma 2

Suppose for a contradiction that there exists a separating equilibrium in which $t^d \neq N\varepsilon$. As the uninformed households’ expectations are correct at equilibrium, the resulting social welfare is $W(t, \varepsilon^d, 0)$ which is strictly lower than $W(N\varepsilon, \varepsilon^d, 0)$. Then, the government would have an incentive to deviate to $t = N\varepsilon$ whatever the households’ inference $\mu$ from observing this tax. Indeed, for any $\mu \in (0, 1]$, we have

$$W(N\varepsilon, \varepsilon^d, 0) = W(N\varepsilon, \varepsilon^d, \mu) + \int_{\mu}^{0} W_{\mu}(N\varepsilon, \varepsilon^d, \rho)d\rho.$$

From the stochastic dominance property, $W_{\mu}(N\varepsilon, \varepsilon^d, \mu) > 0$, thus, $\int_{\mu}^{0} W_{\mu}(N\varepsilon, \varepsilon^d, \rho)d\rho < 0$ and $W(N\varepsilon, \varepsilon^d, 0) < W(N\varepsilon, \varepsilon^d, \mu)$. If $t^d \neq N\varepsilon$, then $W(t^d, \varepsilon^d, 0) < W(N\varepsilon, \varepsilon^d, 0)$. If $t^d = N\varepsilon$, then $W(t^d, \varepsilon^d, 0) = W(N\varepsilon, \varepsilon^d, 0)$. But
and so \( W(t^d, \varepsilon^d, 0) < W(N\varepsilon, \varepsilon^d, \mu) \). Moreover, \( t^d = N\varepsilon \) can be supported as the only separating equilibrium tax when the environment is dirty, given the assumption \( \mu(t) = \hat{\mu} \) for all \( t \neq t^d \) since \( W(t, \varepsilon^d, 0) < W(N\varepsilon, \varepsilon^d, 0) \).

### 5.4 Appendix 4: Proof of proposition 1

Using the definitions of \( t_c \) and \( t_d \), we have

\[
W(t_d, \varepsilon^d, 1) = W(t^d, \varepsilon^d, 0) \tag{26}
\]

\[
W(t_c, \varepsilon^c, 1) = W(0, \varepsilon^c, 0). \tag{27}
\]

Subtracting (27) from (26) yields

\[
\int_{t_c}^{t_d} \int_{\varepsilon^c}^{\varepsilon^d} W(t, \varepsilon, 1) d\varepsilon dt = \int_0^{t^d} \int_{\varepsilon^c}^{\varepsilon^d} W(t, \varepsilon, 0) d\varepsilon dt. \tag{28}
\]

The single-crossing property (9) guarantees that, for all \( \mu \in [0, 1] \) and all \( t \in T \), \( W(t, \varepsilon, \mu) > 0 \). Hence, equality (28) implies that

\[
\int_{t_c}^{t_d} \int_{\varepsilon^c}^{\varepsilon^d} W(t, \varepsilon, 1) d\varepsilon dt > 0, \tag{29}
\]

and so

\[
t_c < t_d. \tag{30}
\]
With a similar argument, it is straightforward to state that $\tilde{t}_c < \tilde{t}_d$. For this, $|Q'|$ must be sufficiently small to fulfill (22), which guarantees that $\tilde{t}_d < \tilde{t}(\varepsilon, 1)$ (see Appendix 1) and so allows to use the single-crossing property. Thus, $\mathcal{T}_c \cap \mathcal{T}_d = [\tilde{t}_c, \tilde{t}_d] \neq \emptyset$ if $\tilde{t}_d < \tilde{t}_c$, and $\mathcal{T}_c \cap \mathcal{T}_d = \mathcal{T}_c$ otherwise. For expositional convenience, we will restrict attention to the case $\tilde{t}_d < \tilde{t}_c$ so that $\mathcal{T}_c \cap \mathcal{T}_d = [\tilde{t}_c, \tilde{t}_d]$.

This leaves the possibility for two different cases depending on whether $\tilde{t}_d$ is lower or higher than 0 (or, equivalently, $W(0, \varepsilon, 1)$ is higher or lower than $W(t^d, \varepsilon, 0)$). If $\tilde{t}_d < 0$, then separation is achieved with a subsidy $t^c < 0$ when the environment is clean. If $\tilde{t}_d > 0$, then the separating equilibrium involves a tax $t^c > 0$. Any $t^c \in [\tilde{t}_d, \tilde{t}_c]$ can be supported as a separating equilibrium in a clean environment by out-of-equilibrium beliefs such that $\mu^*(t) = 0$ for all $t \neq t^c$. Such beliefs make a deviation $t$ from equilibrium unattractive when the environment is clean since $W(t, \varepsilon, 0) \leq W(0, \varepsilon, 0) = W(t_c, \varepsilon, 1) \leq W(t^c, \varepsilon, 1)$, and also when the environment is dirty since $W(t, \varepsilon, 0) \leq W(t^d, \varepsilon, 0)$.

### 5.5 Appendix 5: Proof of proposition 2

Consider the case $W(0, \varepsilon, 1) \geq W(t^d, \varepsilon, 0)$. Then, $\tilde{t}_d \leq 0$ since $\tilde{t}_d$ is the lower root in $t$ of equation $W(t, \varepsilon, 1) = W(t^d, \varepsilon, 0)$.

Suppose now that separation is achieved in equilibrium at $t^c < \tilde{t}_d$. Then, for all $\epsilon > 0$ such that $t^c + \epsilon < \tilde{t}_d$, we have $W(t^c + \epsilon, \varepsilon, 1) < W(\tilde{t}_d, \varepsilon, 1)$.
because \( W(t, \varepsilon^c, 1) \) is increasing in \( t < 0 \). Moreover, \( W(t^c + \epsilon, \varepsilon^d, 1) < W(t_d, \varepsilon^c, 1) = W(t_d, \varepsilon^d, 0) \). Hence, \( t^c + \epsilon \) is a deviation that fulfills both (10) and (11). Thus, any separating equilibrium in which \( t^c < t_d \) fails to survive the intuitive criterion.

Consider now a pooling equilibrium tax \( t^* \). There exist two values \( t' \) and \( t'' \) to the left of \( t \) such that, respectively, \( W(t', \varepsilon^c, 1) = W(t^*, \varepsilon^c, \mu_0) \) and \( W(t'', \varepsilon^d, 1) = W(t^*, \varepsilon^d, \mu_0) \). Using the same technique as that in Appendix 4, one can easily show that \( t' < t'' \). It follows that any deviation \( t \) inside \( (t', t'') \) fulfills the two conditions \( W(t^*, \varepsilon^c, \mu_0) < W(t, \varepsilon^c, 1) \) and \( W(t, \varepsilon^d, 1) < W(t^*, \varepsilon^d, \mu_0) \), which causes elimination of the pooling equilibrium by the intuitive criterion.

Consider now the case \( W(0, \varepsilon^d, 1) < W(t_d, \varepsilon^d, 0) \). Then, \( t_d > 0 \) and separation is achieved with a tax \( t^c \in [t_c, t_d] \). However, a deviation at \( t = 0 \) is potentially attractive for the government faced with the clean environment when this deviation induces the belief that the environment is clean for sure. The intuitive criterion precisely imposes to restrict beliefs in this sense, for \( t = 0 \). As \( W(t^c, \varepsilon^c, 1) < W(0, \varepsilon^c, 1) \) and \( W(0, \varepsilon^d, 1) < W(t_d, \varepsilon^d, 0) \), any separating equilibrium with a positive tax inside \( [t_c, t_d] \) fails to survive the intuitive criterion. Furthermore, the argument to eliminate all the pooling equilibria in this case is similar to that used in the previous case.
5.6 Appendix 6: Proof of corollary 1

Using (8) to differentiate $W(t_d, \varepsilon^d, \mu) - W(t^d, \varepsilon^d, 0) = 0$ with respect to $t_d$
and $I$ yields

$$W_t(t_d, \varepsilon^d, \mu) dt_d + \left[ Q(\varepsilon^d) u(X(t_d, \varepsilon^d)) - (p + N\varepsilon^d)X(t_d, \varepsilon^d) - \hat{Q}(1)u(X(t_d, 1)) + (p + N\varepsilon^d)X(t_d, 1) \right]$$

(31)

which can be rewritten for $I = 0$: $W_t(t_d, \varepsilon^d, \mu) dt_d = -\frac{1}{N} [W(t_d, \varepsilon^d, 0) - W(t_d, \varepsilon^d, 1)] dI$.

Furthermore, the welfare differential in the right-hand side of this equation can be expressed as

$$W(t_d, \varepsilon^d, 1) - W(t_d, \varepsilon^d, 0) = \int_0^1 W_{\mu}(t_d, \varepsilon^d, \rho)d\rho,$$

(32)

which is positive by the stochastic dominance property. As $t_d < t^d$, we have $W_t(t_d, \varepsilon^d, \mu) > 0$ and so

$$\lim_{I \to 0} \frac{dt_d}{dI} = \frac{\int_0^1 W_{\mu}(t_d, \varepsilon^d, \rho)d\rho}{NW_t(t_d, \varepsilon^d, \mu)} > 0.$$

(33)

5.7 Appendix 7: Proof of corollary 1

By (8), we get
\[
W(t, \varepsilon^d, 1) \equiv I \left[ Q \left( \varepsilon^d \right) u(X(t, \varepsilon^d)) - pX(t, \varepsilon^d) \right] \\
+ (N - I) \left[ \tilde{Q}(1) u(X(t, 1)) - pX(t, 1) \right] - \varepsilon^d N \left[ I X(t, \varepsilon^d) + (N - I) X(t, 1) \right],
\]

and

\[
W(t^d, \varepsilon^d, 0) = N \left[ Q \left( \varepsilon^d \right) u(X(t^d, \varepsilon^d)) - pX(t^d, \varepsilon^d) \right] - \varepsilon^d N^2 X(t^d, \varepsilon^d). \quad (35)
\]

These expressions can respectively be rewritten for \( I = 0 \):

\[
W(t^d, \varepsilon^d, 0) = N \left[ Q(\varepsilon) u(X(t^d, \varepsilon^d)) - pX(t^d, \varepsilon^d) \right] - \varepsilon^d N^2 X(t^d, \varepsilon^d) \quad (36)
\]

\[
W(t, \varepsilon^d, 1) = N \left[ Q(0) u(X(t, \varepsilon^c)) - pX(t, \varepsilon^c) \right] - \varepsilon^d N^2 X(t, \varepsilon^c). \quad (37)
\]

Condition \( W(0, \varepsilon^d, 1) > W(t^d, \varepsilon^d, 0) \) then amounts to (16).
References


$W(t, \epsilon^j, \mu), \ j = c, d; \ \mu = 0,1$

**Figure 1**

Welfare functions and separating equilibrium subsidies