CAPACITY DECISIONS WITH DEMAND FLUCTUATIONS AND CARBON LEAKAGE

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Key Words: capacity decisions, demand uncertainty, relocation, climate policy, carbon leakage.

JEL Classification: D24, L13, H23, L74

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Capacity decisions with demand fluctuations and carbon leakage*

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Abstract

This paper investigates the optimal mix between home capacity and imports to face an uncertain demand. It is proved that, if the difference between the home variable cost and the import price is large, the optimal home capacity increases as uncertainty increases, while it decreases if it is small. The model is calibrated using data from the cement sector to study the impact of a unilateral high CO$_2$ price in Europe. The results suggest a higher carbon leakage rate and more relocation of the industry than deterministic models would.

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1 Introduction

A major question for the design of European Union Emissions Trading Scheme$^1$ (EU-ETS) post 2012 concerns its possible impact on ‘sensitive’ sectors. A sector is sensitive under two conditions: (Grubb and Neuhoff, 2006) the impact of the CO$_2$ price is high relative to its value added (*value at stake*), it is highly exposed to international trade (*import intensity*). If both conditions are satisfied the risk of carbon leakage is high, i.e. the reduction of EU production will be partly compensated by imports. Imposing a high CO$_2$ price in the EU will not have the desirable effect of reducing CO$_2$ emissions worldwide. This has triggered a debate: how serious is the risk of leakage in sensitive sectors under pure auctioning of emission permits? If

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$^1$The EU-ETS is a cap and trade system in which firms of major pollutant sectors (electricity, steel, cement...) trade pollution rights (CO$_2$ emission permits) on the European market. Imports are not subject to pollution rights.
it is serious, what are the benefits and pitfalls of measures such as allowing free allocations to these sectors or imposing border tax adjustments to the corresponding trade flows? (Ismer and Neuhoff, 2007; Godard, 2007).

An increasing body of the literature is devoted to these two questions. The first one is more related to industrial organization and the analysis of international trade. The second one draws more on policy studies, such as the compatibility of a specific allocation mechanism with WTO rules. This paper falls into the first category, bringing into the picture a factor that has remained mostly ignored so far: the role of capacity decisions. Indeed, most approaches feature home producers (EU located) competing in a product market with foreign producers (non EU located) in a partial equilibrium analysis. Some of these approaches assume perfect competition to estimate Armington cross elasticities for home and foreign differentiated goods (Fisher and Fox, 2008). Some others assume a homogeneous product but imperfect competition (Smale et al., 2006). These two sets of approaches are essentially short term and capacity decisions are ignored. Contrarily to these general approaches, general in the sense that they have been applied to many different sensitive sectors, specific empirical studies such as the ones for cement detailed in Hourcade et al. (section 3, 2008) point out the crucial role of capacity constraints to explain international trade flows relative to regional economic cycles. Some research contributions specifically concerned by carbon leakage in the cement sector do introduce capacity issues, but only indirectly. Demailly and Quirion (2005) built a world model of the cement sector in which capacity decisions (and constraints) are introduced over a 30 year time horizon. However, their model involves many dimensions and capacity decisions are not analyzed as such. Capacity expansion follows deterministic demand trends, firms do not optimize their sources depending on the economic cycle. Ponssard and Walker (2008) analyze the impact of the spatial dimension in the cement sector, and the role of given capacity constraints in that respect. Proposals to explicitly introduce the rationale for capacity decisions in such models appear worthwhile.

The point made in the paper builds on two premises: capacity decisions are irreversible commitments to face uncertain future market conditions, firms operate networks of plants that allow them to balance the uncertainties that affect each individual plant. Consider a multinational firm that may source its EU market from EU and non EU plants. The short term decision depends on marginal costs, including the transport cost, unless its EU plant is saturated, in which case imports may be used even if more costly. When deciding its EU capacity the firm has to take into consideration the variability of its EU market and the total cost of producing in the EU versus the cost of importing from its non EU plants. The result of the paper is that a unilateral high CO₂ price in Europe affects the level of capacity in the EU in two ways. Firstly, because some of the CO₂ price increase will be passed through in the output price, demand will decline. Secondly, because
imports will become less costly relative to home production, the firm will on average import more over economic cycles. The first effect is the one studied in deterministic models. The second effect, due to the irreversibility of capacity decisions is the crux of the paper. It is shown that it may be as significant than the first one and, if so, it may imply a much larger impact on the carbon leakage than the one derived from deterministic models. The second effect will be referred to as the ”uncertainty effect”.

The analysis is made in a simple framework, assuming linearity in the long run average cost function (investment and production) and in the demand function, the latter one includes an additive random parameter uniformly distributed over a given range. The issue under study concerns the dependence of the optimal capacity upon the range of uncertainty of the demand function. Under this framework, a general result holds. If the CO$_2$ price is below some threshold, the optimal capacity increases as the range of uncertainty increases, while it decreases if it is above. This result has an important empirical implication. An interesting question is to investigate whether or not the EU-ETS may shift a sensitive sector from the first to the second case. If so, it would trigger a further decline in EU investment due to the uncertainty effect.

At the formal level, our approach has some similarities with the literature on the irreversibility effect initiated by Henry (1974) and Arrow and Fisher (1974). This literature draws the attention on the fact that one should possibly under-invest, i.e. commit today only to a lower level of investment, given that some adjustment will be made later on on the basis of future information. The issue of the monotonicity of the ex ante investment with respect to the precision of future information has been analyzed by several authors (Epstein, 1980; Salanié and Treich, 2006; Jones and Ostroy, 1984). The monotonicity results (more information implies lower investment today) holds only in special cases such as the one used by Demers (1991) to analyze capital adaptation over time. Interpreted in terms of capacity and production, within his framework, the firm is constrained to produce as much as its earlier capacity commitment and possibly more with a penalty cost.

In our framework, the issue is not the one of the precision of future information, there is complete information at the production stage. The issue concerns the range of uncertainty at the capacity decision stage. This question is close to the one addressed by Boyer and Moreaux (1989). They exhibit non monotonicity examples when the initial probability distribution on the random parameter varies. In our framework, monotonicity holds with respect to the range of uncertainty and a special class of probability distributions. To be complete, we also prove that, in our context, this result does not yield a monotonicity property with respect to the precision of information.

There exists another trend of economic literature which is related to our analysis. It concerns capacity decisions with demand uncertainty and
short term quantity competition (Gabszewicz and Poddar, 1997; Zoettl, 2008; Murphy and Smeers, 2005). This literature focuses on preemption effects. For instance, Murphy and Smeers (2005) and Zoettl (2008, chap4) consider the case of several technologies and show that preemption increases the incentive to invest in capitalistic technologies with a low variable cost. A side result of this literature provides a comparison between the optimal capacity selected under uncertain demand with the one selected with a known demand. With additive uncertainty and only one technology, firms should select a higher capacity with uncertainty (Gabszewicz and Poddar, 1997). This over-investment result comes from the assumption that firms cannot produce more than capacity but can produce less. In our framework, this assumption is not valid since firms have two technologies (home and foreign). We prove that our monotonicity result obtained in the monopoly case can be extended to the case of Cournot competition between symmetric firms. In this extension firms simultaneously select their capacity and a production plan contingent on the state of demand but not on the capacity decisions made by their competitors. We voluntarily leave aside preemption effects to focus on the role of the uncertainty range.

The empirical relevance of the results are discussed using industry data from the cement sector. The model quantifies the uncertainty effect, i.e. the change in the optimal capacity when going from a low CO\(_2\) to a high CO\(_2\) price. Using cost and demand estimates for this industry, it is shown that it if capacity is reduced by a factor of 5\% due to the pass through of the CO\(_2\) cost into the cement price, it may be reduced by another 10\% due to the uncertainty effect. Consequences in terms of carbon leakage are derived.

The rest of the paper is organized as follows. The model is described in section 2 and analyzed in section 3. Section 4 discusses the leakage issue using the cement sector for illustration. Section 5 gives the limits and the possible extensions. Proofs are in appendix 1. The similarities and differences with the literature on the irreversibility effects are discussed in appendix 2.

2 The model

2.1 Assumptions

The demand function is assumed to be linear: \( p = a + \lambda \theta - bq \)

- in which \( p \) is the price, \( q \) the quantity on the market, \( a \) and \( b \) two positive parameters
- uncertainty is introduced through the random variable \( \theta \) assumed to be uniformly distributed on the interval: \([-1, +1]\) with density \( 1/2 \)
the parameter $\lambda$ measures the range of demand variations, the case of no uncertainty corresponds to $\lambda = 0$.

A firm is in a monopoly situation on the market. It has access to two technologies: a home one and a foreign one. The cost function for the home technology consists of two terms:

- a linear investment cost $c_k$ relative to a capacity choice denoted $k$
- a linear production cost $c_h$ which includes the impact of the CO$_2$ regulation.

The cost function for the foreign technology involves a linear production cost $c_f$ and no investment cost. The production cost should be interpreted as an average delivered cost to the home market from foreign plants that have excess capacity relative to their own home markets. Assuming that the home market is small relative to the foreign market explains that there is no capacity constraint. It is explicitly assumed that the monopoly firm has either direct or indirect control on imports to its home market. This latter assumption may be more or less realistic depending on the sector under analysis.

In case of no uncertainty the home technology would be preferred to the foreign one, $c_h + c_k < c_f$ and the demand would be high enough to make production worthwhile, $a > c_h + c_k$.

Furthermore, the range of demand variations is limited so that in all demand states, in the short term, it is worth producing with the home technology: $0 \leq \lambda \leq a - c_h$.

The decision process takes place in three steps. First, the firm decides its capacity $k$ relative to the home technology. Second, uncertainty unfolds, the realized value of $\theta$ is revealed to the firm. Third, the production decisions $(q_h, q_f)$ using respectively the home and foreign technologies are made by the firm.

Denote $k^*$ the optimal capacity. The question under study concerns the dependence of $k^*$ on $\lambda$ as $c_h$ varies.
3 The solution

The monopoly long term profit $\pi(k)$ for a given capacity choice $k$ is given by:

$$
\pi(k) = \frac{1}{2} \int_{-1}^{+1} \max_{(q_h,k,q_f)} \left[ pq - c_h q_h - c_f q_f \right] d\theta - c_k k
$$

(1)

That is to say, for a given capacity $k$, in each state $\theta$, the firm selects $q_h(k,\theta)$ and $q_f(k,\theta)$ to maximize its short term profit

$$
pq - c_h q_h - c_f q_f
$$

with $q = q_h + q_f$ and subject to $q_h \leq k$.

Introduce two thresholds $\theta^-$ and $\theta^+$ for $\theta$ with $\theta^- < \theta^+$. The values of these thresholds will be precisely defined later on. Three different situations can arise on the short term depending on the position of $\theta$ relative to the two thresholds:

(i) if $\theta < \theta^-$, the firm has excess capacity, it produces the unconstrained monopoly quantity using $c_h$ as its marginal cost: $q_h(k,\theta) = \left( a - c_h + \lambda \theta \right) / 2b$ and $q_f(k,\theta) = 0$,

(ii) if $\theta^- < \theta < \theta^+$, the capacity constraint is binding, $q_h(k,\theta) = k$, and the foreign production is null, $q_f(k,\theta) = 0$,

(iii) if $\theta^+ < \theta$, the total production is the unconstrained monopoly quantity using $c_f$ as its marginal cost: $q(k,\theta) = \left( a + \lambda \theta - c_f \right) / 2b$ with $q_h(k,\theta) = k$ and $q_f(k,\theta) = q(k,\theta) - k$.

All three situations occur if $\lambda$ is sufficiently large as illustrated on figure 1. Marginal revenue of the firm is represented in three demand states, the two extreme ones ($\theta = -1, 1$) and the average one ($\theta = 0$). The optimal production of the firm is at the intercept of the marginal revenue with the marginal cost, which is either $c_h$ or $c_f$. In the low demand state ($\theta = -1$), the capacity constraint is not binding, it is situation (i). In the average demand state ($\theta = 0$), the capacity constraint is binding but it is not worth importing for $a - 2bk < c_f$, it is situation (ii). In the high demand state ($\theta = 1$) firm imports and its aggregate production (capacity and imports) is the unconstrained monopoly production with marginal cost $c_f$, it is situation (iii).
Quantities produced are continuously increasing with respect to $\theta$. At the low threshold state $\theta^-$ the unconstrained monopoly production is precisely equal to the capacity: $a + \lambda \theta^- - c_h = 2bk$. Similarly at the high threshold state $\theta^+$ so that $a + \lambda \theta^+ - c_f = 2bk$. For small $\lambda$ the thresholds are $-1$ or $1$ respectively. This gives:

$$\theta^- = \max \left\{ \frac{(2bk - a + c_h)}{\lambda}, -1 \right\}$$  \hspace{1cm} (2)

$$\theta^+ = \min \left\{ \frac{(2bk - a + c_f)}{\lambda}, 1 \right\}$$  \hspace{1cm} (3)

Using the thresholds and the optimal quantities, the long term profit writes:

$$\pi(k) = \frac{1}{2} \int_{\theta^-}^{\theta^+} \left[ \frac{(a + \lambda \theta - c_h)^2}{4b} \right] d\theta + \frac{1}{2} \int_{\theta^-}^{\theta^+} \left[ \frac{(a + \lambda \theta - bk - c_h) k}{4b} \right] d\theta$$

$$+ \frac{1}{2} \int_{\theta^-}^{\theta^+} \left[ \frac{(a + \lambda \theta - c_f)^2}{4b} + (c_f - c_h) k \right] d\theta - c_k k$$  \hspace{1cm} (4)

Taking the derivative of this expression with respect to $k$ gives the following first order condition (the effects of $k$ on the bounds of the integrals cancel each other if not null):

$$\frac{1}{2} \left[ \int_{\theta^-}^{\theta^+} (a - c_h + \lambda \theta - 2bk) d\theta + \int_{\theta^-}^{\theta^+} (c_f - c_h) d\theta \right] - c_k = 0$$  \hspace{1cm} (5)

From this condition the optimal capacity $k^*$ can be derived. This is done in proposition 1. The marginal cost of a capacity should be equalized with the expected short term marginal profit which is the shadow price of
the capacity constraint \((q_h \leq k)\). This flow of revenue is constituted of two integrals: the first one is the integral of the usual difference \(p + p'k - c_h\) obtained when the capacity sets the price (situation (ii)), the second term is the integral of \(c_f - c_h\) the short term cost reduction when the firm imports (situation (iii)).

Denote \(k^*(\lambda, c_h)\) the optimal capacity as a function of the two parameters \(\lambda\) and \(c_h\). The function \(k^*(\lambda, c_h)\) is certainly decreasing in \(c_h\). Whether it is increasing or decreasing in \(\lambda\) depends upon the position of \(c_f\) relative to \(2c_k + c_h\). It turns out that if \(c_f > 2c_k + c_h\) as \(\lambda\) increases we have \(\theta^- > -1\) before \(\theta^+ < 1\). The optimal capacity is an increasing function of \(\lambda\) because it pays to reduce the opportunity cost imposed by the capacity constraint. And the reverse is true if \(c_f < 2c_k + c_h\).

As the CO\(_2\) price increases the average production cost increases, say, from \(c_h\) to \(c_h + \Delta c_h\). One may go from the first case to the second case. That is,

\[
k^*(\lambda, c_h + \Delta c_h) < k^*(0, c_h + \Delta c_h) < k^*(0, c_h) < k^*(\lambda, c_h)
\]

The central inequality captures the impact of the pass through of the CO\(_2\) cost into the price which reduces the demand. This effect is deterministic. The first and third inequalities capture the impact of uncertainty on the choice of capacity. The fact that they go in opposite directions may make the carbon leakage significant. This is proposition 2.

**Proposition 1** The optimal capacity \(k^*\) writes:

**Case 1:** for \(c_f \geq 2c_k + c_h\):

- If \(0 \leq \lambda \leq c_f\):
  \[k^* = \frac{[a - (c_h + c_k)]}{2b},\]
- If \(c_k \leq \lambda \leq (c_f - c_h)^2/4c_k\):
  \[k^* = \frac{[a - c_h + \lambda - 2(\lambda c_k)^{1/2}]}{2b},\]
- If \((c_f - c_h)^2/(4c_k) \leq \lambda \leq a - c_h\):
  \[k^* = \frac{[a - (c_h + c_f)/2 + \lambda (1 - 2c_k/(c_f - c_h))]}{2b}.\]

**Case 2:** for \(c_f \leq 2c_k + c_h\)

- If \(0 \leq \lambda \leq c_f - (c_h + c_k)\):
  \[k^* = \frac{[a - (c_h + c_k)]}{2b},\]
- If \(c_f - (c_h + c_k) \leq \lambda \leq (c_f - c_h)^2/4(c_f - c_h - c_k)\):
  \[k^* = \frac{(a - c_f - \lambda)}{2b} + \frac{\lambda(c_f - c_k - c_h)^{1/2}}{b}.
\]

\(^2\)In case of increasing return for investment cost one would need to check that a pure import strategy would not be preferred.
\[
if \frac{(c_f - c_h)^2}{4(c_f - c_h - c_k)} \leq \lambda \leq a - c_h:
\]

\[
k^* = \left[ a - \frac{c_h + c_f}{2} + \lambda \frac{1 - 2c_k}{c_f - c_h} \right] / 2b.
\]

**Proposition 2** The optimal monopoly capacity is increasing (decreasing) with respect to \(\lambda\) iff \(c_f \geq 2c_k + c_h\) (iff \(c_f \leq 2c_k + c_h\)).

The \(\text{CO}_2\) price and the demand may be correlated, i.e. with a cap and trade system the \(\text{CO}_2\) price comes from a \(\text{CO}_2\) market which depends on general economic conditions. Assume a positive correlation. In case of high demand, the increase in the \(\text{CO}_2\) price makes imports more profitable. This leads to a further decline into the optimal capacity choice.

A simple way to formalize this idea is to let the increase in variable cost be such that \(\Delta c_I = \alpha_1 + \alpha_2 \theta\) with \(\alpha_1 \geq 0, \alpha_2 \geq 0\) \((\alpha_2 \leq b)\). The effect of correlation is established in the following proposition.

**Proposition 3** The optimal monopoly capacity is decreasing with respect to \(\alpha_1\) and \(\alpha_2\).

The previous analysis is carried on under a monopoly situation. The results can be extended to a Cournot oligopoly of \(n\) firms assuming that firms simultaneously select their capacity and production plan contingent on the state of demand\(^3\). There is a unique equilibrium and this equilibrium is symmetric\(^4\).

**Proposition 4** Under Cournot competition with \(n\) firms, there is a unique symmetric Nash equilibrium. The aggregate equilibrium quantity \(k^*(n)\) is:

\[
k^*(n) = \frac{n}{n + 1} 2k^*(1)
\]

4 The uncertainty effect and the carbon leakage in the cement sector

Cement is an important example to discuss carbon leakage issues. It is a sensitive sector: it ranks second on the scale of value at stake, after lime, and its import intensity is high, some EU States imports as much as 20% of their consumption (Hourcade et al., 2008).

\(^3\)formally, the game is one stage game, each firm strategy is \((k_i, (q_{ih}(\theta), q_{if}(\theta)))_{\theta \in [-1,1]}\).

\(^4\)Preemption issues are left for further research. These issues are important for growing markets which is certainly not the case for the EU.
The current literature on carbon leakage in that industry emphasizes the impact of a unilateral increase in the EU CO\(_2\) price on imports via an increase in international competition. Our model takes a different and complementary point of view, the one that an increase in imports may come from the relocation of EU production by EU firms. The relevance of this view is motivated by the following facts. Cement is typical of regional oligopolies due to a high transport cost. In each region, cement plants operate under tight capacity constraints. That capacity cannot be changed easily: construction of plants takes 2 to 3 years, sometimes much longer in the event a new quarry need be opened. The usual life time of a plant is more than 20 years. On the other hand, consumption may vary considerably from one year to another, because of a change in regional macroeconomic conditions or public policies. To be in equilibrium, a regional market generates either import or, possibly, export flows.

On the supply side, a characteristic of this industry is that most cement firms typically operate a large number of plants, some locally or in adjacent regions but also all over the world (the top 5 cement firms accounted for approximately 20% of worldwide production, in 2007). Under these conditions, a substantial fraction of the imports flows in any one region is directly or indirectly controlled by the firms active in that region. Capacity decisions of a cement firm thus balance local costs in one location versus transport costs plus local costs in another location, giving due considerations to the risks associated with uncertain future market conditions. The fact that some locations (within the EU) may support a CO\(_2\) cost while many others (outside the EU) may not, will impact the carbon leakage for the industry via the uncertainty effect.

Two indicators are traditionally used to quantify carbon leakage. The first indicator, called the pass through rate, measures the price increase in the EU relative to the asymmetric cost increase supported by EU firms. The second indicator, called the leakage rate, measures the increase in CO\(_2\) emissions outside the EU relative to the decrease of CO\(_2\) emissions in the EU.

The following section explicits the role of the uncertainty effect in this quantification.

4.1 Calibration

The data used in the calibration come from Ponssard and Walker (2008).

Cost data for a EU plant (investment cost and variable cost in €/t for a modern plant operating close to a full capacity of 1Mt/year):

\[
c_k = 15 \text{€/t} \\
c_h = 25 \text{€/t}
\]

Cost data for a non EU plant (in €/ton):

\[
c_f = 40 \text{€/t} + t_f
\]
in which $t_f$ stands for the transport cost, this cost would range from 35€/t to 60 €/t depending on whether the delivery takes place in a coastal EU market or in an inland market, to be referred later for convenience as “London” or “Madrid” respectively.

Market data (without uncertainty)
Price elasticity $\varepsilon = -0.27$
Market price $p = 100€/t$.

Assume a linear demand function with no uncertainty. It is a simple matter to check that the cost data and the market data are consistent with 6 (identical) firms competing à la Cournot. For simplicity, the demand can be adjusted so that at the equilibrium each firm produces 1Mt. It writes:

$$p = a - bq = 470 - 61.7q$$

This means that, in the short term, the degree of competition is as if there were 6 firms compete “à la Cournot”. If there were no demand uncertainty the domestic capacity selected by each firm would exactly match its production, there would be no imports. Suppose now that domestic capacities have to be decided under demand uncertainty. The question is: what is the impact of a high CO$_2$ price on the capacities selected by the firms, on their import strategies and, as a consequence, on the pass through and leakage rates?

### 4.2 Discussion

#### 4.2.1 Capacity decisions

With a zero CO$_2$ price, the home technology is preferred both in Madrid and London:

$$c_f(Madrid) = 75 > c_k + c_h = 40$$

$$c_f(London) = 50 > c_k + c_h = 40$$

The optimal capacity of the plant in Madrid would be increasing with uncertainty, but decreasing in London (proposition 2):

$$2c_k + c_h = 55 < c_f(Madrid) = 75$$

$$c_f(London) = 50 < 2c_k + c_h = 55$$

Introduce a cost increase $\Delta c_h = 25€/t$ (a ton of cement generates approximately 0.65t of CO$_2$ so that $\Delta c_h = 25€/t$ corresponds to a CO$_2$ price of 40€/t), the new variable cost of a EU plant is 50€/t. In the short term the London plant is no longer profitable (it may be closed) while the Madrid plant is still profitable.

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5 This calibration procedure is similar to the one used in Smale et al. (2006) and in Ponssard and Walker (2008).
The optimal capacity of the plant in Madrid is now decreasing with uncertainty (proposition 2):

\[ c_0(Madrid) = 75 < 2c_k + c_h + \Delta c_h = 80 \]

Using proposition 1, figure 2 depicts the graphs of the optimal capacities for the Madrid plant as the range for uncertainty \((\lambda/a)\) varies for a \(\text{CO}_2\) price at 0€/t or at 40€/t. For instance, if the uncertainty ranges from 85% to 115% \((\lambda/a = .15)\), three effects can be identified, using the capacities with no uncertainty as benchmarks:

(i) a decrease of 4% due to a larger plant than the benchmark for a \(\text{CO}_2\) price of 0€/t (uncertainty effect)

(ii) a decrease of 5.5% due to higher cement price which reduces the demand (price elasticity effect)

(iii) a decrease of 2.5% due to lower plant than the benchmark for a \(\text{CO}_2\) price of 40€/t (uncertainty effect).

The total impact of the two uncertainty effects is approximately 7.5% which is of the same order of magnitude as the impact of the price elasticity, i.e. 5.5%, the latter being the only effect to be captured if uncertainty were ignored.

In the next two sections we focus on the Madrid situation with \(\lambda/a = .15\).

![Figure 2](image-url)  
**Figure 2:** Capacity with respect to demand fluctuation for two \(\text{CO}_2\) prices.

### 4.2.2 Leakage rate

Assume identical \(\text{CO}_2\) emission rates for EU and non-EU plants, the leakage rate can be evaluated as the decrease in EU production versus the increase in imports.
On the short term there would be no leakage since imports are determined by the capacity constraint.

On the long term leakage occurs because of the lower capacity which triggers imports as soon as \( \theta > 0 \) (see Figure 3 that depicts the EU production and the imports as a function of actual demand). For instance, if the actual demand corresponds to \( \theta = .2 \) the leakage rate would correspond to \( BC/AC = 28\% \). One may also compute an average leakage rate to get a numerical value of 44\%.

In this model, leakage is purely due to the uncertainty effect since there will be no leakage without uncertainty, or assuming capacity decisions were made without taking CO\(_2\) into consideration. It would be interesting to see how robust the model is to the introduction of some form of international competition for imports. Presumably, the decrease in home capacity would persist.

![Figure 3: Changes in home production and import and the leakage rate.](image)

### 4.2.3 Pass through rate

Two pass through rates are considered: the short term one in which the capacity decision is made under the assumption that \( \Delta c_h = 0 \), and the long term one in which it is assumed that \( \Delta c_h = 25\€/t \).

Figure (4) depicts the cement prices in three cases:

- the benchmark case abc\(d\) (\( \text{CO}_2 = 0\€/t \)) using the two thresholds \( \theta^{-}(0) \) and \( \theta^{+}(0) \),
- the short term case e\(h\)\(c\)\(d\) (\( \text{CO}_2 = 40\€/t \)) assuming the capacity decision does not take into consideration the impact of the CO\(_2\) price on capacity,
• the long term case \textbf{efgcd} (\(\text{CO}_2 = 40\text{€/t}\)) assuming the capacity decision takes into consideration the impact of the \(\text{CO}_2\) price on capacity, using the two thresholds \(\theta^- (40)\) and \(\theta^+ (40)\).

Three zones emerge. On the left \((\theta < \theta^- (40))\), capacity is not a constraint. The short term and the long term pass through is the standard one \(n/(n+1)\), in which \(n\) is the number of Cournot competitors (see for instance Kimmel, 1992). In our calibrated model \(n = 6\), it is 6/7. On the right \((\theta > \theta^+ (0))\), capacity is a constraint the price is set by the import cost. The short term and the long term pass through rates are both zero.

What happens in the median zone can be inferred from the graph. The short term pass through rate will remain at 6/7 until \(\theta^- (0)\) and then progressively decline to zero at \(\theta = 0.4\) (the value of \(\theta\) at which capacity is constrained). As for the long term pass through rate, it will start to increase from 6/7 at \(\theta < \theta^- (40)\) to a peak, that can be computed to be 171\%, for all \(\theta\) such that \(\theta^+ (40) < \theta < \theta^- (0)\). From this peak it will then decline progressively to zero at \(\theta^+ (0)\).

The introduction of uncertainty, and its consequence on the optimal capacity, has a major impact on the pass through rate. Short term pass through rate underestimates the long term one.

Interestingly, due to the linearity of our model, it can be proved that the expected long term pass through rate is identical to the static pass through rate, i.e. 6/7=85.7\% (the proof is in the appendix), while the expected value of the short term pass through rate is only 52\%.

![Figure 4: Cement prices for two CO2 prices assuming 15% demand fluctuation.](image-url)
5 Conclusion

The theoretical framework used in this paper to identify an “uncertainty effect” relative to the choice of capacity should be considered only as a first step. The linearity assumptions of the demand and the cost functions need be replaced by more general assumptions. Similarly, the assumption used to formalize the range of uncertainty (a uniform random shock of the demand function) is quite specific. Non uniform and/or multiplicative shocks could be considered. As regards the competitive structure, one may want to integrate preemption motives at the capacity stage (which would seriously complicate the equilibrium analysis) and/or introduce competition from pure foreign players at the production stage.

From an empirical standpoint, it would be worthwhile to integrate the uncertainty effect along with other characteristics of the cement sector such as: the role of spatial competition, the impact of traders on international flows, the actual degree of obsolescence of the EU cement plants... This would be necessary to evaluate the relative weight of each of these factors in the analysis of leakage. Another avenue of research, quite independent of the politics of the EU-ETS, would be to test the empirical reality of the uncertainty effect. Data from the US cement industry could be useful in this respect: the US economy is subject to high economic cycles, regional cement markets are quite independent from one another, the competitiveness of imports vary considerably between the coastal and inland markets. This seems ideal for a natural experiment.

References


Appendix 1: Proofs

Proof of Proposition 1

Proof. The monopoly long term profit is a strictly concave function of \( k \in [0, (a + \lambda - c_h)/2b] \). There is a unique profit maximizing capacity \( k^* \) that solves the first order condition (5). Four situations can arise whether at \( k^* \): \( \theta^- = -1 \) or not and \( \theta^+ = 1 \) or not.

We determine the solution of (5) for each expressions of thresholds and corresponding inequalities satisfied by \( \lambda \). The proposition follows.

1. For \( \theta^- = -1 \) and \( \theta^+ = 1 \): the solution of (5) is:

\[
 k_1 = (a - (c_h + c_k))/2b, \quad \text{and}
\]

\[
\frac{2bk_1 - a + c_h}{\lambda} \leq -1 \iff \lambda \geq c_k,
\]

\[
\frac{2bk_1 - a + c_f}{\lambda} \geq 1 \iff \lambda \leq c_f - c_h - c_k.
\]

2. For \( \theta^- = (2bk - a + c_h)/\lambda \), and \( \theta^+ = 1 \): injecting expressions of \( \theta^- \), \( \theta^+ \) into (5) gives the equation: \( 2c_k = \lambda \int_{\theta^-}^{1} (\theta - \theta^-) d\theta \) i.e. \( 4c_k = \lambda (1 - \theta^-)^2 \) hence, \( \lambda - \theta^- = 2(\lambda c_k)^{1/2} \) so:

\[
 k_2 = \left[a - c_h + \lambda - 2(\lambda c_k)^{1/2}\right]/2b,
\]

\[
\frac{2bk_2 - a + c_h}{\lambda} > -1 \iff \lambda > c_k, 
\]

\[
\frac{2bk_2 - a + c_f}{\lambda} \geq 1 \iff \lambda \leq (c_f - c_h - 4c_k)/4c_k.
\]

3. For \( \theta^- = -1 \) and \( \theta^+ = 2bk - a + cf \), equation (5) is: \( 2c_k = \int_{\theta^-}^{1} (a - c_h + \lambda \theta - 2bk) d\theta + (1 - \theta^+)(c_f - c_h) \). Injecting \( a - 2bk^* = cf - \lambda \theta^+ \) gives \( 2c_k = 2(c_f - c_h) - \lambda (1 + \theta^+)^2/2 \) and the solution is:

\[
 k_3 = \left[a - c_f - \lambda + 2[\lambda (c_f - (c_h + c_k))]^{1/2}\right]/2b,
\]

\[
\frac{2bk_3 - a + c_h}{\lambda} \leq -1 \iff \lambda \geq (c_f - c_h)^2/4(c_f - c_h - c_k), 
\]

\[
\frac{2bk_3 - a + c_f}{\lambda} \leq 1 \iff \lambda > c_f - c_h - c_k.
\]

4. For \( \theta^- = (2bk - a + c_h)/\lambda \), and \( \theta^+ = (2bk - a + cf)/\lambda \) equation (5) is: \( 2c_k = \lambda (\theta^+ - \theta^-)^2/2 + (1 - \theta^+) (c_f - c_h) \) and \( \theta^+ - \theta^- = (c_f - c_h)/\lambda \) so \( \theta^+ = 1 + (c_f - c_h)/2\lambda - 2c_k/(c_f - c_h) \) and replacing \( \theta^+ \) by its expression gives:

\[
 k_4 = \left[ a - (c_f + c_h)/2 + \lambda (1 - 2c_k/c_f - c_h) \right]/2b,
\]

\[
\frac{2bk_4 - a + c_h}{\lambda} > -1 \iff \lambda > (c_f - c_h)^2/4(c_f - c_h - c_k), 
\]

\[
\frac{2bk_4 - a + c_f}{\lambda} \leq 1 \iff \lambda > (c_f - c_h)^2/4c_k.
\]
And finally, as $\lambda$ increases from 0 to $a - c_h$: if $c_f \geq 2c_k + c_h$ (resp. $c_f \leq 2c_k + c_h$) the optimal capacity is successively $k_1$, $k_2$, (resp. $k_3$), $k_4$.

**Proof of Proposition 2**

**Proof.** We use the expression established in proposition 1.

- For $c_f \geq 2c_k + c_h$:
  
  - For $\lambda \leq c_k$ the parameter $\lambda$ has no effect on $k^*$.
  
  - For $c_k \leq \lambda \leq (c_f - c_h)^2/4c_k$ the monopoly capacity is $k^* = \left[a - c_h + \lambda - 2\{\lambda c_k\}^{1/2}\right]/2b$ and derivation gives $[1-(c_f/\lambda)^{1/2}]/2b$ which is positive for $c_k \leq \lambda$.
  
  - For $(c_f - c_h)^2/4c_k \leq \lambda$, the derivative of monopoly capacity with respect to $\lambda$ is $[1-2c_k/(c_f - c_h)]/2b$ positive in that case.

- For $c_f \leq 2c_k + c_h$:
  
  - For $h \leq c_f - (c_h + c_k)$ the parameter $\lambda$ has no effect on $k^*$
  
  - For $c_f - (c_h + c_k) \leq \lambda \leq (c_f - c_h)^2/4(c_f - c_h - c_k)$, the derivative of the monopoly capacity is $\left[(c_f - c_h - \lambda)^{1/2} - 1\right]/2b$ which is negative for $c_f - (c_h + c_k) \leq \lambda$.
  
  - For $(c_f - c_h)^2 / 4(c_f - c_h - c_k) \leq \lambda$, the derivative of monopoly capacity with respect to $\lambda$ is $[1-2c_k/(c_f - c_h)]/2b$

**Proof of Proposition 3**

**Proof.** The analysis can be reproduced with a random permit price if $c_h + \alpha_1 + \alpha_2 \lambda < c_f$ so that the operating cost of the home plant is always lower than the cost of imports.

The low threshold is modified $\theta^- = \max \{ (2bk - a + c_h)/(\lambda - \alpha_2), -1 \}$. Direct calculations are complicated but influences of $\alpha_1$ and $\alpha_2$ can be deduced from the first order condition:

$$\frac{1}{2}\left[ \int_{\theta^-}^{\theta^+} (a - c_h - \alpha(\theta) + \lambda\theta - 2bk) d\theta + \int_{\theta^-}^{\theta^+} (c_f - c_h - \alpha(\theta)) d\theta \right] - c_k = 0$$

Applying the implicit function theorem to the first order condition (the right hand side is differentiable with a strictly positive derivative with respect to $k$) gives the effect of an increase of $\alpha_1$. This effect is $\frac{\partial k^*}{\partial \alpha_1} = \ldots$
\(-\partial^2 \pi / \partial \alpha_1 \partial k\) \((\partial^2 \pi / \partial k^2)^{-1}\) the denominator is \(\partial^2 \pi / \partial k^2 = -b(\theta^+ - \theta^-)\) and the numerator is \(\partial^2 \pi / \partial \alpha_1 \partial k = (1 - \theta^-)/2\) so

\[\partial k^*/\partial \alpha_1 = (1 - \theta^-)/2b(\theta^+ - \theta^-)\]

And the effect of an increase of the degree of correlation between the permit price and market condition is

\[\partial k^*/\partial \alpha_2 = \left[\int_{\theta^-}^{1} \theta d\theta\right] / [2b(\theta^+ - \theta^-)]\]

And \(\int_{\theta^-}^{\theta^+} \theta d\theta = (1 - \theta^-)(1 + \theta^-)/2\) so \(\partial k^*/\partial \alpha_2 = 0.5(1 + \theta^-) \partial k^*/\partial \alpha_1\).

\[\Box\]

Proof of Proposition 4

**Proof.** In order to limit the introduction of notations only a brief sketch of the proof is provided here, a more detailed one can be obtained by request to the authors.

Let assume that there are \(n\) firms with \(n \in \mathbb{N}^*\). Each Firm simultaneously chooses its capacity and a production plan. At an equilibrium: on the short term, in each demand state firms play a constraint Cournot game with two technologies available, and, in the long term, each firm capacity is a solution of a first order equation that equalizes the capacity cost \(c_k\) with expected short term marginal profit. Any equilibrium is symmetric because the expected marginal short term profit of two firms is equal if and only if their capacity are equal. Then the only possible equilibrium is symmetric and the aggregate equilibrium capacity \(k^*(n)\) is the unique solution of equation:

\[\int_{\theta^-((n,k))}^{\theta^+((n,k))} \left(a - c_h + \lambda \theta - \frac{n+1}{n}bk\right) d\theta + \int_{\theta^-((n,k))}^{1} (c_f - c_h) d\theta - 2c_k = 0 \quad (6)\]

where \(\theta^-(n,k)\) and \(\theta^+(n,k)\) are:

\[\theta^- = \max\{((n+1)bk/n - a + c_h)/\lambda, -1\}, \quad \theta^+ = \min\{((n+1)bk/n - a + c_f)/\lambda, +1\},\]

and aggregate equilibrium productions are constrained Cournot one:

\[\begin{align*}
0 \leq \theta \leq \theta^- & : q^*(n, \theta) = n(a + \lambda \theta - c_h)/(n + 1) \\
\theta^- \leq \theta \leq \theta^+ & : q^*(n, \theta) = k^* \\
\theta^+ \leq \theta \leq 1 & : q^*(n, \theta) = n(a + \lambda \theta - c_f)/(n + 1)
\end{align*}\]
By injecting expressions of $\theta^-, \theta^+$ into the first order condition (6) it appears that they are solution of an equation independent of $n$. So equilibrium values of threshold states are independent of $n$, and:

$$k^*(n) = \frac{n}{n+1} 2k^*(1)$$

And finally, the solution of equation (6) and productions (7) are equilibrium strategies because individual profit of each firm is concave and first order conditions are satisfied. The profit of each firm is strictly positive at this equilibrium.

---

**pass through rate**

We establish here that the relationship between concentration and pass through rate obtained with a constant demand still hold when uncertainty is introduced.

**Corollary 1** The long term expected pass through rate is $n/(n+1)$

**Proof.** The pass through rate is defined as the ratio of the change of expected output price and the cost increase. So we analyze here the derivative of expected price denoted $E_p$ with respect to the operating cost $c_h$. The long term derivative is composed of two components a direct one and an indirect one:

$$\frac{dE_p}{dc_h} = \frac{\partial E_p}{\partial c_h} + \frac{\partial E_p}{\partial k} \frac{\partial k^*}{\partial c_h}$$

The direct effect is the short term (i.e. with a fixed capacity) expected pass through: $\frac{\partial E_p}{\partial c_h} = (\theta^- + 1) n/2(n+1)$. The indirect effect is related to the change of capacity. A marginal change of capacity increases expected price of $\frac{\partial E_p}{\partial k} = (\theta^+ - \theta^-) b/2$, and from the first order condition:

$$(\theta^+ - \theta^-) b \frac{n + 1}{n} \frac{\partial k^*}{\partial c_h} + (1 - \theta^-) = 0$$

So finally

$$\frac{dE_p}{dc_h} = \frac{1}{2} \left[ (\theta^- + 1) \frac{n}{n+1} + (\theta^+ - \theta^-) b \frac{1 - \theta^-}{(\theta^+ - \theta^-) b n + 1} \right] = \frac{n}{n+1}$$

---

**Appendix 2: Similarities and differences with the irreversibility effect**

Consider the following timing:
1. the firm chooses its capacity \( k \),
2. the firm receives a signal \( s \) and update its beliefs about the distribution of \( \theta \) and chooses \( q(k,s) \),
3. the firm payoff is:

\[
 u(\theta,q,k) = (a + \lambda \theta - bq)q - c_h \min\{q,k\} - c_f(\max\{k,q\} - k) - c_k k
\]

The prior is an homogeneous distribution on \([-1,1]\). For any signal \( s \) the firm maximizes: \( E[u(\theta,q,k)|s] \) which is simply \( u(E[\theta|s],q,k) \) thanks to the linearity of our model. For any information structure\(^6\) \( S \), the optimal capacity is denoted \( k^\ast(S) \).

In the main text, we consider the case of complete information (denoted \( S_1 \)) for any \( \lambda \): \( k^\ast(S_1) = k^\ast(\lambda) \) but thanks to the linearity of our framework the case of uninformative information structure (denoted \( S_0 \)) is similar to the case \( \lambda = 0 \): \( k^\ast(S_0) = k^\ast(0) \). However, we do not study intermediary situations. With incomplete information, the option value literature analyzed how a change of the precision of the information structure modifies the choice of \( k \). Epstein (1980) established that with \( S \) more informative\(^7\) than \( S' \), if \( \partial u(\theta,q(\theta,k^\ast(S)),k^\ast(S)) \) is convex (resp. concave) then \( k^\ast(S') < k^\ast(S) \), and if \( \partial u(\theta,q(\theta,k^\ast),k^\ast) \) is neither convex nor concave the comparison is ambiguous.

**Proposition 5**

- If \( c_f \geq 2c_k + c_h \) and \( \lambda \leq (c_f - c_h)^2/4c_k \):
  
  the function \( \partial u(\theta,q(\theta,k^\ast(\lambda)),k^\ast(\lambda))/\partial k \) is concave and for any information structure \( S \):

  \[
  k^\ast(S) < k^\ast(S_1)
  \]

- If \( c_f \leq 2c_k + c_h \) and \( \lambda \leq (c_f - c_h)^2/4(c_f - c_h - c_k) \):
  
  the function \( \partial u(\theta,q(\theta,k^\ast(\lambda)),k^\ast(\lambda))/\partial k \) is convex and for any information structure \( S \):

  \[
  k^\ast(S) > k^\ast(S_1)
  \]

- Otherwise, if \( \lambda > \max\{(c_f - c_h)^2/4c_k, (c_f - c_h)^2/4(c_f - c_h - c_k)\} \) the function is neither convex nor concave and the comparison of \( k^\ast(S) \) and \( k^\ast(S_1) \) is ambiguous.

**Proof.** The function \( \partial u(\theta,q(\theta,k^\ast),k^\ast)/\partial k \) is constant and equal to \(-c_k\) for \( \theta < \theta^- \), then linear and increasing for \( \theta^- < \theta < \theta^+ \) then constant and equal to \( c_f - c_h - c_k \) for \( \theta > \theta^+ \). In the first case of the lemma \( \theta^+ = 1 \), in the

\(^6\)An information structure is a random variable \( s \) with a conditional density \( h(s,\theta) \).

\(^7\)"more informative" is defined by Blackwell (1951), a signal \( s \) is more informative than \( s' \) if \( s' \) can be obtained from \( s \). With continuous distribution, there is a function \( g(s',s) \) with \( \int g(s',s)ds' = 1 \) such that \( h(s',\theta) = \int g(s',s)h(s,\theta)ds \).
second case $\theta^- = -1$ and in the last case $-1 < \theta^- < \theta^+ < 1$. Comparisons of $k^\times(S)$ with $k^\times(S_1) = k^\times(\lambda)$ are applications of theorem 1 of Epstein (1980).

Comparing proposition 2 and 5 it can be seen that the monotonicity in $\lambda$ and in the precision of information are only aligned for small values of $\lambda$. In situation 1, the firm never imports, and may be in excess capacity (a case close to Gabszewicz and Poddar 1997). In situation 2, the firm is capacity constrained at all times, and may import (a case close to Demers 1991). For large $\lambda$, the monotonicity over the precision of information is ambiguous while the monotonicity with respect to $\lambda$ is not.

Consider the case of a binary information structure denoted $S_{1/2}$: at the second stage the firm learns whether $\theta \leq 0$ ($s = -1/2$) or $\theta \geq 0$ ($s = 1/2$) with probability 1/2. Side calculations show that for intermediary values of $c_h$ and large $\lambda$ there are situations where:

$$k^\times(S_0) < k^\times(S_1)$$

and:

$$vk^\times(S_{1/2}) > k^\times(S_1),$$

as well as:

$$k^\times(S_0) > k^\times(S_1)$$

and:

$$k^\times(S_{1/2}) < k^\times(S_1).$$