Aid to Poor Resource Exporting Countries: Which Role Should be Played by Resource Taxation?*

Ruxanda Berlinschi†, Julien Daubanes‡

This version, December 15, 2007

Abstract

We use a dynamic, decentralized equilibrium, two-country model to study the transfers between a rich altruistic resource-poor country (North) and a two-class resource-rich country where rich resource holders (southern rich) coexist with poor workers (southern poor). The North typically extracts some mining rents through a distortional tax on the resource. When it uses foreign aid, the North is led to lower its tax rate, which still remains at an inefficient level. Then, we propose the use of a contract between the North government and the resource holders specifying a tax level and a transfer from the latter to the southern poor. We show that the introduction of such an original redistributive instrument corrects the propensity of the North to overtax the resource. It thus improves the global efficiency and appears overall to be Pareto-improving.

JEL classification: Q3; O1; F1

Keywords: Exhaustible resources; Aid; Taxation

*We thank Jean-Paul Azam and André Grimaud for their useful suggestions and encouragements. Thanks are also due to Sjak Smulders, Cees Withagen and Christian Gollier for very valuable comments. The usual disclaimer applies.

†Toulouse School of Economics (MPSE and ARQADE), e-mail address: ruxanda.berlinschi@gmail.com

‡Corresponding Author. Toulouse School of Economics (MPSE and LERNA), e-mail address: daubanes@toulouse.inra.fr. Tel.: +33 561 12 87 69; fax: +33 561 12 85 20
1 Introduction

In some countries endowed with exhaustible resources, powerful rich resource holders coexist with a large population of poor people. Some of these countries are given foreign aid from the developed world, while the latter, through distortional resource taxation, captures part of the mining rent of the resource holders. Is there a more efficient way to help the poor living in resource-rich countries? One solution that was suggested in a 1986 study by the Congressional Budget Office of the United States was to lower the tariff on oil imported from poor countries\(^1\). However, there is no guarantee that the resulting extra rent would have been fairly shared with the poor population. Recently, Sala-i-Martin and Subramanian (2003) propose the drastic option of equally sharing the entire resource revenues with the whole population. Although very appealing, this solution is unlikely to be implementable from a political economy point of view. Our paper aims at designing an efficient and feasible redistribution scheme: a contract with the resource holders on the tax rate and on the share of the resource rent to be redistributed to the poor.

Oil endowments are concentrated in some geologically specific grounds. As a consequence, their distribution across countries is very heterogeneous. For instance, the 19 countries with the largest crude oil reserves per capita represent more than 80% of the current world reserves\(^2\). In a world where countries decide individually on their taxation policy, this heterogeneity provides resource producing countries and resource consuming countries with different incentives to tax the resource. The reason for this is that the taxation of exhaustible resources entails a rent transfer from resource producers to the fiscal authorities (see Bergstrom, 1982, and Daubanes and Grimaud, 2006, for theoretical analysis). Of course, resource taxation can have other rationales. In particular, it may be an economic instrument to correct the pollution externality due to carbon emissions resulting from the resource combustion. Even in that case, a resource-poor country will have a propensity to overtax the resource relatively to a resource-rich country\(^3\). Indeed, a simple empirical study (Bacon, 2001) states that taxes in oil producing countries are lower than those in the non-producing regions. This introduces a bias at the global level: resource taxes are much higher in top-consuming

\(^1\)The objective of this proposal was actually to avoid Mexico’s disability to meet its debt payments.

\(^2\)Sources: The World Bank and Oil & Gas Journal.

\(^3\)Moreover, it seems that world leaders in the debate on mitigating pollution emissions have chosen "cap and trade" instruments based on permits market. In this context, resource taxation would be useless to regulate pollution.
regions. There are two reasons why this bias may not be desirable. On the one hand, this results in very different final resource prices across countries\textsuperscript{4}. The resulting differences in marginal resource productivities cause economic inefficiencies at the world level (see Daubanes and Grimaud, 2006). In other words, the bias in resource taxation due to heterogeneous endowments and non-cooperative local policy design is distortional.

On the other hand, high taxes in developed, top-consuming, regions, lead to huge rent transfers\textsuperscript{5} from some poor countries to the developed world. Indeed, most of the resource exporting countries are low or middle income countries. For instance, the per capita GNI over the 19 countries with the largest crude oil reserves per capita is lower than US$5800\textsuperscript{6}. In particular, among the top oil-producers, some countries are very poor in per capita terms. Typically, in these countries, resource holders receiving the net rent, coexist with numerous very poor workers to which the developed world gives foreign aid\textsuperscript{7}.

Overall, rich developed, top fuel consuming countries capture rents, through distortional resource taxation, supposed to accrue to these poor resource-rich countries, while, at the same time, they transfer aid to these countries. Of course, these inter-country transfers don’t involve the same categories of residents. Aid is supposed to be transferred to poor workers while rents are extracted from rich resource holders’ revenues. The situation of the different groups under the coexistence of these opposite transfers between resource consumers and resource producers and under the related distortional taxes, can be improved. We show that the use of an original redistributive instrument can help mitigate the associated distortions and make all concerned

\textsuperscript{4}The IEA (2001) remarks the extent of this heterogeneity as regards fuels taxation and notes that recent fiscal measures don’t lead to a convergence of these tax rates.

\textsuperscript{5}Given the current high prices of fossil fuels and the high tax rates in most of the top-consuming countries (see IEA, 2001), these rent transfers are indeed supposed to represent huge amounts. A recent document by the OPEC states that over the period 2000-2004, the G7 nations made a total of US$1.600 billion from oil taxation. Another document by the IEA (2001) reports that, in OECD, oil product taxes represent, on average, slightly less than 6% of total tax revenues (6.5% for the EU15).

\textsuperscript{6}Sources: The World Bank and Oil & Gas Journal.

\textsuperscript{7}One of the most suited examples of the countries we refer to is Nigeria. In 2005, fuel exports amounted to 98% of Nigerian merchandise exports. In PPP terms, Nigeria’s per capita GDP was US$1,113 in 1970 and US$1,084 in 2000. Sala-i-Martin and Subramanian (2003) shows how the income distribution deteriorated very sharply since the discovery of oil in Nigeria: "more and more people have been pushed towards poverty and towards extreme wealth". In this country, 70% of the population lived on less than US$1 per day in 2000. In 2005, foreign aid given to Nigeria was equal to about 8% of its GNI and its value was of US$6,437,309,952. Similar examples could be provided by reference to Iran and Algeria. Sources: The World Bank, Sala-i-Martin and Subramanian (2003).
groups better-off.

The issue of redistributing wealth to poor countries endowed with natural resources is related to three different economics literatures. First, for the last decades, the environmental economics literature has developed general equilibrium models of resource depletion. In particular, the standard integrated models representing a homogeneous world have been disaggregated to study the inter-country transfers due to resource taxation. Bergstrom (1982) shows how resource taxation entails rent transfers from resource producing countries to resource consuming countries. The argument is based on the exhaustibility of the resource, that bounds its asymptotic cumulated supply. In Brander and Djajic (1983), both the consuming and the producing countries are represented by local social planners. The consuming country is then limited in extracting the resource rent by the threat of the producing country to use the resource itself. Daubanes and Grimaud (2006) uses a related framework where decisions are decentralized and the resource use fills a stock of atmospheric pollution. It emphasizes that the consuming country is limited by the loss of competitiveness, and the subsequent relocation of the productive activities, resulting from a higher tax rate. It also shows that, even when the taxation of the resource aims at correcting the pollution externality, the bias in the resource-poor economy that consists in taxing at a higher rate than optimally remains. With respect to these contributions, the present paper considers intra-country heterogeneity and a form of inter-country altruism. We consider a two-class resource-rich economy (South) where the poor (southern poor) coexist with rich resource holders (southern rich), who control the government, and a rich resource-poor economy (North), whose agents are altruistic towards the southern poor. We also introduce aid mechanisms.

Second, the development economics literature has examined the particular difficulties of developing countries with abundant natural resources. Some papers try to figure out why these economies grow more slowly than those without natural resources, which is the well known phenomenon of the resource curse. One of the most frequent explanations of this stylized fact is related to the so-called Dutch disease: the fact that important amounts of foreign exchange entering the country lead to the appreciation of the local currency and undermine the competitiveness of other exporting sectors, like manufacturing or agriculture. Other explanations are related to the volatility of the government's revenues caused by the volatility of the international resource prices, which often leads to inflation and hurts growth. Other papers try to understand why resource-rich economies have poorer institutions than the others. One of the explanations of this phenomenon is the fact that the resource revenues remove the need to collect taxes, thus undermining the
accountability of the government. Moreover, important resource revenues encourage rent-seeking behavior. A final stylized fact about these countries is the high income inequality in the population. The fact that "point source" natural resources like oil are more easily controlled by an elite and do not need widespread labor explains why the resource revenues are usually not shared with the population but are concentrated in the hands of a small elite. For a comprehensive description of the resource curse phenomenon in developing countries, see Collier (2007). The question we are interested in is how donor countries can help the population which does not benefit from the resource revenues. A direct solution that comes to one's mind is foreign aid.

The different aspects of foreign aid have been very extensively studied in the development economics literature during the last decades. In particular, the effects of aid on growth, on the policy reforms and on the institutions of the recipient countries, the reasons why foreign aid is given (political and economic), the behavior of donor institutions, the fact that foreign aid is fungible and a large part of it does not reach its intended purpose, are some of the issues that have been hotly debated these last couple of years. For a good survey of the aid literature, see Kanbur (2006). In the present paper, the only objective of foreign aid is to increase the consumption of the poor living in the resource-rich countries. But, additionally to standard foreign aid, we introduce an original redistribution instrument: a contract with the elite that controls the resource rents. By signing this contract, the North commits to lower the tax on the resource imports, thus leaving a larger part of the resource rent to the southern elite, but in exchange the latter commits to redistribute some of this new rent to the poor. We will show that this additional instrument not only increases the welfare of the North, but also that of the southern rich and that of the southern poor.

Finally, the international economics literature has studied the welfare effects of international transfers, a pioneer paper in this field being that of Samuelson (1954). The main focus of this literature has been the effects of a transfer on the terms of trade and its net effects on the welfare of the donor, of the recipient and eventually of a third party not participating in the transaction. In general this literature assumes no inter-country altruism. For example, Lahiri et al (2002) shows that aid can improve the welfare of both the donor and the recipient because it can lead to a reduction in the recipient's optimal trade taxes and thus to an increase in global efficiency. In our paper, the efficiency and welfare improving effects of aid and those of the contract with the South are also related to the indirect effects of these redistribution instruments on the choice of the tariff. But contrarily to this literature, there are no terms of trade effects in our paper as we suppose a
single final good and a single mobile input. The only tariff we consider is the
tax on the imports of the resource by the donor (the North), and it is the
decrease in this tax that improves global efficiency and welfare when foreign
aid and the contract are simultaneously used.

In the two-country, three-group context described above, we use a decen-
tralized general equilibrium model of resource depletion, in order to examine
how the North will choose the resource tax under different redistribution
options. The equilibrium consumption levels of all groups are realized de-
pending on the prior choice of the instruments by the North government.

First, we consider a benchmark case without any redistribution mechanisms.
The bias in the behavior of the North consists in taxing the resource at a
rate greater than optimal. When the North can give foreign aid, it taxes the
resource at a lower rate, still higher than the optimal one. When the North
can use both foreign aid and the contract, it no longer taxes the resource,
thus correcting the global distortion due to strategic taxation policy design
and endowments heterogeneity.

The article is organized as follows. Section 2 presents the model, the
institutions and the redistribution mechanisms. In section 3, we compute the
general equilibrium outcome for a given taxation policy and given amounts
of foreign aid and internal aid. Section 4 examines the choice of the northern
government under three redistribution alternatives. The main results are
presented in this section. Section 5 concludes.

2 The Model

2.1 Technology

At each date \( t \in [0, +\infty) \), the final output is produced in both countries using
labor and a flow of extracted resource. The aggregate production functions
are:

\[
Y_i = (A_i L_i)^{(1-\alpha)} R_i^{\alpha}, \quad i = N, S,
\]

where \( L_i \) is the quantity of labor employed in the production sector, \( R_i \) is
the quantity of natural resource used in country \( i \). The subscripts \( N \) and \( S \)
refer respectively to North and South.

\( A_i, i = N, S, \) is an index of labor productivity in each country. Technical
improvement is given exogenously. For simplicity, these two indexes are

\footnote{For simplicity, the time argument of each variable is dropped as long as this does not create ambiguity.}
supposed to be proportional, thus growing at the same rate\(^9\):

\[
A_N = A, \quad A_S = \phi A, \quad 1 \geq \phi > 0, \quad g_A = x > 0, \quad A(0) = A_0 \text{ given}, \quad (2.2)
\]

where \(\phi\) can be interpreted as a degree of southern development.

The total quantities of labor in the North and the South respectively, \(L_N\) and \(L_S\), are locally fixed and constant over time.

The resource is freely extracted from a finite initial stock and can be used in both countries:

\[
\dot{Q} = -R = -(R_N + R_S), \quad Q(0) = Q_0 > 0, \quad \text{given.} \quad (2.3)
\]

### 2.2 Agents, Preferences and Endowments

In the South, there are two homogeneous groups: a group of poor infinitely-lived workers and a group of rich infinitely-lived resource holders.

Each southern worker is endowed with a unit of labor. The size of this group is \(L_S\). We refer to this group by \(SP\) to mean "southern poor"\(^{10}\). Their preferences are represented by the utility function:

\[
U_{SP} = \int_0^{+\infty} L_S \ln(C_{SP}/L_S) e^{-\rho t} dt, \quad (2.4)
\]

where \(C_{SP}\) is the total consumption of the southern poor people and \(\rho\) is the discount rate common to all groups.

The group of southern resource holders is endowed with the entire stock of the resource \(Q\). The size of this group is normalized to unity and their preferences are given by:

\[
U_{SR} = \int_0^{+\infty} \ln(C_{SR}) e^{-\rho t} dt, \quad (2.5)
\]

where \(C_{SR}\) is the total consumption of this group.

In the North, there is a homogeneous group of households. Each of them are workers endowed with one unit of labor. The size of this group is \(L_N\). They are altruistic in the sense that their utility depends positively on the

\(^9\)For any variable \(X\), its derivative with respect to time \(t\) is denoted by \(\dot{X}\) and its growth rate over time by \(g_X = \frac{\dot{X}}{X}\).

\(^{10}\)For notational convenience, we choose to denote the size of the southern poor population by \(L_S\) instead of \(L_{SP}\).
utility of the southern poor people as in Azam and Laffont (2003)\textsuperscript{11}:

\[ U_N = \int_0^{+\infty} \left( L_N \ln(C_N/L_N) + \delta L_S \ln(C_{SP}/L_S) \right) e^{-\rho t} \, dt, \quad (2.6) \]

where $C_N$ is the total consumption of this group and $\delta$ is the rate of altruism of northern households.

Finally, the binding world budget constraint is as follows:

\[ C_N + C_{SR} + C_{SP} = Y_N + Y_S. \quad (2.7) \]

### 2.3 Markets and other Institutions

There is a world competitive market for the extracted resource\textsuperscript{12}, $R$, and the final good, $Y$. The price of the latter is normalized to unity and the price of the resource is denoted by $p$. There is a world financial market on which the interest rate is denoted by $r$. There are two local competitive labor markets. The respective wages are denoted by $w_N$ and $w_S$.

The northern households (N) are represented by a northern government. The latter maximizes the utility of the representative northern household using several instruments. First, this government can apply a multiplicative tax rate $\tau > 0$ on the use of the natural resource, so that $p\tau$ is the consumer price paid by the firms in the North and $p$ is the producer price received by the resource producers in the South\textsuperscript{13}. The tax revenue is redistributed to the northern households. The tax rate chosen by the North is restricted to be constant over time\textsuperscript{14}. Second, the North government can transfer an

\footnotesize\textsuperscript{11} Actually, these authors assume that the southern rich put a positive weight on the consumption of the southern poor, but this weight is lower than that of the North. For simplicity we focus on the extreme case where this weight is zero (equation (2.4)) but the intuitions of our results would remain true as long as the altruism of the southern rich towards the poor is lower than that of the northern citizens.

\footnotesize\textsuperscript{12} The results are robust to the introduction of market-power in the extraction sector. Indeed, market-power is hardly exercised in the extraction of an explicitly non-renewable resource since the asymptotic cumulative quantity is set exogenously. On this, see Stiglitz (1976).

\footnotesize\textsuperscript{13} In fact, this amounts to setting an \textit{ad valorem} tax rate $\tau - 1 > -1$. Hence, a multiplicative tax rate $\tau > 1$ is equivalent to a strictly positive \textit{ad valorem} tax rate $\tau - 1 > 0$.

\footnotesize\textsuperscript{14} This assumption is made for simplicity. One could show, allowing the tax rate to be time-dependent, that the North government will always set it constant. The main reason why it is so is that the rate at which the resource is used in the North is optimal ($g_{RN} = -\rho$). In this context, a time varying tax rate will distort it. In the presence of an environmental distortion, due for instance to the polluting nature of the resource, the chosen tax rate may not be constant. See Daubanes and Grimaud (2006) for reference to such a framework. However, the introduction of pollution will complicate the analysis a good deal, while not affecting our main results.
amount $F(t)$ of final good, at each date $t$, from the northern citizens to the southern poor people. This instrument is standard foreign aid. Finally, the North government can contract with the South government on the tax level $\tau$. More precisely, it can propose a contract $(\tau, \{I(t)\}_{t \geq 0})$ to the South government, by which the North commits to set the tax level at $\tau$ and the South commits to transfer an amount $I(t)$ at each date $t$, which we call internal aid, from the rich resource holders to the poor people. We call the second and third instruments redistribution instruments, as their role is to redistribute some final good to the poor. As with the tax rate, we restrict the contracted internal aid and the amount of foreign aid proposed by the North to be constant fractions of the gross revenues of the northern citizens and the southern rich at each date$^{15}$. This implies that $F$ and $I$ are growing at the same rate as $Y_N$ and $Y_S$.

The South government represents the group of the rich resource holders (SR), so its objective is to maximize the utility of the representative resource holder. Thus, in the absence of a contract with the North, the South government will not make any redistribution to the poor. We suppose for simplicity that this government cannot tax the poor group (no redistribution in favor of the rich people), but this assumption is not crucial to our results. In principle, the South government could also impose a tax on the export of the resource, but it can easily be shown that it would have no interest in doing that$^{16}$. So the only decisions taken by the South government are to accept or to refuse foreign aid and to accept or to refuse the contract proposed by the North.

To sum up, the game goes as follows. Before date 0, the North government proposes a contract and an amount of foreign aid to the South government. The latter accepts or refuses. If the contract is accepted, the North sets the tax level specified in the contract from date 0 on. If the contract is refused, the North freely chooses the tax rate. Next, the general decentralized equilibrium is realized at each period $t \geq 0$. Since there are no information asymmetries and preferences are time-consistent, the contract proposed at date 0 is renegotiation-proof.

$^{15}$This assumption is not restrictive. Allowing $F$ and $I$ to grow at any rate, one can show that they will be chosen to be constant fractions of the northern and southern revenues. However, this restriction simplifies the computations and the analysis a good deal.

$^{16}$The reason for that is rather intuitive. When taxing the resource consumed in the South, the resource producers earn through tax revenues what they would have earned anyway from selling the resource at a higher price. Moreover, a tax on the resource would have altered the competitiveness in the South and entailed that a larger part of the resource would have been used in the North, where it is effectively taxed. See Daubanes and Grimaud (2007) for a theoretical analysis and refer to the introduction for empirical support.
We will solve this game by backward induction. First, we will determine the equilibrium consumption functions of each representative household, depending on the tax rate and on the transfer levels (section 3). Second, we will determine the optimal choice of the instruments by the North government in different scenarii (section 4).

3 Competitive General Equilibrium

In this section, we solve for the decentralized general equilibrium for a given tax rate and given amounts of foreign aid and internal aid.

3.1 The Agents Behavior

The Northern and Southern Final Sectors

The final sectors maximize their spot profits:

\[ \max_{L_i, R_i} (A_i L_i)^{1-\alpha} R_i^\alpha - w_i L_i - p_{\tau_i} R_i, \quad i = N, S, \]  

where \( \tau_N = \tau \) and \( \tau_S = 1 \). The first-order conditions of these profit-maximization programs are:

\[ (1 - \alpha) \frac{Y_i}{L_i} = w_i, \]  

\[ \alpha \frac{Y_i}{R_i} = p_{\tau_i}, \]  

where \( i = N, S \).

\textbf{Remark 1} Note from the equations above that a condition for world efficiency is \( \tau = 1 \). Indeed, the equalization of the marginal resource productivities (left-hand side of equations (3.3)) is a necessary condition for the maximization of \( Y = Y_N + Y_S \) and implies \( \tau_N = \tau_S \), i.e. \( \tau = 1 \).

The Extraction Sector

The extraction sector maximizes its discounted profits under its exhaustibility constraint:

\[ \max_{R(t), t \in [0, +\infty]} \int_t^{+\infty} pRe^{-\int_t^s r(u) du} ds, \]  

subject to (2.3). This behavior leads to the standard Hotelling rule:

\[ \frac{\hat{p}}{p} = r. \]
Households

The instantaneous budget constraints of the representative northern household, the representative southern rich people and the representative southern poor people are respectively:

\[ C_N + \dot{B}_N \leq w_N L_N + rB_N + H_N, \]  \hspace{1cm} (3.6)
\[ C_{SR} + \dot{B}_{SR} \leq pR + rB_{SR} + H_{SR}, \]  \hspace{1cm} (3.7)
\[ C_{SP} + \dot{B}_{SP} \leq w_S L_S + rB_{SP} + H_{SP}, \]  \hspace{1cm} (3.8)

where \( B_i \) is the group \( i \)'s net stock of financial assets and \( H_i, \ i = N, SR, SP, \) are lump sum transfers to the agents of each group: the northern households receive the tax revenues and pay for the foreign aid, the southern poor people receive the aid from the North and the transfer from the southern government and the resource holders receive the resource rent and pay for the transfers to the southern poor people:

\[ H_N = p(\tau - 1)R_N - F, \]  \hspace{1cm} (3.10)
\[ H_{SR} = -I, \]  \hspace{1cm} (3.11)
\[ H_{SP} = F + I. \]  \hspace{1cm} (3.12)

The respective program of each representative household is:

\[ \max_{\{C_i\}_{i \geq 0}} U_i, \ i = N, SR, SP, \]  \hspace{1cm} (3.13)

subject to their respective budget constraint (3.6)-(3.8) and the no-Ponzi game conditions:

\[ \lim_{t \to +\infty} B_i(t)e^{-\int_0^t r(s)ds} = 0, \ i = N, SR, SP. \]  \hspace{1cm} (3.14)

The first-order conditions of these programs lead to the standard Ramsey-Keynes conditions:

\[ g_{C_N} = g_{C_{SP}} = g_{C_{SR}} = g_C = r - \rho. \]  \hspace{1cm} (3.15)

3.2 Decentralized Equilibrium Outcome

Since we have not considered the mine to be a financial asset, the equilibrium of the financial market at date 0 writes \( B_N(0) + B_{SR}(0) + B_{SP}(0) = 0 \). The
initial debt of one group with respect to the others is arbitrary. For simplicity, let us assume that no group is indebted initially:

\[ B_N(0) = B_{SR}(0) = B_{SP}(0) = 0. \] (3.16)

In the context of this model, the following proposition presents the decentralized equilibrium solution.

**Proposition 1** For a given tax rate \( \tau \) and initial amounts of foreign aid \( F(0) \) and national aid \( I(0) \), the equilibrium path is characterized by the following initial values and rates of growth:

\[
\begin{align*}
Y_N &= L_N \frac{1}{\phi L_S} \tau^{-\alpha/(1-\alpha)}, \\
g_{R_N} &= g_{R_S} = g_R = -\rho, \\
g_{C_N} &= g_{CSR} = g_{CSP} = g_C = g_F = g_I = g_Y = g_{YN} = g_{YS} \\
&= g = (1 - \alpha) x - \alpha \rho, \\
Y_N(0) &= \left( A_0 L_N \right)^{1-\alpha} \left( \frac{\rho Q_0}{1 + \frac{L_N}{\phi L_S} \tau^{1/(1-\alpha)}} \right)^\alpha, \\
Y_S(0) &= \left( \phi A_0 L_S \right)^{1-\alpha} \left( \frac{\rho Q_0}{1 + \frac{L_N}{\phi L_S} \tau^{-1/(1-\alpha)}} \right)^\alpha, \\
C_N(0) &= (1 - \frac{\alpha}{\tau}) Y_N(0) - F(0), \\
C_{SR}(0) &= \frac{\alpha}{\tau} Y_N(0) + \alpha Y_S(0) - I(0), \\
C_{SP}(0) &= (1 - \alpha) Y_S(0) + F(0) + I(0).
\end{align*}
\]

**Proof of proposition 1** See the appendix.

Our study requires to understand the channels through which the northern taxation policy affects the consumption level of each group. Proposition 1 sheds some light on them. First, one can see that the tax rate determines the geographic split of the production. Namely, equation (3.17) tells us that the higher the northern tax rate, the lower the northern production relatively to the southern one. The existence of a world market for the resource implies that the seller price, \( p \), is the same in all countries. Because of the local tax on the resource, however, the price faced by the firms may differ across countries. This way, the level of the tax affects the respective outputs in the North and the South. Eventually, equations (3.22), (3.23) and (3.24) tell us that this channel partly determines the consumption levels of all groups since they depend on the local productions.
Second, from the later equations, one can see that the tax rate enters the northern and the southern rich consumption functions directly: the local outputs being taken as given, the northern tax level influences the respective consumption levels. Actually, the northern tax rate affects the profits from extracting the resource and consequently the northern tax revenues. Those amounts being shared among the local residents, the northern tax rate determines in this way the two consumptions of northern and southern rich households.

One can note that the instruments \((\tau, F, I)\), such as restricted above, have no effect on the dynamics of the economy. Technically, this simplifies a good deal the analysis since the problems of choosing the utility-maximizing instruments can be reduced to static optimization problems.

In the next section, we shall examine how the tax rate on the resource, the foreign aid and the internal aid will be designed by the North government. This study requires to know how these instruments affect the consumption level of each group \((N, SR, SP)\). That is why we define consumption levels as functions of these instruments \((\tau, F, I)\). As the choice of these instruments don’t affect the growth rate of the consumption levels, we can drop their time indexes and denote them by \(C_N(\tau, F)\), \(C_{SR}(\tau, I)\) and \(C_{SP}(\tau, F, I)\)\(^{17}\). Moreover, the utility function of each group can be denoted respectively by \(U_N(\tau, \{F(t)\}_{t \geq 0}, \{I(t)\}_{t \geq 0})\), \(U_{SR}(\tau, \{I(t)\}_{t \geq 0})\) and \(U_{SP}(\tau, \{F(t)\}_{t \geq 0}, \{I(t)\}_{t \geq 0})\)\(^{18}\).

Corollary 1 assesses the net effect of the tax rate on the consumption levels:

**Corollary 1** The northern consumption level is first increasing and then decreasing in the tax rate; the consumption levels of the southern poor and the southern rich groups are respectively increasing and decreasing in the tax rate. Formally,

\[
\frac{\partial C_N(\tau, F)}{\partial \tau} \geq 0 \iff \tau^{1/(1-\alpha)}(\tau - 1) \leq (1 - \alpha) \frac{L_N}{\Phi L_S}, \tag{3.25}
\]

\[
\frac{\partial C_{SP}(\tau, F, I)}{\partial \tau} > 0, \tag{3.26}
\]

\[
\frac{\partial C_{SR}(\tau, I)}{\partial \tau} < 0. \tag{3.27}
\]

**Proof of corollary 1** See the appendix.

\(^{17}\)Note that, from proposition 1, \(C_N\) does not depend on \(I\) and \(C_{SR}\) does not depend on \(F\).

\(^{18}\)As \(C_{SP}\) is function of the three instruments, so are \(U_{SP}\) and \(U_N\), because of altruism. As \(C_{SR}\) does not depend on \(F\), neither does \(U_{SR}\).
An increase in the northern tax rate on the resource has two effects. First, it implies that the North captures through tax revenues a larger part of the resource rent supposed to accrue to the resource producers. This tends to decrease the consumption level of the resource holders and to increase that of the northern citizens. Second, it tends to increase the price of the resource used in the North relatively to the price of the resource used in the South. This contributes to a loss of competitiveness of the northern firms and entails a relocation of the production activities to the South. This tends to decrease the consumption levels of the northern households. Accordingly, the productivity, the wage, and thus the consumption level of the southern poor people tends to increase.

From the northern viewpoint, increasing the tax rate has a positive (rent capture) as well as a negative (loss of competitiveness and relocation) effect. If the tax rate is low (high), the former more (less) than compensates the latter. From the viewpoint of the resource rich people, a higher tax is always worse.

The southern poor people benefit from an increase in the tax rate essentially because part of the final production activities of the North consecutively relocates to the South. Although this effect exists theoretically, it may be irrelevant in the real world. Indeed, in our two-country model northern firms could only relocate to the South, whereas in a setting with more than two countries, firms could relocate elsewhere. Then, the positive effect of an increase of the tax rate on the revenue of the poor people is likely to be marginal.

4 North’s Choice of the Tax under Different Redistribution Mechanisms

In this section, we describe the choice of the tax rate by the North and the determination of the amounts of foreign and internal aid, under different possibilities of redistribution.

The North government chooses the optimal instruments (tax rate on the resource, foreign aid and the contract proposed to the South) taking into account the future realization of the decentralized equilibrium. In order to highlight the role of each instrument, we consider three cases. In the benchmark case, no redistribution instrument is available. The only choice variable of the North government is the tax rate $\tau$. In the second case, this

---

19By relocation, we mean that the resource consumption lowers in one country and increases accordingly in the other, whereas labor is not migrating.
government may use foreign aid, \( \{F(t)\}_{t \geq 0} \), as a redistribution mechanism. It chooses simultaneously the optimal value of the tax rate and the optimal level of foreign aid. Finally, in the third case, the possibility of contracting with the South on \( (\tau, \{I(t)\}_{t \geq 0}) \) is introduced. The North simultaneously chooses the optimal level of foreign aid and the optimal contract (i.e. the utility maximizing tax level and amount of internal aid under the constraint that the contract has to be accepted by the South government).

Accordingly, the chosen values of the instruments will be denoted with superscripts \( NR \) (no redistribution) in the first case, \( A \) (aid) in the second case and \( A,C \) (aid and contract) in the third case.

### 4.1 Benchmark: Rent Capture without Redistribution

Suppose first that the North cannot use either foreign aid or the contract. Then, it solves:

\[
\max_{\tau} U_N(\tau, \{F(t) = 0\}_{t \geq 0}, \{I(t) = 0\}_{t \geq 0}) \quad (4.1)
\]

\[
= \int_0^{+\infty} \left\{L_N \ln \left( \frac{C_N(\tau,0)}{L_N} \right) + \delta L_S \ln \left( \frac{C_{SP}(\tau,0,0)}{L_S} \right) \right\} e^{-\rho t} dt,
\]

subject to (3.19)-(3.24).

According to proposition 1, the choice of the tax rate won’t affect the dynamics of the economy. As a result, the above problem technically reduces to a static maximization problem. The following proposition describes the solution chosen by the North government.

**Proposition 2** In the absence of any redistribution instrument, the North government chooses a strictly positive ad valorem tax rate: \( \tau^{NR} > 1 \).

**Proof of proposition 2** See the appendix.

The North government chooses strategically the tax rate on the resource used in its country. Doing so, it trades-off between levying tax revenues and improving the competitiveness of the northern economy. Proposition 1 tells us that it is optimal for the North to set a positive tax rate. As a consequence, a part of the rent supposed to accrue to resource holders is captured by the North. The strategic choice of the local tax rate in this heterogeneous world introduces a distorting bias at the global level. Indeed, as noted in remark 1, the only tax rate ensuring world efficiency is \( \tau = 1 \).
Remark 2 The greater the northern altruism towards the southern poor people, the higher the tax rate: \( \frac{\partial \tau}{\partial \delta} > 0 \).

One of the effects of an increase in \( \tau \) is the relocation of productive activities from North to South. In our two-country model, the southern poor workers benefit from this relocation as it increases their productivities and thus their wages and consumption levels. Since the North cares about the poor southern workers, it tends to set a tax rate all the higher as it is altruistic. As noted in the comment of corollary 1, this underlying effect is likely to be marginal in a more-than-two-country world.

4.2 Current Situation: Rent Capture and Foreign Aid

When the North is allowed to transfer revenues to the southern poor people through foreign aid, the North agent solves:

\[
\max_{\tau, \{F(t)\}_{t \geq 0}} U_N(\tau, \{F(t)\}_{t \geq 0}, \{I(t) = I(0)\}_{t \geq 0})
\]

\[
= \int_0^{+\infty} \left\{ L_N \ln \left( \frac{C_N(\tau, F)}{L_N} \right) + \delta L_S \ln \left( \frac{C_{SP}(\tau, F, 0)}{L_S} \right) \right\} e^{-\rho t} dt,
\]

subject to (3.19)-(3.24) and \( F(t) \geq 0, \forall t \geq 0 \).

Note that we do not impose the constraint that the South government accepts foreign aid. In fact, we shall see in proposition 4 that foreign aid indirectly benefits the resource holders, so that the South government has no interest in refusing it.

As in the previous subsection, the problem technically reduces to a static problem.

In what follows, we are mainly going to focus on the relevant cases where the North is effectively willing to give some aid. We shall see that a necessary and sufficient condition for the North to give a strictly positive amount of aid, when the tax is set to a given rate \( \tau \) is:

\[
\delta > \frac{C_{SP}(\tau, 0, 0)/L_S}{C_N(\tau, 0)/L_N} \equiv \delta^A(\tau). \tag{4.3}
\]

An immediate interpretation of this condition is that the North is sufficiently altruistic so that it is willing effectively to give aid to the poor it cares about.

Condition (4.3) also writes \( \delta > \frac{1}{C_{SP}(\tau, 0, 0)/L_S} > \frac{1}{C_N(\tau, 0)/L_N} \). When the tax rate is set to a level \( \tau \), in the absence of any redistribution, the consumption of the poor is equal to \( C_{SP}(\tau, 0, 0) \) and that of the northern citizens to \( C_N(\tau, 0) \).

\[\text{\textsuperscript{20}}\text{See proof of proposition 2.}\]
From the utility function of the northern people, one can see that the left-hand side of the latter equation is the instantaneous marginal benefit of giving foreign aid while the right-hand side is the associated instantaneous marginal cost. Condition (4.3) thus tells that, for a given tax rate, when no aid is given, the North has a marginal net interest in transferring a positive amount to the southern poor group.

Another more appealing interpretation of our restriction can be done from the equivalent condition: \( \delta C_N(\tau, 0)/L_N > C_{SP}(\tau, 0, 0)/L_S \). This writing tells that (4.3) amounts to a restriction to cases where the level of \( \text{per capita} \) consumption of the poor does not reach the \( \text{endogenous poverty line} \) given by \( \delta C_N(\tau, 0)/L_N \).

The following proposition describes the chosen tax rate and level of foreign aid.

**Proposition 3** If standard foreign aid is possible and if the northern citizens are sufficiently altruistic, the North government chooses a strictly positive ad valorem tax rate and a strictly positive amount of aid. However, the tax rate is then strictly lower than in the absence of any redistributive instrument. Formally, if \( \delta > \delta^A(\tau^{NR}) \), then \( 1 < \tau^A < \tau^{NR} \) and \( F^A > 0 \), otherwise, \( \tau^A = \tau^{NR} \) and \( F^A = 0 \).

**Proof of proposition 3** See the appendix.

We saw in the previous section that beyond the willingness to capture part of the resource rent, the North has also an incentive to tax high in order to alter its own competitiveness in favor of the southern workers’ productivity. This is a sort of sacrifice.

Proposition 3 tells us that the North sets a lower tax rate when it uses foreign aid. Indeed, foreign aid allows the North to increase the consumption level of the poor without enduring any cost in terms of efficiency, because it is a pure lump sum transfer.

Nevertheless, the North keeps setting a strictly positive tax rate on the resource. Although the introduction of foreign aid mitigates the incentives to choose a high tax rate, it does not completely offset the North’s propensity to capture a part of the mining rent through taxation, at the expense of economic efficiency. This is because the North government does not internalize all the effects of the tax on the global output. In particular, it does not take into account its effects on the consumption of the resource holders. As a result, the original bias in the northern choice of a tax rate, due to strategic behavior and resource endowments heterogeneity, remains present.

The use of foreign aid results in a lower tax rate and a positive transfer to the southern poor. Thus, additionally to its positive effect on global
efficiency, it changes the split of the world output among the different groups. The following proposition assesses the welfare impacts of the aid instrument.

**Proposition 4** The equilibrium allocation, when foreign aid is used by the strategic North government, Pareto-dominates the allocation when foreign aid is not used. Formally, if $\delta > \delta^A(\tau^{NR})$, then

$$
\begin{cases}
U_N(\tau^A, \{F^A(t)\}_{t \geq 0}, \{I(t) = 0\}_{t \geq 0}) > U_N(\tau^{NR}, \{F(t) = 0\}_{t \geq 0}, \{I(t) = 0\}_{t \geq 0}), \\
U_{SR}(\tau^A, \{I(t) = 0\}_{t \geq 0}) > U_{SR}(\tau^{NR}, \{I(t) = 0\}_{t \geq 0}), \\
U_{SP}(\tau^A, \{F^A(t)\}_{t \geq 0}, \{I(t) = 0\}_{t \geq 0}) > U_{SP}(\tau^{NR}, \{F(t) = 0\}_{t \geq 0}, \{I(t) = 0\}_{t \geq 0}).
\end{cases}
$$

**Proof of proposition 4** See the appendix.

The welfare of the North is obviously improved when an additional choice instrument is available. The southern resource holders are better-off since a lower part of their resource rent is captured by the North (lower tax rate). The effect of foreign aid on the southern poor is twofold. On the one hand, they lose competitiveness relatively to the North (lower tax rate). On the other hand, they are given a pure transfer from the North (strictly positive foreign aid). Proposition 4 tells that the latter effect more than compensates the former. Thus, the use of foreign aid by the North government makes everybody better-off.

This subsection illustrates the current interaction between developed resource consuming countries and poor resource producing countries: the former simultaneously extract some mining rent through distortional resource taxation and transfer aid to the latter.

Next subsection examines the effects of introducing the contract with the resource holders as a complementary redistribution scheme.

### 4.3 A Proposal: Contracting on the Tax Level and the Split of the Rent

Suppose now that, additionally to the aid instrument, the North proposes a contract $(\tau, \{I(t)\}_{t \geq 0})$ to the South government. By this contract, the North commits to set the tax rate equal to $\tau$ in all periods, and the southern rich commit to make a transfer $I(t)$ in each period $t \geq 0$ to the poor. The South government can refuse the contract, in which case we are back to the problem of the previous subsection, i.e. the North will implement the tax rate $\tau^A$ and transfer an amount $F^A(t)$ of foreign aid. As the South government represents the resource holders and no internal aid is given to the poor in the absence of the contract, a necessary condition for the contract to be accepted is $\tau < \tau^A$. 

17
So basically, the contract consists of a commitment by the North to lower the tax rate from \( \tau^A \) to \( \tau \) in exchange of a positive amount of internal aid.

Formally, the problem of the North government is the following:

\[
\max_{\tau, \{F(t)\}_{t \geq 0}, \{I(t)\}_{t \geq 0}} U_N(\tau, \{F(t)\}_{t \geq 0}, \{I(t)\}_{t \geq 0})
\]

\[
= \int_0^{+\infty} \{ L_N \ln \left( \frac{C_N(\tau, F)}{L_N} \right) + \delta L_S \ln \left( \frac{C_{SR}(\tau, F, I)}{L_S} \right) \} e^{-\rho t} dt,
\]

subject to (3.19)-(3.24), \( F(t) \geq 0, I(t) \geq 0, \forall t \geq 0 \), and \( U_{SR}(\tau, \{I(t)\}_{t \geq 0}) \geq U_{SR}(\tau^A, \{I(t) = 0\}_{t \geq 0}) \).

Here again, the problem reduces to a static problem. The parallel of condition (4.3) in this subsection is:

\[
\delta > \left( \frac{C_{SR}(\tau, 0, 0) + C_{SR}(\tau, 0) - C_{SR}(\tau^A, 0)}{C_N(\tau, 0)/L_N} \right) \equiv \delta^{A,C}(\tau),
\]

where \( C_{SR}(\tau, 0) - C_{SR}(\tau^A, 0) \) is the additional rent that accrues to the resource holders when the tax decreases from \( \tau^A \) to \( \tau \). The numerator of \( \delta^{A,C}(\tau) \) is thus the per capita consumption level of the southern poor when all this extra rent is redistributed to them and no foreign aid is given. Then, similarly to the previous subsection, we will see that \( \delta^{A,C}(\tau) \) is the minimum rate of altruism such that the North is willing to give a positive amount of foreign aid when the contract with the South stipulates that the tax rate on the resource will decrease from \( \tau^A \) to \( \tau \) and that all the resulting extra rent of the resource holders will be redistributed to the poor. Other interpretations made in subsection 4.2 remain valid here.

The following proposition assesses the choice of the tax rate, of the amount of foreign aid and that of internal aid by the North government.

**Proposition 5** If, beyond standard foreign aid, the North can use the contract defined above, and if it is sufficiently altruistic, then the North government no longer taxes the resource, foreign aid is strictly positive and internal aid is strictly positive. Formally, if \( \delta > \delta^{A,C}(1) \), then \( \tau^{A,C} = 1 \), \( F^{A,C} > 0 \) and \( I^{A,C} > 0 \).

**Proof of proposition 5** See the appendix.

This result is quite interesting. It says that if the North is altruistic enough, in the presence of the two redistribution instruments, it chooses the tax rate that maximizes global output, which is \( \tau = 1 \), as we have seen in remark 1. This is quite puzzling, given that the North is a strategic
The intuition for this result is the following. The contract signed with the South allows to manipulate the split of the world production among the different groups. With this new instrument, the North’s objective amounts to maximizing global production minus a rent that it cannot credibly threaten to capture from the resource holders, which is their rent in the absence of the contract. Thus, the optimal tax level is the one that maximizes global output as well.

Another way of understanding this result is related to the qualitative difference between the possibilities that the North has to capture the rent of the resource holders. Without the possibility of contracting, the North tends to catch a part of the rents of the resource holders and to redistribute part of it to the southern poor. The two transfers are made through different channels: capturing the rents requires the setting of a distortional tax while foreign aid is a direct lump sum transfer with no consequence on the efficiency of the northern economy. The introduction of the possibility to contract with the South offers a way to avoid the distortional tax aimed at capturing the resource rent. Indeed it renders possible to lower the extent of the distortion while ensuring that the revenues not captured through taxation are transferred to the southern poor. Since, we are dealing with cases where the North is indeed willing to give money to the poor people, the opportunity offered by the contract is then taken. Contracting then allows to correct the bias in the northern strategic behavior and restores world efficiency.

The introduction of the contract leads to a change in the tax rate and a change in the amounts of foreign and internal aid. Thus, additionally to its efficiency effects, the contract alters the split of the world output. The following proposition assesses who wins and who looses from this new possibility.

**Proposition 6** The equilibrium allocation, when both aid and the contract are used by the strategic North government, Pareto-dominates the one when only foreign aid is used. Formally, if $\delta > \max\{\delta^A(\tau^{NR}), \delta^{AC}(1)\}$, then

\[
\begin{cases}
U_N(\tau^{AC}, \{F^{AC}(t)\}_{t \geq 0}, \{I^{AC}(t)\}_{t \geq 0}) > U_N(\tau^A, \{F^A(t)\}_{t \geq 0}, \{I(t) = 0\}_{t \geq 0}), \\
U_{SR}(\tau^{AC}, \{I^{AC}(t)\}_{t \geq 0}) = U_{SR}(\tau^A, \{I(t) = 0\}_{t \geq 0}), \\
U_{SP}(\tau^{AC}, \{F^{AC}(t)\}_{t \geq 0}, \{I^{AC}(t)\}_{t \geq 0}) > U_{SP}(\tau^A, \{F^A(t)\}_{t \geq 0}, \{I(t) = 0\}_{t \geq 0}).
\end{cases}
\]

**Proof of proposition 6** See the appendix.

The welfare of the North is obviously improved when it has the possibility to use an additional instrument. The southern resource holders have higher rents due to the lowering of the tax, but all the additional rents are
redistributed to the poor. Indeed, the participation constraint of the South is binding (see proof of proposition 5). Thus, the resource holders have the same payoff as without the contract. Finally, the introduction of the contract has three effects on the consumption of the poor. First, it decreases their direct revenue because of the decrease in the tax rate. Second, it changes the amount of foreign aid given by the North. Third, it introduces a positive amount of internal aid. Proposition 6 tells that the overall effect on the welfare of the poor is positive.

Propositions 5 and 6 thus show that the introduction of the contract as an additional redistributive instrument for the North government not only increases global efficiency but is Pareto-improving.

5 Conclusion and Final Remarks

In this paper we set up a North-South exogenous growth model in which a privileged group in the South owns the entire stock of an essential, non-renewable resource, while the rest of the southern population and the northern population are only endowed with labor. We assumed that the northern citizens have a higher labor productivity, that they are altruistic towards the southern workers and that they are represented by a North government. We also assumed that the South government only represents the interests of the resource holders.

We allowed the North government to use different instruments in order to maximize the welfare of the northern citizens. First, the available instruments are a tax on the imports of the resource and foreign aid to the poor southern workers. Second, we introduced a contract with the South government on the tax level and on the amount of internal rent transfer to be redistributed from the resource holders to the poor.

We saw that although global output is maximized when there is no tax on the resource, the North will generally set a positive tax in order to capture some of the rent supposed to be earned by the resource holders.

We have shown that the simultaneous use of foreign aid and the contract with the South Pareto-dominates the allocation when only foreign aid is used, and importantly, it leads to a maximization of the global output (the resource is no longer taxed by the North). This result is quite interesting. Although the North government does not directly internalize the effects of the tax on the revenue of the resource holders, the combination of the two redistributive instruments (foreign aid and the contract) provides him with incentives to minimize the extent its distortional actions.

Our results are thus quite optimistic about the scope of the contract with
the resource holders. To our knowledge, this type of contract does not exist in the real world and it hasn’t been studied in the economics literature. But our setting abstracted from some potential problems that could arise in the real world. First, we supposed that foreign and internal aid could be collected and redistributed at no cost. Allowing some cost of public funds would keep the basic intuitions true, but would restrain the cases where foreign aid and the contract are worth being used. Second, we supposed that foreign aid actually reaches the poor and is not diverted by the South government. Unfortunately, we know that a large part of foreign aid does not reach its destination. This is related to the problem of foreign aid fungibility. The same is true with internal aid. Our assumptions on the commitment power of the South are not too restrictive given our dynamic setting if the North observes the consumption of the poor. Indeed, if the South government deviates, the North will no longer wish to transfer foreign aid or to contract with that government, and the resource holders would be worse-off because the North would set a higher tax on the resource. Thus the possibility of a prolonged cooperation, which is in everybody’s interest given that it is Pareto-improving, would serve as a commitment device. This is less obvious if the North doesn’t observe the consumption of the poor. It would be interesting to check the robustness of our results to the introduction of information asymmetries.
A Appendix

Proof of proposition 1 • Let us begin with the proof of equation (3.17). The production functions (2.1) imply $Y_N/Y_S = (A_N L_N/(A_S L_S))^{1-\alpha}(R_N/R_S)^\alpha$, where, from (2.2), $A_N/A_S = \phi^{-1}$. Thus, $Y_N/Y_S = (L_N/(\phi L_S))^{1-\alpha}(R_N/R_S)^\alpha$. Besides, equations (3.3) imply $R_N/R_S = Y_N/(\tau Y_S)$. Substituting the latter equation in the former, rearranging and simplifying leads to equation (3.17).

• Let us show here equations (3.18) and (3.19). (3.17) implies $g_{YS} = g_Y$. Since, in (2.7), $g_F = g_t = g_{YN} = g_{YS}$, one deduces $g_Y = g_C$. Moreover, from (3.15), $g_Y = g_C = r - \rho$. In addition, by (3.3), $g_{YN} = g_{YS}$ implies $g_{R_N} = g_{R_S} = g_R = g_Y - g_p$, which, by (3.5), leads to $g_R = g_Y - r$.

Besides, from equations (2.1) and (2.2), $g_Y = (1 - \alpha)x + \alpha g_R$. From above, this gives $g_Y = (1 - \alpha)x + g_Y - \alpha r$. Rearranging, one obtains $g_Y = x - \alpha r/(1 - \alpha)$. Recalling from above $g_Y = r - \rho$ and using the two later equations, one gets $r = (1 - \alpha)(x + \rho)$. Substituting this expression of $r$ in $g_Y = x - \alpha r/(1 - \alpha)$ and using $g_{CS} = g_{SR} = g_{SP} = g_C = g_Y = g_{YN} = g_{YS}$ leads to (3.19). Using finally $g_{R_N} = g_{R_S} = g_R = g_Y - r$ leads to (3.18).

• Next, let us show equations (3.20) and (3.21). From (3.18), $g_R = -\rho$. Then, by (2.3), $Q_0 = \int_0^{+\infty} R(t) dt = \int_0^{+\infty} R(0)e^{-\rho t} dt = R(0)/\rho$. Hence, $R(0) = \rho Q_0$. In addition, equation (3.3) imply $R_N/R_S = Y_N/(CN Y_S)$. Using (3.17), this leads to $R_N(0)/R_S(0) = (L_N/(\phi L_S))^{1/(1-\alpha)}$. Finally, using the latter ratio and $R_N(0) + R_S(0) = R(0) = \rho Q_0$, one easily gets: $R_N(0) = \rho Q_0/(1 + (\phi L_S/L_N))^{1/(1-\alpha)}$ and $R_S(0) = \rho Q_0/(1 + (L_N/(\phi L_S))^{1/(1-\alpha)})$. Substituting respectively these expressions in equations (2.1), written at date 0, one obtains equations (3.20) and (3.21).

• Let us eventually turn to the proof of equations (3.22), (3.23) and (3.24). In budget constraints (3.6), (3.7) and (3.8), respectively with equations (3.10), (3.11) and (3.12), let us substitute as follows. From (3.2), $w_N L_N = (1 - \alpha)Y_N$ and $w_S L_S = (1 - \alpha)Y_S$. From (3.3), $p(\tau - 1)R_N = \alpha(\tau - 1)Y_N/\tau = \alpha Y_N(\tau - 1/\tau)$ and $pR = p(R_N + R_S) = \alpha Y_N/\tau + \alpha Y_S$.

Substituting this way and rearranging, one obtains the following expressions of the budget constraints. $B_N + C_N = Y_N - \alpha Y_N/\tau - F + r B_N$, $B_{SR} + C_{SR} = \alpha Y_N/\tau + \alpha Y_S - I + r B_{SR}$ and $B_{SP} + C_{SP} = (1 - \alpha)Y_S + F + I + r B_{SP}$. Note, from the above proof of equations (3.18) and (3.19), that $r$ is constant. Solving those three instantaneous budget constraints as first-order linear differential equations respectively is $B_N$, $B_{SR}$ and $B_{SP}$, one gets three intertemporal
budget constraints that hold at any date $T \geq 0$. One then obtains:

$$B_N(T)e^{-rT} + \int_0^T C_N(t)e^{-rt} dt$$

$$= (1 - \alpha/\tau) \int_0^T Y_N(t)e^{-rt} dt - \int_0^T F(t)e^{-rt} dt + B_N(0),$$

$$B_{SR}(T)e^{-rT} + \int_0^T C_{SR}(t)e^{-rt} dt$$

$$= (\alpha/\tau) \int_0^T Y_N(t)e^{-rt} dt + \alpha \int_0^T Y_S(t)e^{-rt} dt - \int_0^T I(t)e^{-rt} dt + B_{SR}(0),$$

$$B_{SP}(T)e^{-rT} + \int_0^T C_{SP}(t)e^{-rt} dt$$

$$= (1 - \alpha) \int_0^T Y_S(t)e^{-rt} dt + \int_0^T F(t)e^{-rt} dt + \int_0^T I(t)e^{-rt} dt + B_{SP}(0).$$

Since, from the transversality conditions (3.14), $\lim_{T \to +\infty} B_i(T)e^{-rT} = 0, i = N, SR, SP$, these equations imply:

$$\int_0^{+\infty} C_N(t)e^{-rt} dt$$

$$= (1 - \alpha/\tau) \int_0^{+\infty} Y_N(t)e^{-rt} dt - \int_0^{+\infty} F(t)e^{-rt} dt + B_N(0),$$

$$\int_0^{+\infty} C_{SR}(t)e^{-rt} dt$$

$$= (\alpha/\tau) \int_0^{+\infty} Y_N(t)e^{-rt} dt + \alpha \int_0^{+\infty} Y_S(t)e^{-rt} dt - \int_0^{+\infty} I(t)e^{-rt} dt + B_{SR}(0),$$

$$\int_0^{+\infty} C_{SP}(t)e^{-rt} dt$$

$$= (1 - \alpha) \int_0^{+\infty} Y_S(t)e^{-rt} dt + \int_0^{+\infty} F(t)e^{-rt} dt + \int_0^{+\infty} I(t)e^{-rt} dt + B_{SP}(0).$$

Let us recall that variables $C_N, C_{SR}, C_{SP}, Y_N, Y_S, F$ and $I$ increase at the constant rate $r - \rho$. For each variable $X$ growing at this rate, $\int_0^{+\infty} X(t)e^{-rt} dt = \int_0^{+\infty} X(0)e^{(r-\rho)t}e^{-rt} dt = X(0) \int_0^{+\infty} e^{-rt} dt = X(0)/\rho$. This way, the above asymptotically integrated budget constraints and assumption (3.16) lead to equations (3.22), (3.23) and (3.24).

**Proof of proposition 2** Let us first of all simplify problem (4.1). To do this, denote the consumption level of groups $N$ and $SP$ at date $t \geq 0$ respectively by $C_N(\tau, 0)(t)$ and $C_{SP}(\tau, 0, 0)(t)$. Consider any $T \geq 0$. Using equation (3.19), $L_N \ln(C_N(\tau, 0)(T)/L_N) + \delta L_S \ln(C_{SP}(\tau, 0, 0)(T)e^{(t-T)\rho}/L_S) =$
One can show that problem (4.2) reduces to the static problem of maximizing,

\[ \mathcal{L} = L_N \ln(C_N(\tau, 0)(T)/L_N) + \delta L_S \ln(C_{SP}(\tau, 0, 0)(T)e^{(t-T)g}/L_S) \]

for \( T \geq 0 \), where from (3.19), (3.22) and (3.24),

\[ C_N(\tau, 0)(t) = C_N(\tau, 0)(0)e^{\rho t} = (A_0L_N)^{-\alpha}(\rho Q_0)^{\alpha} (1 + (\Phi L_S/L_N)^{1/(1-\alpha)})^{1/(1-\alpha)} \]

and

\[ C_{SP}(\tau, 0, 0)(t) = C_{SP}(\tau, 0, 0)(0)e^{\rho t} = (1 - \alpha)(\Phi A_0L_S)^{-\alpha}(\rho Q_0)^{\alpha} (1 + (L_S/(L_N)^{1/(1-\alpha)})^{1/(1-\alpha)} \]

Since the problem can be solved at any date, let us drop from now on the time argument of the consumption functions.

- The first-order condition of this problem is: \( L_N(\partial C_N(\tau, 0)/\partial \tau)/C_N(\tau, 0) + \delta L_S(\partial C_{SP}(\tau, 0, 0)/\partial \tau)/C_{SP}(\tau, 0, 0) = 0 \). Using the above expressions of the consumption functions and simplifying, one gets the following equivalent condition: \( \tau^{-2}(1 + (\Phi L_S/L_N)^{1/(1-\alpha)}) - (\Phi L_S/L_N)(1/(1 - \alpha)/(1 - \alpha/\tau) + (\delta L_S/L_N)(1/(1 - \alpha))(1 - \alpha/\tau) = 0 \). This gives eventually:

\[
Z^{NR}(\tau) = \frac{\tau^{-1/(1-\alpha)}}{(1 - \alpha)} + \frac{\Phi L_S}{L_N} \frac{1}{(1 - \alpha)} - \frac{\Phi L_S}{L_N} \frac{1}{(1 - \alpha)} + \frac{\delta L_S}{L_N} \frac{1}{(1 - \alpha)} = 0.
\]

(A.1)

In this condition, one could show that \( Z^{NR}(\tau) \) is decreasing in \( \tau \),

\[ \lim_{\tau \to +\infty} Z^{NR}(\tau) = -(\Phi L_S/L_N)/(1 - \alpha) < 0 \] and \( Z^{NR}(1) = (1 + \delta L_S/L_N)/(1 - \alpha) \). This implies that there exists a unique \( \tau = \tau^{NR} \) satisfying the above condition and that \( \tau^{NR} > 1 \).

- Moreover, for a given \( \tau \), \( Z^{NR}(\tau) \) is increasing in \( \delta \). It implies that \( \tau^{NR} \) is increasing in \( \delta \).

**Proof of proposition 3** • The same way as in the proof of proposition 2, one can show that problem (4.2) reduces to the static problem of maximizing, for any \( T \geq 0 \),

\[ L_N \ln(C_N(\tau, 0)(T)/L_N) + \delta L_S \ln(C_{SP}(\tau, 0, 0)(T)e^{(t-T)g}/L_S) + L_N(t-T)g + \delta L_S(t-T)g \]

Then, the objective of problem (4.1) rewrites:

\[ U_N(\tau, \{F(t) = 0\}_{t \geq 0}, \{I(t) = 0\}_{t \geq 0}) = L_N \ln(C_N(\tau, 0)(T)/L_N) \int_0^T e^{-\rho t} dt + \delta L_S \ln(C_{SP}(\tau, 0, 0)(T)/L_S) \int_0^T e^{-\rho t} dt - \delta L_ST \int_0^T e^{-\rho t} dt + L_N \int_0^T e^{-\rho t} dt + \delta L_S \int_0^T e^{-\rho t} dt. \]

So maximizing \( U_N(\tau, \{F(t) = 0\}_{t \geq 0}, \{I(t) = 0\}_{t \geq 0}) \) amounts to maximizing \( L_N \ln(C_N(\tau, 0)(t)/L_N) + \delta L_S \ln(C_{SP}(\tau, 0, 0)(t)e^{(t-T)g}/L_S) \), at any date \( T \geq 0 \), where, from (3.19), (3.22) and (3.24),

\[ C_N(\tau, 0)(t) = C_N(\tau, 0)(0)e^{\rho t} = (A_0L_N)^{-\alpha}(\rho Q_0)^{\alpha} (1 + (\Phi L_S/L_N)^{1/(1-\alpha)})^{1/(1-\alpha)} \]

and

\[ C_{SP}(\tau, 0, 0)(t) = C_{SP}(\tau, 0, 0)(0)e^{\rho t} = (1 - \alpha)(\Phi A_0L_S)^{-\alpha}(\rho Q_0)^{\alpha} (1 + (L_S/(L_N)^{1/(1-\alpha)})^{1/(1-\alpha)} \]

Let us then solve

\[ (A.1) \]

where from (3.19), (3.22) and (3.24),

\[ C_N(\tau, F) = C_N(\tau, 0)(0)e^{\rho t} = (A_0L_N)^{-\alpha}(\rho Q_0)^{\alpha} (1 + (\Phi L_S/L_N)^{1/(1-\alpha)})^{1/(1-\alpha)} \]

and

\[ C_{SP}(\tau, 0, 0)(F) = C_{SP}(\tau, 0, 0)(0)e^{\rho t} = (1 - \alpha)(\Phi A_0L_S)^{-\alpha}(\rho Q_0)^{\alpha} (1 + (L_S/(L_N)^{1/(1-\alpha)})^{1/(1-\alpha)} \]

Let us then solve

\[ (A.2) \]

where from (3.19), (3.22) and (3.24),

\[ C_N(\tau, F) = C_N(\tau, 0)(0)e^{\rho t} = (A_0L_N)^{-\alpha}(\rho Q_0)^{\alpha} (1 + (\Phi L_S/L_N)^{1/(1-\alpha)})^{1/(1-\alpha)} \]

and

\[ C_{SP}(\tau, 0, 0)(F) = C_{SP}(\tau, 0, 0)(0)e^{\rho t} = (1 - \alpha)(\Phi A_0L_S)^{-\alpha}(\rho Q_0)^{\alpha} (1 + (L_S/(L_N)^{1/(1-\alpha)})^{1/(1-\alpha)} \]

Let us then solve

\[ (A.2) \]
\[ C_{SP}(\tau, F, 0) = C_{SP}(\tau, F, 0)e^{gt} = [(1 - \alpha)(\Phi A_0 L_S)^{1-\alpha}(\rho Q_0)^\alpha(1 + (L_N/(\Phi L_S))^{\tau^{-1/(1-\alpha)}) - \alpha + F]e^{gt}. \]

The first-order conditions of the problem are:

\[
\begin{align*}
\partial \mathcal{L}/\partial \tau &= 0, \quad \text{(A.3)} \\
\partial \mathcal{L}/\partial F &= 0, \quad \text{(A.4)} \\
\pi F &= 0, \quad \text{(A.5)} \\
\pi &\geq 0, \quad \text{(A.6)} \\
F &\geq 0, \quad \text{(A.7)}
\end{align*}
\]

where \( \mathcal{L} \) is given by (A.2).

• Case i) \( \pi > 0 \)

It implies by (A.5) \( F = 0 \). Then, (A.3) writes \( L_N(\partial C_N(\tau, 0)/\partial \tau)/C_N(\tau, 0) + \delta L_S(\partial C_{SP}(\tau, 0, 0)/\partial \tau)/C_{SP}(\tau, 0, 0) = 0 \), which is the same condition that gives \( \tau^{NR} \) (see proof of proposition 2). Hence, \( \tau^A = \tau^{NR} \).

But, from (A.4), \( \pi > 0 \) is equivalent to \( \delta < (C_{SP}(\tau^{NR}, 0, 0)/L_S)/(C_N(\tau^{NR}, 0)/L_N) \).

• Case ii) \( \pi = 0 \)

From (A.4), it implies that:

\[ F = \frac{\delta L_SC_N(\tau, 0) - L_SC_{SP}(\tau, 0, 0)}{L_N + \delta L_S}. \quad \text{(A.8)} \]

Then, \( F \geq 0 \) is equivalent to the condition:

\[ \delta \geq \frac{C_{SP}(\tau, 0, 0)/L_S}{C_N(\tau, 0)/L_N}. \quad \text{(A.9)} \]

Replacing \( F \), as expressed in (A.8), in (A.3), we have that the solution \( \tau^A \) must solve:

\[ \frac{\partial C_N(\tau, 0)}{\partial \tau} + \frac{\partial C_{SP}(\tau, 0, 0)}{\partial \tau} = 0. \quad \text{(A.10)} \]

For this solution to be valid, we must have that \( \tau \) that solves (A.10) satisfies (A.9).

Let us first show that (A.10) has a unique solution. Replacing \( C_N(\tau, 0) \) and \( C_{SP}(\tau, 0, 0) \) by their above expressions and simplifying, (A.10) is equivalent to \( 1 + (\Phi L_S/L_N)^{\tau^{1/(1-\alpha)}} - (1 - \alpha/\tau)(\Phi L_S/L_N)^{\tau^{-1/(1-\alpha)}} + (\Phi L_S/L_N)^{\tau^{1/(1-\alpha)}} = 0 \), which is also equivalent to

\[ Z^A(\tau) \equiv \frac{L_N}{\Phi L_S} \tau^{-1/(1-\alpha)} + \frac{2 - \alpha}{1 - \alpha} - \frac{\tau}{1 - \alpha}. \quad \text{(A.11)} \]
From this condition, one can get that \( Z^A(\tau) \) is continuously decreasing in \( \tau \), \( \lim_{\tau \to +\infty} Z^A(\tau) = -\infty \) and \( Z^A(1) = L_N/\Phi L_S + 1 > 0 \). This implies that equation (A.10) has a unique solution, \( \tau^A > 1 \).

Now, let us show that case ii) requires \( \delta \geq \delta^A(\tau^{NR}) \), where \( \delta^A(\tau) \) is such as defined in (4.3). By the way, we are going to show that, in this case, \( \tau^A < \tau^{NR} \).

Suppose, on the contrary, that \( \delta < \delta^A(\tau^{NR}) \). \( \tau^A \) is defined by (A.10) such that \( (\partial C_N(\tau,0)/\partial \tau)/(\partial C_{SP}(\tau,0,0)/\partial \tau) = -1 \). From the proof of proposition 2, \( \tau^{NR} \) is characterized by

\[
(\partial C_N(\tau,0)/\partial \tau)/(\partial C_{SP}(\tau,0,0)/\partial \tau) = -\delta(L_L/L_N)(C_N(\tau^{NR})/C_{SP}(\tau^{NR},0,0)).
\]

Since \( \delta < \delta^A(\tau^{NR}) \), then,

\[
\frac{\partial C_N(\tau,0)/\partial \tau}{\partial C_{SP}(\tau,0,0)/\partial \tau} \bigg|_{\tau=\tau^A} < \frac{\partial C_N(\tau,0)/\partial \tau}{\partial C_{SP}(\tau,0,0)/\partial \tau} \bigg|_{\tau=\tau^{NR}},
\]

where, from the above expressions of the consumption functions, one can find \( (\partial C_N(\tau,0)/\partial \tau)/(\partial C_{SP}(\tau,0,0)/\partial \tau) = 1+\tau^{-1(1-\alpha)}L_N/(\Phi L_S)-T(\tau-\alpha)/(1-\alpha) \), which is a decreasing function of \( \tau \). Thus, (A.12) implies \( \tau^A > \tau^{NR} \). \( C_N(\tau,0) \) is decreasing at \( \tau = \tau^{NR} \) and \( \tau = \tau^A \). Indeed, \( (\partial C_N(\tau,0)/\partial \tau) \bigg|_{\tau=\tau^{NR}} = -(\delta L_L/L_N)(C_N(\tau^{NR},0)/C_{SP}(\tau^{NR},0,0)) \),

\[
\frac{\partial C_N(\tau,0)/\partial \tau}{\partial C_{SP}(\tau,0,0)/\partial \tau} \bigg|_{\tau=\tau^{NR}} < 0 \text{ and (A.9) implies that } \tau^A < \tau^{NR} \implies C_N(\tau^A,0,0) < C_N(\tau^{NR}).
\]

Considering that \( C_N(\tau,0,0) \) is increasing in \( \tau \), this also implies:

\[
(C_{SP}(\tau^A,0,0)/L_L) / (C_N(\tau^A,0,0)/L_N) > (C_{SP}(\tau^{NR},0,0)/L_L) / (C_N(\tau^{NR},0,0)/L_N) > \delta.
\]

But this in contradiction with (A.9). So we have shown that a necessary condition for the case ii) is \( \delta \geq \delta^A(\tau^{NR}) \).

Let us now show that it is also sufficient, and that it implies \( \tau^A < \tau^{NR} \). Suppose that \( \delta \geq \delta^A(\tau^{NR}) \). Using the same reasoning as above, this implies \( \tau^A < \tau^{NR} \), which implies \( (C_{SP}(\tau^A,0,0)/L_L) / (C_N(\tau^A,0,0)/L_N) < (C_{SP}(\tau^{NR},0,0)/L_L) / (C_N(\tau^{NR},0,0)/L_N) < \delta \) and that \( \tau^A \) satisfies (A.9).

To resume, if \( \delta \leq (C_{SP}(\tau^{NR},0,0)/L_L) / (C_N(\tau^{NR},0,0)/L_N) \), then case i) works, \( F = 0 \) and \( \tau^A = \tau^{NR} \), otherwise case ii) holds, \( F > 0 \) and \( 1 < \tau^A < \tau^{NR} \).

Proof of proposition 4 • The proposition

\[
U_N(\tau^A, \{F^A(t)\}_{t \geq 0}, \{I(t) = 0\}_{t \geq 0}, \{I(t) = 0\}_{t \geq 0}) > U_N(\tau^{NR}, \{F(t) = 0\}_{t \geq 0}, \{I(t) = 0\}_{t \geq 0})
\]

is immediate as \( \tau^A \) and \( \{F^A(t)\}_{t \geq 0} \) are the solutions to problem (4.2) and \( \tau^{NR} \) is the solution to the more constrained problem (4.1).

• From corollary 1, \( C_{SR}(\tau,0) \) is decreasing in \( \tau \). From proposition 3, \( \tau^A < \tau^{NR} \). Hence, at all periods, \( C_{SR}(\tau^A,0) > C_{SR}(\tau^{NR},0) \). Hence,

\[
U_{SR}(\tau^A, \{I(t) = 0\}_{t \geq 0}) > U_{SR}(\tau^{NR}, \{I(t) = 0\}_{t \geq 0}).
\]
First, note that, in order to show that \( \mu_{SP}(\sigma, \{F^A(t)\}_{t \geq 0}, \{I(t) = 0\}_{t \geq 0}) > \mu_{SP}(\tau^{NR}, \{F(t) = 0\}_{t \geq 0}, \{I(t) = 0\}_{t \geq 0}) \), it is sufficient to show that \( \mu_{SP}(\tau^{NR}, 0, 0) < \mu_{SR}(\sigma, F^A, 0) \) at all periods.

Under assumption \( \delta > \delta(\tau^{NR}) \), we know, from the above proof of proposition 3, \( F^A = [\delta L_S C_N(\tau^A, 0) - L_S C_{SP}(\tau^A, 0, 0)]/([L_N + \delta L_S]). \) Hence, \( C_{SP}(\tau^A, F^A, 0) = C_{SP}(\tau^A, 0, 0) + F^A = (\delta L_S/([L_N + \delta L_S]) C_{SP}(\tau^A, 0, 0) + C_N(\tau^A, 0)]. \) So we need show \( C_{SP}(\tau^{NR}, 0) < (\delta L_S/([L_N + \delta L_S]) C_{SP}(\tau^A, 0, 0) + C_N(\tau^A, 0)] \), which is equivalent to

\[
C_{SP}(\tau^{NR}, 0, 0) + (L_N/\delta L_S) C_{SP}(\tau^{NR}, 0, 0) < C_{SP}(\tau^A, 0, 0) + C_N(\tau^A, 0). \tag{A.13}
\]

Let us show now that a sufficient condition for the later one to hold is \((L_N/\delta L_S) C_{SP}(\tau^{NR}, 0, 0) < C_N(\tau^{NR}, 0). \) To do so, we are going to show that \( C_N(\tau^{NR}, 0) + C_{SP}(\tau^{NR}, 0, 0) < C_N(\tau^A, 0) + C_{SP}(\tau^A, 0, 0). \)

From proof of proposition 2, \( \tau^{NR} \) solves

\[
- \frac{\partial C_N(\tau, 0)}{\partial \tau} = \delta \frac{L_S}{L_N} C_N(\tau, 0) \frac{\partial C_{SP}(\tau, 0, 0)}{\partial \tau}. \tag{A.14}
\]

From proof of proposition 3, if \( F^A > 0, \tau^A \) solves

\[
- \frac{\partial C_N(\tau, 0)}{\partial \tau} = \frac{\partial C_{SP}(\tau, 0, 0)}{\partial \tau}. \tag{A.15}
\]

As we suppose in proposition 4 that \( \delta > \delta(\tau^{NR}) \), then

\[
\delta \frac{L_S}{L_N} C_N(\tau^{NR}, 0) > 1. \tag{A.16}
\]

(A.14) and (A.16) imply that \(- \partial C_N(\tau^{NR}, 0)/\partial \tau > \partial C_{SP}(\tau^{NR}, 0, 0)/\partial \tau, \) while (A.15) implies \(- \partial C_N(\tau^A, 0)/\partial \tau > \partial C_{SP}(\tau^A, 0, 0)/\partial \tau. \) Then, as \( \tau \) moves from \( \tau^{NR} \) to \( \tau^A < \tau^{NR}, \) the resulting increase in \( C_N \) is larger than the decrease in \( C_{SP}. \) Hence, \( C_N(\tau^{NR}, 0) + C_{SP}(\tau^{NR}, 0, 0) < C_N(\tau^A, 0) + C_{SP}(\tau^A, 0, 0). \)

Thus, in order to show (A.13), it is sufficient to show that \( C_N(\tau^{NR}, 0) + C_{SP}(\tau^{NR}, 0, 0) > C_{SP}(\tau^{NR}, 0, 0) + (L_N/\delta L_S) C_{SP}(\tau^{NR}, 0, 0), \) which is equivalent to \((L_N/\delta L_S) C_{SP}(\tau^{NR}, 0, 0) < C_N(\tau^{NR}, 0) \) and eventually to \( \delta > \delta(\tau^{NR}). \)

Proof of proposition 5 - The same way as in the proof of propositions 2 and 3, one can show that problem (4.4) reduces to the static problem of maximizing, for any \( T \geq 0, \ L_N \ln(C_N(\tau, F(T)) (T)/L_N) + \delta L_S \ln(C_{SP}(\tau, F(T), I(T)) (T)/L_S) \) with respect to \( \tau, \ F(T) \) and \( I(t), \) under
the same constraints at the exception of the individual rationality constraint that reduces to \( C_{SR}(\tau, I(T))(T) \leq C_{SR}(\tau^A, 0)(T) \). The solutions \( \{ F^{A,C}(t) \}_{t \geq 0} \) and \( \{ I^{A,C}(t) \}_{t \geq 0} \) are then obtained from \( F^{A,C}(t) = F^{A,C}(T)e^{(t-T)}g \) and \( I^{A,C}(t) = I^{A,C}(T)e^{(t-T)}g \). Let us then solve the static problem at any date and drop the time arguments.

- Note that the individual rationality constraint, \( C_{SR}(\tau, I(T))(T) \geq C_{SR}(\tau^A, 0)(T) \), will be binding at the solution since \( I \) enters positively the North’s objective function. Hence, \( C_{SR}(\tau, I) = C_{SR}(\tau^A, 0) \), which is equivalent to \( C_{SR}(\tau, 0) - I = C_{SR}(\tau^A, 0) \) and finally leads to \( I = C_{SR}(\tau, 0) - C_{SR}(\tau^A, 0) \). As, from corollary 1, \( \partial C_{SR}(\tau, 0)/\partial \tau < 0 \), then \( I \geq 0 \) and \( \tau \leq \tau^A \). The problem of the North government can thus be written as the maximization of \( L \ln(C_N(\tau, F)) + \delta L \ln(C_{SP}(\tau, F, 0) + C_{SR}(\tau, 0) - C_{SR}(\tau^A, 0)) \) subject to \( F \geq 0 \) and \( \tau \leq \tau^A \). Let us denote by \( \pi \) and \( \chi \) respectively the co-state variables associated to the former and the latter constraints.

Let us denote by \( \Sigma \) the Lagrangian of this problem. Then,

\[
\Sigma = L \ln(C_N(\tau, F)) + \delta L \ln(C_{SP}(\tau, F, 0) + C_{SR}(\tau, 0) - C_{SR}(\tau^A, 0)) + \pi F + \chi(\tau^A - \tau),
\]

(A.17)

where from (3.19)-(3.24), \( C_N(\tau, F) = C_N(\tau, F)(0)e^{\pi t} \)

\[
= [(A_0 L_N)^{1-\alpha}(\rho Q_0)^{\alpha}(1 + (\Phi L_S/L_N)(\tau^{1/(1-\alpha)})^{\alpha}(1-\alpha/\tau) - F]e^{\pi t},
\]

\( C_{SP}(\tau, F, 0) = C_{SP}(\tau, F, 0)e^{\pi t} \)

\[
= [(1 - \alpha)(\Phi A_0 L_S)^{1-\alpha}(\rho Q_0)^{\alpha}(1 + (L_N/(\Phi L_S))(\tau^{-1/(1-\alpha)})^{\alpha} + F]e^{\pi t}
\]

and \( C_{SR}(\tau, 0) = C_{SR}(\tau, 0)(0)e^{\pi t} \)

\[
= [(\alpha/\tau)(A_0 L_N)^{1-\alpha}(\rho Q_0)^{\alpha}(1 + (\Phi L_S/L_N)(\tau^{1/(1-\alpha)})^{-\alpha} + \alpha(\Phi A_0 L_S)^{1-\alpha}(\rho Q_0)^{\alpha}(1 + (L_N/(\Phi L_S))(\tau^{-1/(1-\alpha)})^{-\alpha} - I]e^{\pi t}.
\]

The first-order conditions of the problem are:

\[
\frac{\partial \Sigma}{\partial \tau} = 0,
\]

(A.18)

\[
\frac{\partial \Sigma}{\partial F} = 0,
\]

(A.19)

\[
\chi(\tau^A - \tau) = 0,
\]

(A.20)

\[
\pi F = 0,
\]

(A.21)

\[
\tau^A \geq \tau,
\]

(A.22)

\[
F \geq 0,
\]

(A.23)

\[
\pi \geq 0,
\]

(A.24)

\[
\chi \geq 0.
\]

(A.25)

where \( \Sigma \) is given by (A.17).

- Case i) \( \pi > 0, \chi > 0, \tau = \tau^A, F = 0 \)

Then, \( \partial \Sigma/\partial \tau = 0 \) is equivalent to:

\[
\frac{L_N}{C_N(\tau^A, 0)} \frac{\partial C_N(\tau^A, 0)}{\partial \tau} + \delta \frac{L_S}{C_{SP}(\tau^A, 0)} \left( \frac{\partial C_{SP}(\tau^A, 0, 0)}{\partial \tau} + \frac{\partial C_{SR}(\tau^A, 0)}{\partial \tau} \right) = \chi > 0.
\]

(A.26)
For what follows, note that it can be easily shown that \( \partial C_{SP}(\tau, 0, 0)/\partial \tau + \partial C_{SR}(\tau, 0)/\partial \tau, \forall \tau > 0 \), thus implying that \( \partial C_N(\tau^A, 0)/\partial \tau > 0 \).

Either \( \delta \leq \delta^A(\tau_{NR}) \) and then \( \tau^A = \tau_{NR} \). Hence, \( \tau^A \) is such that \((L_N/C_N(\tau^A, 0, 0))\partial C_N(\tau^A, 0, 0)/\partial \tau + \delta(L_S/C_{SP}(\tau^A, 0, 0))\partial C_{SP}(\tau^A, 0, 0)/\partial \tau = 0 \) (See proof of propositions 2 and 3). This is in contradiction with (A.26) because \( \partial C_{SR}(\tau^A, 0)/\partial \tau < 0 \).

Either, \( \delta > \delta^A(\tau_{NR}) \) and \( \tau^A < \tau_{NR} \). Then, \( \tau^A \) is such that \( \partial C_N(\tau^A, 0)/\partial \tau + \partial C_{SP}(\tau^A, 0, 0)/\partial \tau = 0 \), which is also in contradiction with (A.26) since \( \partial C_{SP}(\tau^A, 0, 0)/\partial \tau > 0 \) and it has been shown above that \( \partial C_N(\tau^A, 0)/\partial \tau > 0 \).

This case is not possible.

- **Case ii)** \( \pi = 0, \chi > 0, F > 0, \tau = \tau^A \)

Then, \( \partial \Sigma/\partial \tau = 0 \) is equivalent to

\[
\frac{\partial C_N(\tau, 0)}{\partial \tau} + \delta \frac{L_S}{L_N} \frac{C_N(\tau, 0)}{C_{SP}(\tau, 0, 0) + C_{SR}(\tau, 0) - C_{SR}(\tau^A, 0)} \left[ \frac{\partial C_{SP}(\tau, 0, 0)}{\partial \tau} + \frac{\partial C_{SR}(\tau, 0)}{\partial \tau} \right] = 0
\]  
(A.27)

and \( \partial \Sigma/\partial F = 0 \) implies \( \pi = L_N/C_N(\tau, 0) - \delta L_S/\left[ C_{SP}(\tau, 0, 0) + C_{SR}(\tau, 0) - C_{SR}(\tau^A, 0) \right] \). Hence, \( \pi > 0 \) is equivalent to \( \delta < \delta^{AC}(\tau) \), for \( \tau \) satisfying equation (A.27).

- **Case iii)** \( \pi = 0, \chi = 0, \tau < \tau^A, F = 0 \)

Then, \( \partial \Sigma/\partial \tau = 0 \) is equivalent to:

\[
\frac{\partial C_N(\tau, 0)}{\partial \tau} + \delta \frac{L_S}{L_N} \frac{C_N(\tau, F)}{C_{SP}(\tau, F, 0) + C_{SR}(\tau, 0) - C_{SR}(\tau^A, 0)} \left[ \frac{\partial C_{SP}(\tau, 0, 0)}{\partial \tau} + \frac{\partial C_{SR}(\tau, 0)}{\partial \tau} \right] = 0
\]  
(A.28)

and \( \partial \Sigma/\partial F = 0 \) is equivalent to:

\[
F = \frac{1}{L_N + \delta L_S} \left[ \delta L_SC_N(\tau, 0) - L_N \{ C_{SP}(\tau, 0, 0) + C_{SR}(\tau, 0) - C_{SR}(\tau^A, 0) \} \right].
\]  
(A.29)

Hence, \( F > 0 \) is equivalent to \( \delta > \delta^{AC}(\tau) \) for \( \tau \) satisfying (A.28).

Replacing (A.29) in (A.28) gives \( \partial C_N(\tau, 0)/\partial \tau + \partial C_{SP}(\tau, 0, 0)/\partial \tau + \partial C_{SR}(\tau, 0)/\partial \tau = 0 \), which is equivalent to \( \partial Y_N/\partial \tau + \partial Y_S/\partial \tau = \partial Y/\partial \tau = 0 \). From (3.17), one gets...
\[
\frac{\partial Y_S}{\partial \tau} = (\Phi L_S/L_N)(\partial Y_N/\partial \tau)\tau^{\alpha/(1-\alpha)} + (\Phi L_S/L_N)Y_N(\alpha/(1-\alpha))\tau^{(2\alpha-1)/(1-\alpha)}
\]

while, from (3.20), one gets \(\partial Y_N/\partial \tau = (-\alpha/(1-\alpha))(\Phi L_S/L_N)\tau^{\alpha/(1-\alpha)}(1 - (\Phi L_S/L_N)\tau^{1/(1-\alpha)})^{-1}Y_N\). Hence, \(\partial Y_N/\partial \tau + \partial Y_S/\partial \tau = 0\) is equivalent to

\[
(-\alpha/(1-\alpha))(\Phi L_S/L_N)^{1/(1-\alpha)}(1 + (\Phi L_S/L_N)\tau^{1/(1-\alpha)})^{-1} - (\alpha/(1-\alpha))(\Phi L_S/L_N)^{2\alpha/(1-\alpha)}(1 + (\Phi L_S/L_N)\tau^{1/(1-\alpha)})^{-1} + (\alpha/(1-\alpha))(\Phi L_S/L_N)\tau^{(2\alpha-1)/(1-\alpha)} = 0,
\]

which can be easily simplified to \(\tau = 1\).

**Proof of proposition 6** • The proposition

\(U_N(\tau^{A,C}, \{F^{A,C}(t)\}_{t \geq 0}, \{I^{A,C}(t)\}_{t \geq 0}) > U_N(\tau^{A}, \{F^{A}(t)\}_{t \geq 0}, \{I^{A}(t)\}_{t \geq 0})\) is immediate as \(\tau^{A,C}, \{F^{A,C}(t)\}_{t \geq 0}\) and \(\{I^{A,C}\}_{t \geq 0}\) are the solutions to problem (4.4) and \(\tau^{A}\) and \(\{F^{A}(t)\}_{t \geq 0}\) is the solution to the more constrained problem (4.2).

- We have proved in the above proof of proposition 5 that the participation constraint of the southern rich people is binding instantaneously at each date, so that: \(U_{SR}(\tau^{A,C}, \{I^{A,C}(t)\}_{t \geq 0}) = U_{SR}(\tau^{A}, \{I(t) = 0\}_{t \geq 0})\).

- Let us now show that \(U_{SP}(\tau^{A,C}, \{F^{A,C}(t)\}_{t \geq 0}, \{I^{A,C}(t)\}_{t \geq 0}) > U_{SP}(\tau^{A}, \{F^{A}(t)\}_{t \geq 0}, \{I(t) = 0\}_{t \geq 0})\). For this, it is sufficient to show that

\[
C_{SP}(\tau^{A,C}, 0, 0) + F^{A,C} > C_{SP}(\tau^{A}, 0, 0) + F^{A},
\]

at all dates.

In the above proof of proposition 5, we have shown that \(F^{A,C} = [\delta L_S C_{N}(\tau^{A,C}, 0) - L_N (C_{SP}(\tau^{A,C}, 0, 0) + C_{SR}(\tau^{A,C}, 0) - C_{SR}(\tau^{A}, 0))] / (L_N + \delta L_S)\) and \(I^{A,C} = C_{SR}(\tau^{A,C}, 0) - C_{SR}(\tau^{A}, 0)\). And we have shown in the proof of proposition 3 that \(F^{A} = [\delta L_S C_{N}(\tau^{A}, 0) - L_N C_{SP}(\tau^{A}, 0, 0)] / (L_N + \delta L_S)\).

Replacing these expressions in (A.30), we have to show that

\[
C_{SP}(\tau^{A,C}, 0, 0) + [\delta L_S C_{N}(\tau^{A,C}, 0) - L_N (C_{SP}(\tau^{A,C}, 0, 0) + C_{SR}(\tau^{A,C}, 0) - C_{SR}(\tau^{A}, 0))] / (L_N + \delta L_S)\]

is equivalent to \(C_{SP}(\tau^{A,C}, 0, 0) + C_{N}(\tau^{A,C}, 0) + C_{SR}(\tau^{A,C}, 0) - C_{SR}(\tau^{A}, 0)] / (L_N + \delta L_S)\). This condition rewrite \(C_{N}(\tau^{A,C}, 0) + C_{SR}(\tau^{A,C}, 0, 0) + C_{SP}(\tau^{A,C}, 0, 0) > C_{N}(\tau^{A}, 0) + C_{SR}(\tau^{A}, 0) + C_{SP}(\tau^{A}, 0, 0)\). Defining the world output as a function of the tax rate, this condition is also equivalent to: \(Y(\tau^{A,C}) > Y(\tau^{A})\), which is satisfied since \(\tau^{A,C}\) maximizes \(Y(\tau)\).
References


Kanbur, R., 2006. The Economics of International Aid. In Handbook of the Economics of Giving, Altruism and Reciprocity Vol. 2; Editors: Kolm, S.-C., Ythier, J. M.


