Cooperation among Overlapping Generations

for a Public Project

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Abstract

Many public projects initiated by one generation need to be main-

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tained by the next generation. With no future prospects, such projects are susceptible to free-riding between selfish generations. Only in an ongoing relationship is there a possibility to overcome free-riding behaviours. This paper investigates the pattern of tax payments that are likely to finance both implementation and maintenance of a public project in an ongoing economy with overlapping generations of two-period lived agents. The study identifies conditions under which cooperation among generations can be achieved as a steady state subgame perfect equilibrium.

Keywords: cooperation, ongoing economy, overlapping generations, public project.

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1 Introduction

“And there is no future

In England’s dreaming

No future, no future,

No future for you

No future, no future,

No future for me” (The Sex Pistols)

This somewhat provocative song was released in 1977 by the famous English punk rock band to coincide with the Queen’s Silver Jubilee. At the time, it was regarded by the old generation as a threat uttered by the young generation to the British society. Paradoxically enough, the threat of “no future” can be regarded as an effective mechanism of providing the old generation with correct incentives to behave cooperatively. Part of the motivation for this article is to develop this argument more formally.

The bulk of the analysis concerns the pattern of tax payments used to fi-
nance public projects that benefit two overlapping generations. Every young
generation must live willy-nilly with public projects that have been under-
taken by the previous old generation. Such projects have several prominent
features. First, the contribution made by the old to the project will not be
reversed since there is no tomorrow for this generation. When initiating the
project, the old generation incurs costs that are sunk once they are borne
and hence create a commitment value. This commitment value is all the
higher as the public project is more specific to the generation that initiates
it. Second, the young generation is excluded from the genesis of the public
project but not from its external effects on welfare, whether they be benefi-
cial or detrimental. Furthermore, contrary to the old, the young have future
prospects and so future benefits are more important to them than they are
to the old.

Obvious examples are given by all the public buildings and equipment
inherited from the previous generation, such as hospitals, libraries, highway-
road networks, military sites and weapon stocks accumulated for national
defense. The commitment value of public constructions will be high to the
extent that they have no value on a second-hand market\footnote{For weapon stocks, however, this may not be the case.} and cannot be
allocated to another use than that for which they have been designed. Similarly, some of the public policies implemented by one generation may have high commitment values and lasting effects on the next generation. Such is the case for nuclear programs or wars involving costs that can hardly be recouped. Sunk costs also characterize human investments such as education: every child is stuck with her mother tongue and the level of education provided by his parents. On one hand, children benefit from sharing the same language as their parents and going to the college chosen by their parents. On the other hand, children may be cornered by some slangy language daily used in family, too specific features of the old’s culture or too narrow investments made by their parents for education.

Public projects of common concern for two overlapping generations are susceptible to free-riding problems between generations which are exacerbated by the absence of tomorrow. One problem deals with the young’s temptation to shirk financing the maintenance cost of the public project once sunk contributions have been made by the old. This temptation may be hard to resist when the young know the old to place a higher value on the public project. Another problem will arise from the old’s incentive to shirk paying for the implementation cost of the public project. This incentive may
be strong either because the old value the public project less than the young, or because the old anticipate the free-riding of the young and are unwilling to finance alone the public project. This paper addresses such free-riding problems and shows how to mitigate them in an ongoing economy.

For this, we investigate whether overlapping generations are likely to cooperate in implementing public projects and finance both implementation and maintenance. The problem of cooperation among generations is formalized in a dynamic game with infinite horizon and overlapping generations of two-period-lived agents motivated by narrow self-interest. Every period both generations choose how much to spend on a public project financed with taxes. The decisions about tax payments are sequential, with the old moving first and the young moving second. This captures a situation in which the old’s contributions to the public project are sunk at the time they are made. Assuming complete and perfect information, the analysis seeks steady state subgame perfect equilibria of the game.

The short-term version of the model is closely related to Varian’s (1994) sequential contributions game. It obtains a unique subgame perfect equilibrium in which maintenance of the public project fails to be financed. With no future prospects, if the young know that the old’s valuation of the public
project is significantly higher, they will free-ride and let the old pay for the whole cost. Conversely, the old may shirk financing implementation even if they value the public project more than the young. Knowing that they will live shorter, the old have stronger incentives than the young to squander money for private consumption and free-ride on the young to implement the public project.

Under infinite horizon, the use of a trigger strategy is shown to sustain cooperative behaviours between the two overlapping generations and achieve both implementation and maintenance of the public project. The intuition behind the result is outlined as follows. As every young generation can observe how many resources the old generation has devoted to the public project, the young are in position to mete out a reward or a punishment to the old. The reward is the higher amount of public good and services provided by maintenance of the public project and the punishment is the reversion to the worst sustainable outcome, i.e., the equilibrium with no future prospects. Moreover, by rewarding the old, the young reward themselves as future old. This self-policing mechanism proves to be sufficiently powerful to achieve cooperation among generations and overcome the temptation to free-ride of the two overlapping generations. As long as tax payments are kept up
to the cooperative level, the economy presumes that generations abide by implicit cooperation and the reward ensues: there is both implementation and maintenance of the public project. If this reward is large enough, then cooperation among generations will persist in equilibrium.

The present result that overlapping generations of finitely-lived agents succeed in cooperating when they engage in an ongoing relationship departs from the usual Folk Theorems which have been previously established in the economics literature. For example, Kandori (1992) presents a Folk Theorem for infinitely repeated games played by overlapping generations of finitely-lived agents. However, this author assumes the agents to choose their actions simultaneously. Wen (2002) formalizes a Folk Theorem for a class of repeated games in which agents do not move simultaneously in every subgame. While this Folk Theorem explains cooperation in organizations run by the same set of infinitely-lived agents, it does not provide direct insight into ongoing relationships between finitely-lived agents. Technically, our framework combines the sequential structure of Varian (1984) for the stage game of tax payments, and the infinitely lived overlapping generations structure of Samuelson (1958) for the ongoing economy. A standard result is that cooperation fails to be sustained in an ongoing organization with only two members
moving simultaneously (see Cremer (1986)). By contrast, we derive conditions under which cooperation among the two overlapping generations can be achieved as a subgame perfect equilibrium of the ongoing game. Sequentiality of moves between generations is crucial for this result. Knowing that their tax payment is observable, the old anticipate that their decision will be responded optimally before they retire, which would not be the case if the moves were simultaneous. Due to sequentiality, the prospect of benefiting from a public project of higher size or quality becomes relevant from the old’s point of view: the latter have now an incentive to cooperate since they can be rewarded for their cooperation. Furthermore, the possibility that intergenerational relationship will go on is likely to make the young feel some altruistic concern for the old, even though the young are selfish. Indeed, the next young generation is expected by the current young generation to abide by the reward given to the old generation. And so it goes, the forthcoming young will be rewarded in turn by their descendants for their cooperative tax payment. Thus, the future is more valued, the higher the reward that can be expected by the young from contributing the cooperative amount to the public project.

The ability to cooperate in the provision of a public good has been previ-
ously discussed, but not, for the most part, in overlapping generation models. Mac Millan (1979) analyzes how trigger strategies can support a cooperative equilibrium by providing firms with the correct incentives for supplying public intermediate goods. Pecorino (1999) investigates the free-rider problem within a generation in a dynamic setting and shows that cooperation can be achieved even in large economies. Admati and Perry (1991) depart from the traditional formulation that deals with the scale of provision of the public good and address a problem where a public project is either provided or not. They develop a model in which agents alternate in making contributions to the public project until the project is completed. They show that only low-cost projects are completed and many socially efficient projects are not completed. Unlike Admati and Perry (1991), the focus here is on the scale of the public good that is provided and the agents do not alternate in making sequential contributions.

Overlapping generations models have been widely used to address the free-rider problem central to many studies concerning the exhaustible resources or the environmental protection. Considering environmental quality as a particular instance of public good, a number of articles highlight the difficulty to internalize polluting externalities between generations (see Howarth
and Norgaard (1992), John et al. (1995) or Rangel (2003)). The present work can be linked to this literature in the case of an environmental friendly public project, for instance a flow of pollution abatement goods and services, which would significantly reduce the environmental damages borne by old and young people currently alive. This paper provides the insight that there may be incentives for two overlapping generations to cooperate in paying environmental maintenance and improvement taxes, even when generations are selfish. This statement differs from that in Rangel (2003) in the sense that there is no need here to force a link between social security and investment in an environmental public good to foster cooperation among generations. The main reason is that, in the model developed here, contributions to a public project are assumed to benefit the generations that make them.

Section 2 presents the short-term version of the game and the equilibrium outcomes with no future prospects. Section 3 investigates the ongoing economy with two overlapping generations and identifies a class of subgame perfect equilibria. Section 4 concludes.
2 The equilibrium with no future prospects

This section investigates the design of an optimal public project that benefits two overlapping generations in the short term. For this, we consider a game $G$ with the same sequential structure as that in Varian (1994). The game $G$ will be the one-period stage game of the dynamic game introduced in the next section.

The economy is consisting of two overlapping selfish generations. The first generation to move, $i = 1$, is called “old”, and the second one, $i = 2$, is called “young”. The old decide to pay a tax $t_1$ used to finance the implementation of the public project. The young observe $t_1$, hence the size and existence of the public project, and decide in turn how many resources to devote to the public project. This contribution will finance either maintenance or implementation of the public project depending on whether the old have already financed implementation or not. The young choose to pay a tax $t_2$ for their expenditures on the public project. When determining how much they spend on the implementation of the public project, the old act as a Stackelberg leader anticipating the likely responses of the young, that is, how much they will spend on the maintenance of the public project. Once the public project has been constructed, the young cannot be excluded from
the benefits that are generated. Sequentiality captures the idea that the old’s choices are somewhat irreversible.

Each generation divides its wealth $w_i$ between private consumption of a good, $x_i \geq 0$, and tax payment $t_i \geq 0$ in return for the public project of size $g$ which is a direct conveyer of utility to both generations. Let $p$ denote the price of the private consumption good. The public project is produced under constant return to scales and its level is determined by $g = ct_1 + mt_2$ where $c, m > 0$. Parameters $c$ and $m$ are efficiency characteristics of the old and the young respectively ($c$ stands for construction/implementation which is expected to be the old’s task and $m$ for maintenance which should be assigned to the young). Following Barro (1991), the assumption of constant returns suggests a broad view of public project that encompasses human investments such as education and nonhuman investments in buildings and equipment.

Generation $i$ faces the budget constraint $w_i = px_i + t_i$ and has quasi-linear preferences described by $x_i + a_i u(g)$, hence:

$$x_i + a_i u(g) = (w_i - t_i)/p + a_i u(ct_1 + mt_2).$$

(1)

Parameter $a_i$ represents the concern of generation $i$ for the public project.
and the sign of $a_i$ indicates whether the project has beneficial or detrimental effects on generation $i$’s well-being. For instance, parameter values of $a_2 < 0$ allow for the possibility that the young expect the public project to exert negative externalities on their well-being, such as a war or a nuclear project.

Let us introduce two useful definitions. Generation $i$’s tax payment $\bar{t}_i$ at generational autarky is defined as the tax paid by generation $i$ when the other generation spends none on the public project. Thus, using budget constraints and assuming interior solutions, $\bar{t}_1$ solves equation

$$ca_1 u'(c\bar{t}_1) = 1/p,$$

and $\bar{t}_2$ solves equation

$$ma_2 u'(m\bar{t}_2) = 1/p. \tag{3}$$

When parameter values are such that $m\bar{t}_2 < c\bar{t}_1$, the old care more about the public project than the young since the former would contribute more if they were the only contributor. Moreover, the inequality is equivalent to $u'(c\bar{t}_1) < u'(m\bar{t}_2)$ since $u(.)$ is concave. Hence, from (2) and (3), $m\bar{t}_2 \leq c\bar{t}_1 \Leftrightarrow a_2 \leq a_1$. Assume that $w_i > \bar{t}_i$ so that consumption of the private good is always strictly positive.
Game G unfolds as follows. The old choose to pay a tax $t_1$ for the implementation of a public project. The young observe $t_1$ and then decide to finance further the public project by paying a tax $t_2$. Hence, the young’s choice of tax payment $t_2$ if it is positive must satisfy

$$ma_2 u'(ct_1 + mt_2) = 1/p.$$  \hspace{1cm} (4)

Using (3), it follows from (4) that the young are better off spending $\max \{0, \tilde{t}_2 - ct_1/m\}$ on the public project. Let $T_2(t_1)$ denote the young’s reaction function to the old’s choice of tax payment $t_1$:

$$T_2(t_1) = \begin{cases} \tilde{t}_2 - ct_1/m & \text{if } t_1 \leq m\tilde{t}_2/c, \\ 0 & \text{otherwise.} \end{cases}$$  \hspace{1cm} (5)

The following proposition characterizes the Stackelberg equilibrium pair of tax payments $(t_1^*, X_2(t_1^*))$ which is associated with the backward-induction outcome of the game G. It turns out that maintenance of the public project cannot be achieved with no future prospects.

**Proposition 1:** The short-term equilibrium pair of tax payments $(t_1^*, T_2(t_1^*))$
is such that:

\[ (t_1^*, T_2(t_1^*)) = \begin{cases} 
(0, \bar{t}_2) & \text{if } a_1 < a_2 \text{ or } \{ a_2 < a_1 \text{ and } a_1u(c\bar{t}_1) - \bar{t}_1/p < a_1u(m\bar{t}_2) \}, \\
(\bar{t}_1, 0) & \text{otherwise.} 
\end{cases} \]

**Proof:**

Assuming that the old move first, their utility as a function of their tax payment is given by

\[ (w_1-t_1)/p + a_1u(ct_1 + mT_2(t_1)) = \begin{cases} 
(w_1-t_1)/p + a_1u(m\bar{t}_2) & \text{if } t_1 \leq m\bar{t}_2/c, \\
(w_1-t_1)/p + a_1u(ct_1) & \text{otherwise.} 
\end{cases} \]

This function is quasi-concave and strictly decreasing in \( t_1 \) provided that \( c\bar{t}_1 < m\bar{t}_2 \) (or, equivalently \( a_1 < a_2 \)). Otherwise, the function is not quasi-concave and has two local optima, namely 0 and \( \bar{t}_1 \). If \( m\bar{t}_2 < c\bar{t}_1 \) and \( a_1u(c\bar{t}_1) - \bar{t}_1/p < a_1u(m\bar{t}_2) \), then 0 is a global optimum, i.e., the old free-ride on the young and achieve utility \( w_1/p + a_1u(m\bar{t}_2) \). If \( m\bar{t}_2 < c\bar{t}_1 \) and \( a_1u(c\bar{t}_1) - \bar{t}_1/p > a_1u(m\bar{t}_2) \), then \( \bar{t}_1 \) is a global optimum, hence the old finance alone the public project and achieve utility \( (w_1 - \bar{t}_1)/p + a_1u(c\bar{t}_1) \).

**End of proof.**
When $a_1 < a_2$, the young value more the public project than the old and, not surprisingly, the latter set $t_1^*$ at zero. That is, they spend none on the public project and free-ride on the young to finance its implementation. As a result, the young choose the tax payment that would prevail at generational autarky to finance the public project.

When $a_2 < a_1$, the old are more concerned about the public project than the young and two contrasting results can emerge depending on whether the old are scarcely more concerned or far more concerned than the young about the public project.

Firstly, if $a_1 u(c\tilde{t}_1) - \tilde{t}_1/p < a_1 u(m\tilde{t}_2)$, the old are scarcely more concerned than the young about the public project in the sense that the old would prefer the young to finance alone the public project than finance it themselves. Thus, in equilibrium the old let the young undertake the public project and pay themselves for its costs. This seems somewhat paradoxical since the young contribute everything while they care less about the public project. Such a result would not arise if both generations were making simultaneous decisions: the threat to free-ride by the old who are known to value more the public project would not be credible. By contrast, in the sequential game, the young may observe the “fait accompli” that the old have spent none on
implementation of the public project.

Secondly, consider the case where the old are far more concerned than the young about the public project so that \( \max \{ a_2, \bar{t}_1/p(u(c\bar{t}_1) - u(m\bar{t}_2)) \} < a_1 \). Then, the old find the public project more valuable if they finance its implementation themselves than if this expenditure were left to the young alone. In equilibrium, the old set \( t^*_1 \) at \( \bar{t}_1 \), that is, the old choose the tax payment at generational autarky and the young fail to finance further maintenance of the public project.

### 3 The ongoing economy

To model long-term relationships between the old and the young generations, we consider now the ongoing economy with an overlapping generations structure that is described by the following game \( G(\infty) \): in each of infinitely many periods \( t = 1, 2, \ldots \), a young generation enters the economy and replaces the old generation that has left at the end of the previous period; consumers’ life is assumed to last two periods, hence, at each date, the economy matches an old generation staying here for one period and a young generation staying here for two periods; every period, generation \( i \) is endowed with wealth
consumes a private good and decides how much to spend on the public project. Let $t_{ik}$ denote the tax paid by generation $i$ in period $k$ and $h_k$ the entire history through period $k$:

$$h_k = ((t_{11}, t_{21}), \ldots, (t_{1k-1}, t_{2k-1})).$$

In each period, the old decide on their own contribution to the public project by choosing their tax payment without knowing the young’s contribution. By contrast, the young decide on their tax payment after observing the old’s contribution. As we shall see, the young’s knowledge of the old’s choice of tax is crucial to induce cooperative behaviours between generations. Knowing that their behaviour is observable, the old have an incentive to cooperate since they can be rewarded for this cooperation. Moreover, by rewarding the old, the young reward themselves as future old.

In period $k$, the old’s utility reduces to $v_1(t_{1k}, t_{2k}) \equiv (w_1 - t_{1k})/p + a_1 u(ct_{1k} + mt_{2k})$ since they disappear at the end of the period. Defining $v_2(t_{1k}, t_{2k}) \equiv (w_2 - t_{2k})/p + a_2 u(ct_{1k} + mt_{2k})$, the present value of the lifetime utility for consumers born at the beginning of period $k$ is given by

$$v_2(t_{1k}, t_{2k}) + \delta v_1(t_{1k+1}, t_{2k+1}),$$

(6)
where $\delta$ represents the discount factor.

A strategy for generation $i = 1$ which takes a decision in period $k$ is a function $\varphi_{1k}(h_k) = t_{1k}$ that specifies the tax payment at any possible history. Moreover, as the young move second, their strategy in period $k$ is a function $\varphi_{2k}(h_k, t_{1k}) = t_{2k}$. Consequently, the subgames of $G(\infty)$ can be grouped into two classes: those beginning after $h_k$ and those beginning after $(h_k, t_{1k})$.

We use the steady state subgame-perfect equilibrium (SPE) as the solution concept. Hence, the generation’s strategies must satisfy the two following conditions:

1. Along the equilibrium path, the tax paid by any generation $i$ in period $k$ is the same as that paid by generation $i$ in period $k + 1$,

2. The generation’s strategies must constitute a Nash equilibrium on every subgame of $G(\infty)$.

To model the possibility of cooperation among generations in $G(\infty)$ and tackle at the same time the problem of the infinite variety of SPE, we shall restrict attention to a simple variant of trigger strategies. Formally, let $(t_1, t_2)$ be an agreed-upon profile of tax payments and define strategies $\varphi_{ik}(t_1, t_2) =$
for generations by:

\[
\varphi_{1k}(h_k) = t_1, \quad \varphi_{21}(h_1, t_1) = t_2, \quad \text{(7)}
\]

\[
\varphi_{1k}(h_k) = \begin{cases} 
  t_1 & \text{if } h_k = ((t_1, t_2), \ldots, (t_1, t_2)), \\
  t_1^* & \text{otherwise,}
\end{cases} \quad \text{(8)}
\]

and

\[
\varphi_{2k}(h_k, t_{1k}) = \begin{cases} 
  t_2 & \text{if } h_k = ((t_1, t_2), \ldots, (t_1, t_2)) \text{ and } t_{1k} = t_1, \\
  T_2(t_{1k}) = \max \{0, \bar{t}_2 - ct_{1k}/m\} & \text{otherwise.}
\end{cases} \quad \text{(9)}
\]

Note that the strategies \( \varphi_{ik} (t_1, t_2) \) closely follow the logic of subgame perfection by requiring, first, that the old respond optimally to any deviation that is detected at the end of some previous period, and second, that the young respond optimally to a deviation by the old during a period. Hence, the old pay the agreed-upon tax \( t_1 \) as long as in every past period both the old and the young have paid their agreed-upon tax; if any generation has failed to pay the agreed-upon tax in the previous period, then the old revert to the equilibrium tax payment \( t_1^* \) with no future prospects. Furthermore, the young match the old’s decision to pay the agreed-upon tax as long as no
generation has reneged the agreed-upon tax payment in both the past and
the present period; otherwise, given an observed $t_{1k}$, the young choose the
no-future equilibrium strategy $T_2(t_{1k})$.

As choices of tax payments are sequential, the young can provide the old
with an incentive to cooperate by responding optimally to their defection.
If instead the moves in the stage game were simultaneous, the old couldn’t
be given any incentive to pay an agreed-upon tax since they are in their last
period of life. Knowing that their behaviour cannot be observable, they would
be better off shirking, and so they would choose the same tax payment as
that emerging in the Nash equilibrium of the simultaneous-move stage game.
This argument directly follows from the result in Cremer (1986) that the
threat to withhold cooperation cannot work when the ongoing organization
has only two members.

Let us now determine the conditions under which the trigger strategies
$\varphi_{ik}(t_1, t_2)$ are subgame perfect.

**Lemma 1:** Suppose that both generations follow the trigger strategies
$\varphi_{ik}(t_1, t_2)$. Then, whatever the previous history, the best possible deviation
from \( t_1 \) for the old is \( t_1^* \) yielding utility

\[
v_1(t_1^*, T_2(t_1^*)) = \begin{cases} 
  a_1 u(m\tilde{t}_2) & \text{if } a_1 < a_2 \text{ or } \{ a_2 < a_1 \text{ and } a_1 u(c\tilde{t}_1) - \bar{t}_1/p < a_1 u(m\tilde{t}_2) \}, \\
  a_1 u(c\tilde{t}_1) - \bar{t}_1/p & \text{if } a_2 < a_1 \text{ and } a_1 u(c\tilde{t}_1) - \bar{t}_1/p > a_1 u(m\tilde{t}_2). 
\end{cases}
\]

**Proof:**

Let \( t_1^d \neq t_1 \) denote the best possible tax payment for the old when both generations play the trigger strategies \( \varphi_{ik}(t_1, t_2) \). If the old shirk paying the agreed-upon tax \( t_1 \), then a subgame of \( G(\infty) \) begins, in which the young decide on their tax payment after observing the old’s decision. By requiring that the young respond optimally to the old, the trigger strategies \( \varphi_{ik}(t_1, t_2) \) satisfy subgame perfection. Hence, the old anticipate the young’s best response to their choice \( t_1^d \) and maximize \( v_1(t_1^d, \varphi_{2k}(h_k, t_1^d)) = v_1(t_1^d, T_2(t_1^d)) \), for a given period \( t \), where

\[
T_2(t_1^d) = \begin{cases} 
  \bar{t}_2 - c t_1^d/m & \text{if } t_1^d \leq m\tilde{t}_2/c, \\
  0 & \text{otherwise.}
\end{cases}
\]

Thus, the optimal solution for the deviating old is given by the equilibrium
stated in Proposition 1. The old obtain at most

\[
v_1(t_1^d, T_2(t_1^d)) = \begin{cases} 
(w_1 - t_1^d)/p + a_1 u(m\tilde{t}_2) & \text{if } t_1^d \leq m\tilde{t}_2/c, \\
(w_1 - t_1^d)/p + a_1 u(ct_1^d) & \text{otherwise}.
\end{cases}
\]

and we know from Proposition 1 that

\[
t_1^d = t_1^* = \begin{cases} 
(0, \tilde{t}_2) & \text{if } a_1 < a_2 \text{ or } \{ a_2 < a_1 \text{ and } a_1 u(c\tilde{t}_1) - \tilde{t}_1/p < a_1 u(m\tilde{t}_2) \}, \\
(\tilde{t}_1, 0) & \text{otherwise}.
\end{cases}
\]

**End of proof.**

The following lemma gives, at the same time, the best possible deviation from the agreed-upon tax payment \( t_2 \) and the worst SPE outcome for the young in \( G(\infty) \).

**Lemma 2:** Suppose that both generations follow the trigger strategies \( \varphi_{ik}(t_1, t_2) \). Given \( h_k = ((t_{11}, t_{21}), \ldots, (t_{1k-1}, t_{2k-1})) \) and \( t_{1k} = t_1 \), the best possible deviation from \( t_2 \) for the young is \( T_2(t_1) \) with associated lifetime utility of

\[
v_2(t_1, T_2(t_1)) + \delta v_1(t_1^*, T_2(t_1^*)�.
\]

**Proof:**
Suppose that the young shirk after observing the old’s agreed-upon tax payment $t_1$ and spend $t_2'$ on the public project. Then, they trigger a punishment phase in the next period, which brings them down to $v_1 \left( \varphi_{1t+1}(h_{t+1}), \varphi_{1t+1}(h_{t+1}, t_1) \right) = v_1(t_1^*, T_2(t_1^*))$ when old. From Lemma 1, this is a SPE outcome. Following the trigger strategies $\varphi_{ik}(t_1, t_2)$, any deviation that is detected at the end of a period $k$ must be responded optimally by both the old and the young in period $k + 1$. Anticipating this, the young obtain a lifetime utility of $v_2(t_1, t_2') + \delta v_1(t_1^*, T_2(t_1^*))$ which depends on their deviation $t_2'$. Thus, the best possible deviation for the young is to pay a tax $T_2(t_1)$ which yields $v_2(t_1, T_2(t_1)) + \delta v_1(t_1^*, T_2(t_1^*))$. This measures the greatest lifetime utility in the absence of cooperation among generations.

**End of proof.**

Given that the old pay the agreed-upon tax $t_1$ to finance the public project, the greatest lifetime utility for the young reneging cooperation is given by $v_2(t_1, T_2(t_1)) + \delta v_1(t_1^*, T_2(t_1^*))$. Furthermore, this is the worst SPE outcome for the young in $G(\infty)$ since, from Lemma 1, $v_1(t_1^*, T_2(t_1^*))$ is a SPE outcome for the old in $G(\infty)$.

Suppose the young observe a tax payment set by the old at the agreed-upon level $t_1$. Then, by paying the agreed-upon tax $t_2$, the young can earn
\( v_2(t_1, t_2) + \delta v_1(t_1, t_2) \), which represents their lifetime utility of spending \( t_2 \) on the public project while young and \( t_1 \) while old, and so benefiting in exchange from an even provision of public good during lifetime. Denote \( V(t_1, t_2) \) as the function measuring the net value of cooperation among generations from the profile \((t_1, t_2)\), that is,

\[
V(t_1, t_2) \equiv v_2(t_1, t_2) + \delta v_1(t_1, t_2) - [v_2(t_1, T_2(t_1)) + \delta v_1(t_1^*, T_2(t_1^*))], \quad (11)
\]

where \( v_2(t_1, T_2(t_1)) + \delta v_1(t_1^*, T_2(t_1^*)) \) measures the greatest lifetime utility for the young in absence of cooperation among generations.

Proposition 2 below characterizes the agreed-upon profiles of tax payments \((t_1, t_2)\) that can be achieved as a SPE using strategies \( \varphi_{ik}(t_1, t_2) \).

**Proposition 2:** A profile of tax payments \((t_1, t_2)\) can be achieved as a SPE using strategies \( \varphi_{ik}(t_1, t_2) \) if and only if

\[
V(t_1, t_2) \geq 0.
\]

**Proof:**

We argue that the profile \((t_1, t_2)\) can be achieved as a SPE of \( G(\infty) \).
following strategies $\varphi_{ik}(t_1, t_2)$ if and only if, for any $k$,

$$v_1(t_1, t_2) \geq v_1(t_{1k}, \varphi_{2k}(h_k, t_{1k}))$$

(12)

and

$$v_2(t_1, t_2) + \delta v_1(\varphi_{1k+1}(h_{k+1}), \varphi_{2k+1}(h_{k+1}, t_1)) \geq$$

$$v_2(t_1, t_{2k}) + \delta v_1(\varphi_{1k+1}(h_{k+1}), \varphi_{2k+1}(h_{k+1}, t_1)),$$

(13)

for all $t_{1k} \in [0, w_1]$ and $t_{2k} \in [0, w_2]$.

Using (9), the right-hand side of (12) can be rewritten $v_1(t_{1k}, T_2(t_{1k}))$ for any $t_{1k} \neq t_1$. From Lemma 1, $v_1(t_1^*, T_2(t_1^*)) = \max_{t_{1k} \neq t_1} v_1(t_{1k}, T_2(t_{1k}))$ is what the old obtain at most when they deviate and anticipate the young to respond optimally to their deviation. Thus, condition (12) is equivalent to

$$v_1(t_1, t_2) \geq v_1(t_1^*, T_2(t_1^*)),$$

(14)

which requires that, in period $k$, the old’s utility from abiding by the agreed-upon tax payment $t_1$ is higher than their utility from being at the no-future equilibrium.

Similarly, from (13), it should be worth the young’s while to contribute
in period $k$, thereby inducing both the old and the young generations to pay the agreed-upon taxes for the public project in period $k + 1$. The left-hand side of inequality (13) measures the lifetime utility for any generation of following the trigger strategies $\varphi_{ik}(t_1, t_2)$ along the equilibrium path. Indeed, the young generation in period $k$ is sure to enjoy utility $v_1\left(\varphi_{1k+1}(h_{k+1}), \varphi_{2k+1}(h_{k+1}, t_1)\right) = v_1(t_1, t_2)$ when old. The right-hand side of inequality (13) measures what can be obtained during lifetime in the absence of cooperation among generations. If the young deviate to $t_{2k}$ in period $k$, then we know from Lemma 2 that they will get $v_1\left(\varphi_{1k+1}(h_{k+1}), \varphi_{2k+1}(h_{k+1}, t_1)\right) = v_1(t_1^*, T_2(t_1^*))$ when old: the deviation is responded optimally by both of the generations in period $k+1$. From Lemma 2, $v_2(t_1, T_2(t_1)) = \max_{t_{2k}} v_2(t_1, t_{2k})$ measures what the young receive at most by responding optimally to the old’s agreed-upon tax payment $t_1$.

Finally, inequality (13) is equivalent to $V(t_1, t_2) \geq 0$, which can be rewritten

$$\delta [v_1(t_1, t_2) - v_1(t_1^*, T_2(t_1^*))] \geq v_2(t_1, T_2(t_1)) - v_2(t_1, t_2).$$

(15)

By definition of $T_2(t_1)$, $v_2(t_1, T_2(t_1)) - v_2(t_1, t_2) \geq 0$ for all $t_1$ and $t_2$, and so, inequality (15) implies (14).

We have so far shown that if condition $V(t_1, t_2) \geq 0$ holds then strategies
\( \varphi_{ik}(t_1, t_2) \) are a Nash equilibrium of \( G(\infty) \).

To show that strategies \( \varphi_{ik}(t_1, t_2) \) are subgame-perfect, we must prove that they constitute a Nash equilibrium in every subgame of \( G(\infty) \).

Firstly, consider the subgames beginning after an agreed-upon history, i.e., those beginning after \( \bar{h}_k = ((t_1, t_2), \ldots, (t_1, t_2)) \) and those beginning after \( (h_k, t_1) \). Then, strategies \( \varphi_{ik}(t_1, t_2) \) have previously been shown to be a Nash equilibrium provided that condition \( V(t_1, t_2) \geq 0 \) is satisfied.

Secondly, consider the subgames beginning after any history in which the outcome of at least one earlier stage differs from the agreed-upon one. Then, either a deviation has been detected at the end of a period, or a deviation by the old is detected during a period. In both cases, strategies \( \varphi_{ik}(t_1, t_2) \) involve that the deviation is responded optimally. Indeed:

1. If a deviation has been detected at the end of a period, say \( k - 1 \), then a punishment phase begins in the next period \( k \). Following (8) and (9), the old choose their no-future equilibrium tax payment \( \varphi_{1k}(h_k) = t^*_1 \), anticipating that the young will choose to pay the tax \( \varphi_{2k}(h_k, t^*_1) = T_2(t^*_1) \) in the same period. Moreover, the young anticipate that they will face, next period, the same problem as the old in the present period. Hence, the young in period \( k \) anticipate that they will choose
\[ \varphi_{1k+1}(h_{k+1}) = t^*_1 \] next period, which will be responded optimally by the young in the same period, i.e., \[ \varphi_{2k+1}(h_{k+1}, t^*_1) = T_2(t^*_1). \] Thus, starting from any point in the considered subgame, the trigger strategies \( \varphi_{ik}(t_1, t_2) \) constitute a Nash equilibrium since the generation to move chooses to pay its no-future equilibrium tax which maximizes its intertemporal utility given the subsequent strategies of the other generation and itself.

2. If a deviation has been detected within a period, after observing the old’s tax choice, then the young begin a punishment phase during the same period and the same reasoning as above applies.

**End of proof.**

After observing the old’s contribution to the public project, every young generation needs to decide how much to pay for this project, and in exchange the young benefit from a public project of same scale when old, to the extent that the next young generation will choose the same level of tax payment as that chosen by the young in the previous period. The sequentiality of the overlapping generations moves is the key to the difference between the present model and that in Cremer (1986). As the old’s behaviour is observable,
the young can mete out rewards or punishments depending on what they have learnt. Even though they are about to leave, the old can be given positive incentives to spend more on the public project than \( t_1^* \), that is, what they would spend if there were no future prospects. If the old’s tax payment is seen to reach the agreed-upon level \( t_1 \), the young reward them by paying, in turn, the agreed-upon tax \( t_2 \). It follows that the public project benefiting both generations is improved in the sense that it will be maintained by the young once constructed by the old. Moreover, by rewarding the old, the young reward themselves as future old. They anticipate that, when old, they will earn the same reward discounted at the present date, namely \( \delta [v_1(t_1, t_2) - v_1(t_1^*, T_2(t_1^*))] \), provided that they choose the agreed-upon tax payment \( t_2 \). However, the young incur a loss \( v_2(t_1, T_2(t_1)) - v_2(t_1, t_2) \) from rewarding the old, hence rewarding themselves as future old. To achieve cooperation, the young’s prospect of reward must outweigh the opportunity cost of rewarding the old. This is the meaning of inequality \( V(t_1, t_2) \geq 0 \): the reward for abiding by the agreed-upon level in both periods of life must outweigh the temptation to shirk and revert to the equilibrium behaviours in generational autarky, which would prevail with no future prospects. When this inequality holds for positive tax payments, an implicit contract can be
conceived in which the young reward the old by financing maintenance of the public project in exchange for the old financing implementation.

Obviously, for such cooperative behaviours to emerge in equilibrium, agents need to be patient. Indeed, the future is more valued, the higher the reward that can be expected from contributing the cooperative amount to the public project. More precisely, consider a pair of tax payments \((t_1, t_2)\) such that inequality (14) is strictly met and denote by \(\delta^*\) the critical value of \(\delta\) such that (15) holds as an equality

\[
\delta^* \equiv \frac{v_2(t_1, T_2(t_1)) - v_2(t_1, t_2)}{v_1(t_1, t_2) - v_1(t_1^*, T_2(t_1^*))}.
\]  

(16)

For all pairs of tax payments \((t_1, t_2)\) satisfying (14) as a strict inequality, the index \(\delta^*\) measures the difficulty of maintaining cooperation among generations: the lower is \(\delta^*\), the more likely it is that both generations will select the agreed-upon tax payments \((t_1, t_2)\). If \(\delta^* = 1\), then cooperation fails whatever \(\delta\) and we are back in the situation with no future prospects: one generation free-rides on the contribution of the other and no one pays for maintenance of the public project. By contrast, if \(\delta^* = 0\), then inequality (15) is always satisfied provided that \(v_1(t_1, t_2) > v_1(t_1^*, T_2(t_1^*))\). Whatever
\( \delta \), it will be in both generations’ interests to choose the agreed-upon taxes \((t_1, t_2)\) rather than the equilibrium taxes that would prevail with no future prospects.

Moreover, it can be seen from (16) that \( \lim_{t_2 \to T_2(t_1)} \delta^* = 0 \). That is, among all pairs of tax payments \((t_1, t_2)\) satisfying condition (14), cooperation will be easier to achieve as \( t_2 \) approaches \( T_2(t_1) \). The idea of cooperation here is to provide the generation that is more willing to free-ride with an incentive to make a positive contribution to the public project. The commitment value of the old’s choice of tax together with the fact that the old don’t have so long to live impose a strong constraint: the old must be rewarded for cooperation regardless of whether they are more inclined to free-ride or not, otherwise they will behave as if there were no future prospects. The old’s reward for cooperation is given by \( v_1(t_1, t_2) - v_1(t_1^*, T_2(t_1^*)) \). The choice of the old’s tax \( t_1 \) must guarantee that this reward is positive, hence the old will enjoy the public project more than that undertaken with no future prospects. The numerator in (16) can be interpreted as the cost of cooperation borne by the young, which is null at \( t_2 = T_2(t_1) \). Finally, selecting tax payments \( t_2 \) close to \( T_2(t_1) \) facilitates cooperation by reducing the cost of rewarding the old. In other words, cooperation among generations is all the more successful as the
young adopt a behaviour closer to that prevailing with no future prospects.

To provide more insight on how tax payments should be designed to ensure the existence of cooperative SPE involving both implementation and maintenance of the public project, let us now distinguish between the two cases that yield the opposite results in Proposition 1:

1. Case 1: either $a_1 < a_2$ or \{ $a_2 < a_1$ and $a_1 u(c\bar{t}_1) - \bar{t}_1/p < a_1 u(m\bar{t}_2)$ \} meaning that the old are less concerned or scarcely more concerned about the public project than the young. With no future prospects, the old are better off free-riding on the young’s contribution.

2. Case 2: $\max \{ a_2, \bar{t}_1/p (u(c\bar{t}_1) - u(m\bar{t}_2)) \} < a_1$, that is, the old are far more concerned than the young about the public project. With no future prospects, the young are better off free-riding on the old’s contribution.

Proposition 3: *Cooperation cannot be achieved as a SPE with a profile of tax payments* $(t_1, T_2(t_1))$ *such that* $t_1 \in (0, m\bar{t}_2/c)$.

Proof:

Suppose that the old’s agreed-upon tax payment for implementation is
such that \( t_1 \in (0, m\bar{t}_2/c) \). Then \( T_2(t_1) = \bar{t}_2 - ct_1/m > 0 \). Replacing \( t_2 \) by this expression in the quasi-linear form of 

\[
v_1(t_1, t_2) \text{ gives the following reward to the old}
\]

\[
v_1(t_1, T_2(t_1)) - v_1(t_1, T_2^*(t_1)) = (t_1^* - t_1)/p + a_1 \left[ u(m\bar{t}_2) - u(ct_1^* + mT_2(t_1^*)) \right].
\]

(17)

In case 1, the no-future equilibrium involves \( t_1^* = 0 \) and \( T_2(t_1^*) = \bar{t}_2 \) and (17) can be rewritten

\[
v_1(t_1, T_2(t_1)) - v_1(t_1, T_2^*(t_1)) = -t_1/p.
\]

(18)

Thus, condition (14) is not met with \( t_1 > 0 \).

In case 2, the equilibrium tax payments with no future prospects are 

\( t_1^* = \bar{t}_1 \) and \( T_2(t_1^*) = 0 \), which gives 

\[
v_1(t_1, T_2(t_1)) - v_1(t_1^*, T_2^*(t_1^*)) = (\bar{t}_1 - t_1)/p + a_1 \left[ u(m\bar{t}_2) - u(c\bar{t}_1) \right].
\]

(19)

Moreover, parameter values in case 2 are such that \( \bar{t}_1/p + a_1 \left[ u(m\bar{t}_2) - u(c\bar{t}_1) \right] < 0 \). Again, condition (14) is not met with \( t_1 > 0 \).

**End of proof.**
Proposition 3 states that there is no cooperative equilibrium in which both implementation and maintenance of the public project are financed by positive taxes of $t_1$ and $T_2(t_1)$ paid by the old and the young respectively. Consequently, observing in the ongoing economy that the old pay a positive tax $t_1$ to finance implementation of the public project, the young should not react by spending on maintenance the same amount of resources as that they would spend with no future prospects, i.e., $T_2(t_1)$. This level of expenditures would fail to provide the old with more utility than what they would get in equilibrium at generational autarky with no future prospects, that is, condition (14) would not hold.

Proposition 4 shows that, in case 1, there is always a possibility to overcome the old’s free-riding behaviour arising in equilibrium with no future prospects provided that the young spend more resources on maintenance of the public project than what they would spend at generational autarky.

**Proposition 4:** In case 1, for any $t_2 > \bar{t}_2$ satisfying

$$p(\delta a_1 + a_2)u'(mt_2) > \frac{1}{m},$$

there exists some $t_1 \in (0, m\bar{t}_2/c)$ such that $(t_1, t_2)$ is sustainable in SPE.
Proof:

Using the quasi-linear form of \( v_1(t_1, t_2) \), the old’s reward for cooperation can be expressed as

\[
v_1(t_1, t_2) - v_1(t_1^*, T_2(t_1^*)) = (t_1^* - t_1)/p + a_1 [u(ct_1 + mt_2) - u(ct_1^* + mT_2(t_1^*))].
\]

(21)

In case 1, the no-future equilibrium involves \( t_1^* = 0 \) and \( T_2(t_1^*) = \tilde{t}_2 \). Replacing in (21) yields

\[
v_1(t_1, t_2) - v_1(t_1^*, T_2(t_1^*)) = -t_1/p + a_1 [u(ct_1 + mt_2) - u(m\tilde{t}_2)].
\]

(22)

Similarly, using the quasi-linear form of \( v_2(t_1, t_2) \), the cost of cooperation given by the right-hand side of (15) can be rewritten

\[
v_2(t_1, T_2(t_1)) - v_2(t_1, t_2) = (t_2 - T_2(t_1))/p + a_2 [u(ct_1 + mT_2(t_1)) - u(ct_1 + mt_2)].
\]

(23)

Consider now that the old’s tax payment for implementation is such that
\[ t_1 < m\bar{t}_2/c. \] Then \[ T_2(t_1) = \bar{t}_2 - ct_1/m \] which gives

\[ n_2(t_1, T_2(t_1)) - n_2(t_1, t_2) = (t_2 - \bar{t}_2 + ct_1/m)/p + a_2 \left[ u(m\bar{t}_2) - u(ct_1 + mt_2) \right]. \]  

(24)

From (22) and (24), condition (15) can now be written as follows:

\[ \delta \left[-t_1 + pa_1 \left[u(\delta t_1 + mt_2) - u(m\bar{t}_2)\right]\right] \geq t_2 - \bar{t}_2 + ct_1/m + pa_2 \left[u(m\bar{t}_2) - u(\delta t_1 + mt_2)\right], \]

(25)

which is thus equivalent to \( V(t_1, t_2) \geq 0 \) under the assumptions of case 1.

If \( V(0, t_2) > 0 \), then there will exist a positive tax payment \( t_1 \) satisfying \( V(t_1, t_2) \geq 0 \). Setting \( t_1 = 0 \) in (25) yields that the following condition

\[ \bar{t}_2 - t_2 + p(\delta a_1 + a_2) \left[u(mt_2) - u(m\bar{t}_2)\right] > 0 \]  

(26)

is sufficient to get \( V(t_1, t_2) \geq 0 \) here. Moreover, since \( u(.) \) is a concave function, we can write

\[ \bar{t}_2 - t_2 + p(\delta a_1 + a_2) \left[u(mt_2) - u(m\bar{t}_2)\right] \geq \bar{t}_2 - t_2 + p(\delta a_1 + a_2) mu'(mt_2) (t_2 - \bar{t}_2). \]  

(27)
The following inequality

\[ (\bar{t}_2 - t_2) [1 - p(\delta a_1 + a_2) mu'(mt_2)] > 0 \]  

(28)

is stronger than (26). Condition (28) is thus sufficient to meet requirement \( V(t_1, t_2) \geq 0 \). From (3), we get that \( u'(mt_2) = \frac{1}{pa_2m} \). If \( t_2 < \bar{t}_2 \), then \( 1 - pa_2mu'(mt_2) < 0 \) and condition (28) could not be satisfied. Thus, condition (28) requires that both conditions \( t_2 > \bar{t}_2 \) and \( 1 - p(\delta a_1 + a_2) mu'(mt_2) < 0 \) hold.

**End of proof.**

Proposition 4 identifies, in case 1, a class of SPE in which positive levels of tax payments allow to finance both implementation and maintenance of the public project. The young’s tax payments belonging to this class meet two requirements. First, they are set above the level of generational autarky which prevails in equilibrium with no future prospects. Second, at this level of tax, the lifetime marginal valuation of the public project would exceed the marginal cost of implementing the public project borne by the young if they were contributing everything. To sum up, when the old are less concerned or scarcely more concerned about the public project than the young, the
latter can induce the old to spend resources on implementation of the public project by paying more taxes than they would pay in generational autarky with no future prospects.

Let us now turn to case 2.

**Proposition 5:** In case 2, for any $t_1 \in (0, m\overline{t}_2/c)$ satisfying

$$p(\delta a_1 + a_2) u'(ct_1) < \delta/c + 1/m,$$

there exists some $t_2 > 0$ such that $(t_1, t_2)$ is sustainable in SPE.

**Proof:**

In case 2, the equilibrium tax payments with no future prospects are $t_1^* = \overline{t}_1$ and $T_2(t_1^*) = 0$. Replacing in (21) yields

$$v_1(t_1, t_2) - v_1(t_1^*, T_2(t_1^*)) = (\overline{t}_1 - t_1)/p + a_1 [u(ct_1 + mt_2) - u(c\overline{t}_1)].$$

Let us now consider an old’s tax payment for implementation such that
$t_1 < m\bar{t}_2/c$. From (30) and (24), condition (15) can be written as follows:

$$
\delta \left[ (\bar{t}_1 - t_1) + p a_1 \left[ u(ct_1 + mt_2) - u(c\bar{t}_1) \right] \right] \geq t_2 - \bar{t}_2 + ct_1/m + pa_2 \left[ u(m\bar{t}_2) - u(ct_1 + mt_2) \right]
$$

(31)

which is thus equivalent to $V(t_1, t_2) \geq 0$ under the assumptions of case 2.

Setting $t_2 = 0$ in (31) gives that the following condition

$$
\delta \left[ (\bar{t}_1 - t_1) + p a_1 \left[ u(ct_1) - u(c\bar{t}_1) \right] \right] + \bar{t}_2 - ct_1/m + pa_2 \left[ u(ct_1) - u(m\bar{t}_2) \right] > 0
$$

(32)

is sufficient to get $V(t_1, t_2) \geq 0$ here.

The concavity of $u(\cdot)$ gives the following inequalities

$$
\begin{align*}
&\begin{cases}
    u(ct_1) - u(c\bar{t}_1) \geq u'(ct_1) c \left( t_1 - \bar{t}_1 \right) \\
    u(ct_1) - u(m\bar{t}_2) \geq u'(ct_1) \left( ct_1 - m\bar{t}_2 \right)
\end{cases} \\
\end{align*}
$$

(33)

Consequently, inequality

$$
(\bar{t}_1 - t_1) (\delta - p\delta a_1 u'(ct_1)c) + (\bar{t}_2 - ct_1/m) (1 - p\alpha_2 u'(ct_1)) > 0
$$

(34)

is stronger than (32). Since, in case 2, we have $m\bar{t}_2/c < \bar{t}_1$, the previous
inequality is weaker than

\[(\bar{t}_2 - ct_1/m) (\delta/c + 1/m - p(\delta a_1 + a_2) u'(ct_1)) > 0. \quad (35)\]

where \(\bar{t}_2 - ct_1/m > 0\). It follows that (29) is a sufficient condition to meet requirement \(V(t_1, t_2) \geq 0\).

End of proof.

A cooperative equilibrium is possible in case 2 with the tax payments stated in Proposition 5 involving both implementation and maintenance of the public project. The old’s tax payments belonging to this set satisfy \(t_1 < m\bar{t}_2/c\), hence they are sufficiently low to ensure that the young will finance maintenance of the public project even if they deviate from cooperation, i.e., \(T_2(t_1) > 0\). As \(m\bar{t}_2/c < \bar{t}_1\) in case 2, the old’s tax payments \(t_1\) lay below their level of tax payment at generational autarky, which would prevail with no future prospects. Moreover, condition (29) means that, at the level of tax payments \(t_1\), the lifetime marginal valuation of the public project would be lower than its discounted marginal cost if the old were the only ones to finance expenditures. Finally, when the old are far more concerned than the young about the public project, they can induce the young to cooperate and...
maintain the public project by reducing the expenditures for implementation below either of the levels $m \bar{t}_2$ and $c \bar{t}_1$ prevailing at generational autarky.

4 Conclusion

This paper demonstrates that cooperation for public projects benefiting two overlapping generations can spontaneously be enforced as an “implicit” contract between the old and the young generations even though they are selfish. The result obtains because, on one hand, generations are linked in an ongoing relationship, and on the other hand, every young generation can observe the contribution made by the previous generation to the public project. In such a context, the young have an ability to reward the old, and thus a carrot to motivate cooperation among generations. A collapse of this cooperation carries automatic costs in the form of welfare losses stemmed from the absence of maintenance of the public project. Furthermore, by rewarding the old, the young reward themselves as future old. The analysis derives sufficient conditions under which the young’s threat to behave as if there were no future endows both generations with the correct incentives to overcome their temptation to free-ride. The results provide new insight into the possibility
of cooperation among overlapping generations of finitely-lived agents making sequential contributions to a public project. Two cases can be distinguished depending on the valuation of the public project by the young versus the old.

First, in an economy with no future prospects, the old shirk paying for implementation of the public project when they know that the young are sufficiently willing to pay for it. However, in an ongoing economy, the old have the opportunity to do better by contributing even a slight amount to the public project. Indeed, observing this positive contribution, the young will pay more for the project than if they had to finance the whole cost themselves. Such cooperative behaviours achieve a better result for both generations.

Second, in an economy with no future prospects, the old contribute everything when they value more the public project than the young. However, in an ongoing economy, the old can get more utility from the project by reducing their expenditures below their contribution at generational autarky. Observing this reduction, the young will have an incentive to pay something for the maintenance of the public project. As a result, the project is better than that at generational autarky in the sense that it benefits more both generations. This is a somewhat paradoxical case of cooperation among gen-
erations: reducing the old’s spending for a public project induces the young
to contribute to this project even though it has less appeal for them.
References


