The Impact of Intermediary Remuneration in Differentiated Insurance Markets

Annette Hofmann* and Martin Nell
Institute for Risk and Insurance, University of Hamburg, Germany

April 15, 2008

Abstract

This article deals with the impact of intermediaries on insurance market transparency and performance. In a market exhibiting product differentiation and coexistence of perfectly and imperfectly informed consumers, competition among insurers leads to non-existence of a pure-strategy market equilibrium. Consumers may become informed about product suitability by consulting an intermediary. We explicitly model two intermediary remuneration systems: commissions and fees. We find that social welfare under fees is first-best efficient but fees lead to lower profits of insurers and non-existence of a pure-strategy market equilibrium. Commissions, in contrast, cause 'overinformation' of consumers relative to minimal social cost, but yield a full-information equilibrium in pure strategies associated with higher profits of insurers. This might explain why intermediaries are generally compensated by insurers in practice.


Keywords: product differentiation, intermediation, insurance oligopoly

* Corresponding author. e-mail: ahofmann@econ.uni-hamburg.de
1 Introduction

Despite many potential sources of information about insurance products available today – like the internet, financial magazines, informative advertising or simply word-of-mouth information – a fraction of consumers looking for insurance coverage is often not well-informed about the insurance product or provider which best suits their individual preferences. A reason for this phenomenon might be that information is sometimes not easy or too costly to obtain or simply that some consumers do not trust in information publicly available. This paper considers an oligopolistic insurance market with product differentiation where some consumers decide to remain uninformed about product ‘fit’.

Common economic models of insurance markets often consider either a monopolistic insurer or a perfectly competitive insurance market. These contrarian analyses are due to two implicit assumptions. The first assumption is that insurance is a homogeneous good. The second is that any insurer can potentially serve the whole market demand. Given these assumptions, if an insurer offers a marginally smaller price than its competitors, it gains (and serves) the whole market demand. Therefore, if at least two insurers compete in premiums, Bertrand competition yields zero profits and the premium to the insureds is actuarially fair. This is the so called Bertrand Paradox.¹

The Bertrand Paradox may ‘explain’ that monopolistic and competitive insurance markets are subject to vast research in insurance economics, while very few theoretical work examines the intermediate case of oligopolistic insurers sustaining positive profits in equilibrium. Polborn (1998) considers an insurance oligopoly with two risk-averse insurers engaged in Bertrand competition. The insurers face a trade-off between profit and risk, so that in equilibrium premiums tend to exceed marginal cost. Schlesinger and v. d. Schulenburg (1991) study oligopolistic competition of insurers in a product differentiation framework exhibiting search and switching costs. They show that introducing search and switching costs of insureds provides some market power to incumbent insurers and reduces market shares to new entrants.

In contrast to the small theoretical literature on insurance oligopolies, empirical evidence suggests that insurance markets are rather oligopolistic than perfectly

¹ The Bertrand Paradox is named after Bertrand (1883) who argued that if duopolistic firms compete via price, the noncooperative equilibrium price would fall to marginal cost since the firms would keep undercutting each other.
competitive. It is widely known that insurance providers tend to make positive profits. It seems thus interesting to develop a theoretical model of an oligopolistic insurance market where insurers actually make profits in equilibrium.

Most insurance models interpret an 'insurance contract' as a pair of only two parameters, the insurance premium paid by the insured regardless of state and the indemnification payment paid by the insurer in case of loss. As pointed out by Schlesinger and v. d. Schulenburg (1991), in reality such an insurance contract is rather a long description of contingencies in which the contract pays out and in which it does not. One should make a reasonable distinction between the "insurance contract" and the "insurance product". The insurance product represents a service that may differ from other services. The actual "fit" of the product to the needs of a consumer therefore also differs. The service may include claims settlement and risk management services, the availability of local agents or method-of-payment options. Warranties and embedded options in life insurance are also a means by which insurance products may be differentiated. We may consider life insurance policies with the same premium but different maturities or embedded options. Product characteristics differ between insurers, even though insurance contracts seem to be identical. For Germany, for instance, empirical evidence of perceived product heterogeneity in insurance markets is found by Schlesinger and v. d. Schulenburg (1993).

Taking these arguments into account, insurance is a rather complex and multidimensional product about which characteristics consumers are often poorly informed. Although consumers might be well-informed about existence and prices of insurance products (and maybe even some attributes differentiating them), information about which product is actually best suited for them is not easy to obtain. Consumers may observe prices and product characteristics of existing insurance products but they may not easily determine their best matching product due to lack of information. Our framework highlights this lack of information and aims at explaining the existence of intermediaries in insurance.

---

2 See, for instance, Nissan and Caveny (2001) and Murat, Tonkin, and Jüttner (2002). Murat, Tonkin, and Jüttner (2002) also find that their evidence is consistent with a trade-off between competition and industry stability. See Murat, Tonkin, and Jüttner (2002), p. 477. Interestingly, our findings suggest that more intense competition (due to existence of imperfectly informed consumers in the market) makes the market less stable.

3 According to the Insurance Information Institute, for instance, property-casualty insurers in the US earned net income after taxes of about 44 billion dollars in 2005 and 64 billion dollars in 2006.

markets. An intermediary plays a matchmaking role of ‘market maker’ in recommending the appropriate insurance product, i.e. in matching buyers with appropriate insurance providers.\(^5\) Thus ‘uninformed’ consumers can be informed about their best matching insurance product by consulting a broker. Therefore, in our model the role of brokers is to provide information about optimal product match. This information, however, comes at a cost. We analyze two compensation systems: commissions and fees. We show that fees lead to higher social welfare than commissions but imply lower profits of insurance providers and non-existence of a pure-strategy market equilibrium. Commissions, in contrast, cause ‘overinformation’ of consumers relative to minimal social cost, but yield a full-information market equilibrium in pure strategies.

The paper is organized as follows. The next section presents a short literature review. Section 3 introduces our model in a Hotelling framework. As a point of reference, we first study a differentiated insurance market without intermediation in section 4 and show that no equilibrium exists in pure strategies. Section 5 introduces intermediation. We show that intermediation may lead to a pure strategy full-information equilibrium under commissions, but not under fees. We also analyze efficiency of both remuneration systems from a social welfare viewpoint. Conclusions are summarized in the final section.

## 2 Related Literature


insurers and consumers can be of either high or low search cost type. Insurers may transfer the search task to brokers. Brokers then fulfill a search function by looking for potential customers at a commission fee to be paid by the insurer. Seog (1999) examines an insurance market where consumers are poorly informed about the price distribution and focuses on dynamic aspects of price search by intermediaries in order to find a long-run equilibrium where dependent and independent brokers might coexist. Cummins and Doherty (2006) argue that in an insurance market where insurers cannot observe loss probabilities but do know overall average loss probability, brokers might prevent a market failure due to adverse selection by informing insurers about loss probabilities of their customers.

In insurance markets, the brokers’ most important function can be seen in a matchmaking function. Gravelle (1994) offers a model for a competitive insurance market with intermediation. He assumes that the brokers’ function lies in determining the best-matching insurance product for consumers. However, insurers offer only one type of insurance product. They engage in Bertrand competition via brokers. Consumers are heterogeneous in their preferences for the insurance product and uninformed about whether the product actually offers a good match with their individual preferences. They know the overall distribution of mismatch in society and the expected mismatch but are unable to determine their individual degree of mismatch from buying the insurance product. A broker can inform a consumer about his individual degree of mismatch. In competitive equilibrium, marginal cost pricing and thus zero profits of insurers result. This is also true under both the commission system and the fee-for-advice system. Hence, neither compensation system affects profitability for insurers. Gravelle also finds that neither fees nor commissions might achieve an even second-best efficient equilibrium solution in his framework. Unlike Gravelle, we look at an oligopolistic insurance market with differentiated products where insurance providers interact strategically and may exercise market power due to the uniqueness of their products.

Focht, Richter, and Schiller (2006) build upon a product differentiation frame-

---

6 Although there may be other functions of intermediaries in insurance markets, such as providing risk management consulting, loss mitigation or assistance with claims settlement, the matchmaking role actually seems to be the most important.

7 Gravelle (1994) builds upon similar papers he wrote in 1991 and 1993. Gravelle (1991) and Gravelle (1993) studies the implications of the commission system with regard to advice quality provided by brokers in a life insurance market.
work by Schultz (2004). There are some uninformed consumers who can neither observe prices nor product varieties in the market. The authors show that a pure-strategy equilibrium might exist given that all consumers have rational and identical expectations about prices and product characteristics. Without intermediation, equilibrium profits of insurance providers are higher compared to profits given intermediation. From the insurers’ viewpoint, intermediation is associated with lower profits but higher social welfare. We differ from this view. In our framework prices are observable by all consumers in the insurance market. The rationale underlying this assumption is that unobservability of prices seems not very common. Price information is actually quite easy to obtain (for instance via the internet) while information about individual product match is less easily determined. This perhaps more realistic information structure leads to different results.

3 The Model

Our model follows the well-known product differentiation approach by Hotelling (1929) who formulated the following model of location and price choice in duopoly: Consumers are uniformly distributed on the unit line. A single good is produced at zero cost by two firms, each of which selects a location in the unit line and a price. Consumers have travel cost proportional to the distance to firms, and buy one unit of the good from the firm for which price plus travel cost is lowest. The model is a two-stage game between the two firms. In the first stage, each firm (simultaneously) selects a location on the unit line, in the second stage, having observed the locations selected, each firm (simultaneously) offers a price. The Hotelling model has been discussed in different contexts by many authors. Our setting follows D’Aspremont, Gabszewicz, and Thisse (1979).

Consider an insurance market with a continuum of consumers. The number of consumers in the market is normalized to one. Consumers are uniformly distributed on a line of unit length. There are two insurers in the market, \( i \in \{1, 2\} \), offering each some variant of the insurance product. Insurers are located at the extreme points of the unit line: Provider 1 is located at 0 and

---

8 The Hotelling model is a model of horizontal product differentiation. For a detailed discussion of horizontal product differentiation models, see Martin (2002), pp. 84-105.

9 Our results generalize to the case of \( N \) firms in an insurance market. Maintaining the symmetric variety pattern of the Hotelling duopoly market, the \( N \) firm case can be ana-
provider 2 at 1. This simplification of 'maximal differentiation' represents no restriction to our analysis as is shown in the appendix. The assumption only serves to simplify our exposition. The position of a consumer on the Hotelling line represents his individual preferences for the insurance products offered. A consumer located at $x \in (0, 1)$ cannot have a perfectly matching product, so there is some disutility (called transportation cost) involved in purchasing this product.

We make the following basic assumptions concerning the distribution of information in the insurance market: All consumers can observe prices of all existing insurance products, but some consumers cannot observe product varieties in order to determine which insurance product actually offers the best match for their type.\textsuperscript{10} We will refer to these consumers as 'uninformed' because, technically, they do not know their position on the Hotelling line.\textsuperscript{11} In particular, we assume that only a fraction $\delta \in (0, 1)$ of consumers is able to determine the best matching insurance product to their type. The uninformed consumers, however, following the 'principle of insufficient reason', have rational expectations of being in the middle of the market and thus tend to purchase insurance at the cheapest provider.\textsuperscript{12} Both types of consumers are uniformly distributed on locations. This is common knowledge.

We assume that all consumers are risk-averse with respect to the insurable risk and risk-neutral with respect to mismatch risk. Consumers differ in expected loss and in their preferences for insurance products. Thus, individual suitability of insurance products to consumers also differs. Insurers offer premiums $P_i =$

---

\textsuperscript{10} Alternatively, we might assume that all consumers observe prices and varieties, but some consumers are unable to process this information in order to find their best matching insurance product.

\textsuperscript{11} Since those consumers do have information about insurance premiums, they are in fact "partially informed" consumers. However, for simplicity we refer to them as "uninformed" since they differ from the other group in that they do not know their position on the Hotelling line and thus cannot determine optimal product match.

\textsuperscript{12} The 'principle of insufficient reason' was first expressed by Jacob Bernoulli. It states that if an agent is ignorant of the ways an event might occur (and therefore has no reason to believe that one way will occur rather than another), the event will occur equally likely. Keynes referred to the principle as the 'principle of indiffERENCE', formulating it as "if there is no known reason for predicating of our subject one rather than another of several alternatives, then relatively to such knowledge the assertions of each of these alternatives have an equal probability." See Keynes (1921), pp. 52-53.
\( E + p_i \), where \( E \) is expected loss of the policy and \( p_i \) represents the loading. Since premiums are marginally fair, i.e. include a fixed loading fee, full insurance is optimal for consumers as long as the cost of insurance net of expected loss (i.e. the sum of premium loading and individual disutility of mismatch) does not exceed their individual risk premium \( r \). The amount paid by policyholders above actuarial cost is the price of insurance. Hence, we refer to \( p_i \) as the price for an insurance product of provider \( i \). We assume there is no moral hazard problem.

We might introduce a zero stage into the game where insurers first simultaneously compete in locations (i.e. product characteristics) on the Hotelling-line before competing in prices. We do not explicitly consider this stage. Locations of insurers are not very interesting from our point of view since we are mainly interested in gaining insight into the performance of broker remuneration systems. Note, however, that for \( \delta = 1 \), it is easy to show that insurance providers choose their locations 0 and 1, respectively.

### 4 Insurance Market without Intermediation

Given some positive transportation cost \( t > 0 \), a consumer located at \( x \in [0,1] \) faces disutility \( tx^2 (t(1-x)^2) \) from purchasing an insurance product from insurer 1 (2). The heterogeneity or transportation cost parameter \( t \) represents the marginal disutility of mismatch and thus measures the intensity of product differentiation. In our setting, we refer to it as disutility to emphasize the utility loss from purchasing an insurance product which does not perfectly match a consumer’s type. Consumers are sufficiently risk-averse so that the market is

---

13 Expected loss plus risk premium may be interpreted as the individual willingness to pay for full risk reduction of a consumer. See the seminal articles by Pratt (1964), Arrow (1963), Mossin (1968), Smith (1968) or Doherty (1975).

14 D’Aspremont, Gabszewicz, and Thisse (1979) have shown that if all consumers are informed about product characteristics and firms are restricted to locate in \([0,1]\), firms will locate in 0 and 1, respectively. In the same way, we may derive optimal product characteristics for insurers in our framework. Since our focus is on an equilibrium in prices (second stage of the original Hotelling game), we suppose locations 0 and 1 in the following. This is without any loss of generality as we show in the appendix.

15 It seems plausible that unit disutility rises more than proportionately with distance to an insurance provider. Given linear transportation cost, the so called "principle of maximum differentiation" (i.e. firms locate as far from each other as possible in equilibrium of a two-stage game) is replaced by the "principle of minimum differentiation". See, for
completely covered. A consumer purchases insurance from provider 1 if his net utility is higher, i.e.

\[ r - (p_1 + tx^2) \geq r - (p_2 + t(1 - x)^2) \iff x \leq \frac{p_2 - p_1 + t}{2t}, \]  

(1)

where \( p_1 + tx^2 \) \( (p_2 + t(1 - x)^2) \) represent the total cost of purchase for a \( x \)-type consumer when purchasing from provider 1(2). A consumer is indifferent between purchasing from provider 1 or 2 if he is located at

\[ \hat{x} = \frac{p_2 - p_1 + t}{2t}. \]  

(2)

We assume that only a fraction \( \delta \) of consumers is aware of the "fit" of insurance products to their type. Uninformed consumers, represented by \( (1 - \delta) \), minimize expected cost of purchase by choosing the cheapest provider (since both product varieties are equally likely to be associated with each price). Hence, for uninformed consumers, we have classic Bertrand competition. The fraction \( \delta \) of informed consumers might be interpreted as a measure of market transparency: the higher \( \delta \), the more transparent is the insurance market.

The market demand of insurer 1 is therefore given by

\[ D_1(p_1, p_2, \delta) = \delta \left( \frac{p_2 - p_1 + t}{2t} \right) + (1 - \delta) \begin{cases} 1 & \text{if} \; p_1 < p_2 \\ 1/2 & \text{if} \; p_1 = p_2 \\ 0 & \text{if} \; p_1 > p_2 \end{cases} \]  

(3)

and demand for insurer 2 is then

\[ D_2(p_1, p_2, \delta) = \delta \left( 1 - \frac{p_2 - p_1 + t}{2t} \right) + (1 - \delta) \begin{cases} 1 & \text{if} \; p_2 < p_1 \\ 1/2 & \text{if} \; p_1 = p_2 \\ 0 & \text{if} \; p_2 > p_1 \end{cases} \]  

(4)

Given the market shares of providers 1 and 2, profits are

instance, Martin (2002), p. 99. However, one should not expect minimum differentiation as advocated by Hotelling. This is because the price subgame in Hotelling’s model fails to have a pure strategy equilibrium if firms are located "too close" to each other (but not at the same point). Correcting the non-existence problem in Hotelling’s original model, D’Aspremont, Gabszewicz, and Thisse (1979) verify that with quadratic transportation cost a price equilibrium exists for all possible locations.

\[ \Pi_1(p_1, p_2, \delta) = p_1 \cdot D_1(p_1, p_2, \delta) \]  
(5)

\[ \Pi_2(p_1, p_2, \delta) = p_2 \cdot D_2(p_1, p_2, \delta). \]  
(6)

Intuition suggests that the price game has no \((\text{Bertrand-Nash-})\) equilibrium in pure strategies. This seems intuitively clear since the presence of uninformed consumers provides the insurers with an incentive to slightly undercut the rival in order to capture all uninformed consumers without losing informed ones. Undercutting seems worthwhile as long as the price is not "too low". Then, it may be profitable for an insurer to become a 'niche player' and set a high premium supplying informed consumers only, which, again, makes undercutting worthwhile for the rival insurer, and so on.\(^{17}\) To illustrate this for our framework, note first that the usual Bertrand result cannot be an equilibrium. One of the providers might increase its price by some amount \(\varepsilon > 0\) in order to increase its profit: In particular, assume provider 1 increases its price from \(p_1 = p_2 = 0\) to \(\tilde{p}_1 = p_2 + \varepsilon = \varepsilon\). Of course, at \(p_1 = p_2\) profits are zero. At \(\tilde{p}_1 = \varepsilon\), however, due to (3) we obtain

\[ \tilde{\Pi}_1(\tilde{p}_1, p_2, \delta) = \tilde{p}_1 \cdot \delta \frac{p_2 - \tilde{p}_1 + t}{2t} = \frac{\varepsilon \delta (t - \varepsilon)}{2t} > 0 \quad \text{for } t > \varepsilon > 0. \]  
(7)

Thus one of the two insurers could always increase its profit by increasing its price, meaning that fair insurance \(p_1 = p_2 = 0\) could not be an equilibrium.

It seems intuitively clear that symmetric pricing \(p_1 = p_2 > 0\) cannot be an equilibrium since a provider may slightly undercut its rival and gain all uninformed consumers without losing informed ones. We may illustrate equilibrium behavior graphically in order to see that there is no other (asymmetric) equilibrium. We have\(^{18}\)

\[ \frac{\partial \Pi_1}{\partial p_1} |_{p_1 \neq p_2} = 1_{[0, p_2]}(p_1) \cdot \left[ \delta \frac{p_2 - 2p_1 + t}{2t} + (1 - \delta) \right] \]

\[ + 1_{(p_2, \infty)}(p_1) \cdot \left[ \delta \frac{p_2 - 2p_1 + t}{2t} \right]. \]  
(8)

\(^{17}\) The proof is due to Polo (1991) who first showed that the game under imperfect information has indeed no pure-strategy equilibrium in prices. See Polo (1991), p. 708.

\(^{18}\) Here \(1_{[0, p_2]}(p_1)\) etc. represent the indicator functions meaning \(1_A(y)\) which for every \(y \in A\) has value 1 and for \(y \notin A\) has value 0.
and
\[
\frac{\partial \Pi_2}{\partial p_2} \bigg|_{p_1 \neq p_2} = 1_{[0, p_2)}(p_2) \cdot \left[ \frac{\delta}{2t} (p_1 - 2p_2 + t) + (1 - \delta) \right] + 1_{(p_1, \infty)}(p_1) \cdot \left[ \frac{\delta}{2t} (p_1 - 2p_2 + t) \right].
\] (9)

The price reaction functions of the two insurers are then given by
\[
p_1^R(p_2) \bigg|_{p_1 \neq p_2} = 1_{[0, p_2)}(p_2) \cdot \left[ \frac{1}{2} p_2 + t \left( \frac{1}{\delta} - \frac{1}{2} \right) \right] + 1_{(p_2, \infty)}(p_1) \cdot \left[ \frac{1}{2} p_2 + \frac{t}{2} \right]
\] (10)

and
\[
p_2^R(p_1) \bigg|_{p_1 \neq p_2} = 1_{[0, p_2)}(p_1) \cdot \left[ \frac{1}{2} p_1 + t \left( \frac{1}{\delta} - \frac{1}{2} \right) \right] + 1_{(p_1, \infty)}(p_2) \cdot \left[ \frac{1}{2} p_1 + \frac{t}{2} \right]
\] (11)

These reaction functions are represented graphically in Figure 1. As can easily be seen from the figure, the reaction functions don’t intersect. There is no equilibrium in pure strategies.

If some consumers are uninformed about their best product match, discontinuities in the reaction functions imply that the game will have no pure-strategy equilibrium. Thus any game structure leaving some consumers uninformed
about their optimal product match cannot lead to a market equilibrium in pure strategies. Building upon a result by Dasgupta and Maskin (1986), it can be shown that there is a symmetric equilibrium in mixed strategies.\footnote{Mixed strategy equilibria in Hotelling’s original model are examined by Dasgupta and Maskin (1986) and Osborne and Pitchik (1987). Osborne and Pitchik (1987) show that the range of possible outcomes with mixed strategy equilibria may be large. Dasgupta and Maskin (1986) demonstrate that under full information there may be infinite equilibria in mixed strategies. Introducing imperfect information, Polo analyzes the second stage of Hotelling’s game. He considers a slightly different setting than we do. For our framework and results, it is sufficient to assume that the market is completely covered which means that we assume $t_j^- := 0$ and $t_j^+ := 1$ in Polo’s framework. See Polo (1991), pp. 703, 708-711. This assumption is standard and avoids the possibility of an issue discussed by Wang and Yang (1999) which may be interpreted as follows. When risk aversion of consumers reaches a sufficiently low level, less than maximum differentiation might result in the pure-strategy equilibrium since insurers might have an incentive to move towards the middle of the market in order to capture consumers in the central area. Since our focus is on the second stage of the original game, this problem is not interesting from our view and we thus avoid it.}

Real-world decisions of insurance providers concerning their prices seem, however, not randomly made. A stable pure-strategy equilibrium seems a more convincing concept in order to offer a plausible hypothesis on empirical market behavior of firms in oligopolistic interaction. The unsatisfactory instability of prices within a certain range (given mixed strategies) could be overcome when intermediation is added to our analysis. We suggest this solution in the next section.

\section{Insurance Market with Intermediation}

As we have shown in the previous section, the existence of uninformed and thus price-sensitive consumers plays an important role since those consumers provoke a failure of market equilibrium. In this situation, it seems interesting and reasonable to introduce an honest intermediary who improves the matching of uninformed consumers and insurers in the market. We assume that an intermediary incurs some variable cost $k > 0$ of performing a risk analysis and reveals the position of a consumer on the Hotelling-line perfectly.\footnote{A risk analysis is usually expensive since it requires expertise not only in finance, but also in actuarial science, law and engineering. See Cummins and Doherty (2006), p. 392.} Thus a broker transforms a previously uninformed consumer into an informed consumer.

Generally, there are two remuneration systems how the broker may be paid: the broker might be paid directly by consumers (we refer to this system as "fee"
or "fee-for-advice" system) or the broker might be paid for his service by the insurer (we refer to this system as "commission" remuneration system). We first study the latter and then compare our results with the former system. We analyze each remuneration system separately by taking it as exogenously given. Under commissions, insurance products are sold via intermediaries who earn a commission paid by insurers. Under fees, products are sold by insurers and intermediaries are paid by consumers. The case where an insurer may use both remuneration systems for different groups of consumers is discussed in the conclusion. The sequence of play is depicted in Figure 2.

5.1 Commissions

Under the commission remuneration system, the broker is paid by insurance providers. Since information from a broker is costless for consumers at stage three of the game, all uninformed consumers prefer to become informed by the broker. The insurer has to pay the brokers’ fee $m$ for each risk analysis. Given the broker market is competitive, marginal cost pricing leads to the brokers’ fee $m = k$. In the US, for example, there is indeed intense competition in the intermediary market, especially for small and medium-sized risks. Overall competitiveness tends to vary by market segment.21 An alternative interpretation of marginal cost pricing might be that the participation constraint of a broker must hold and implies that he participates if he is indifferent between offering his service and not offering. Therefore, insurance providers would pay him $m = k$ (given the broker is indifferent between offering his service and not offering, he still offers his service). This leads us to

---

Proposition 1 Under commissions, there is a full-information market equilibrium in pure strategies. Equilibrium profits of insurance providers are $\frac{t}{2}$.

Proof: Profits of insurers are

$$\Pi_1^c = (p_1 - (1 - \delta)k) \cdot (\frac{p_2 - p_1 + t}{2t})$$  \hfill (12)

and

$$\Pi_2^c = (p_2 - (1 - \delta)k) \cdot (1 - \frac{p_2 - p_1 + t}{2t}).$$  \hfill (13)

First-order conditions for a profit maximum are

$$\frac{d\Pi_1^c}{dp_1} = \frac{p_2 - 2p_1 + t + (1 - \delta)k}{2t} = 0$$  \hfill (14)

and

$$\frac{d\Pi_2^c}{dp_2} = \frac{p_1 - 2p_2 + t + (1 - \delta)k}{2t} = 0.$$  \hfill (15)

Hence, price reaction functions are well-behaved and given by

$$p_1^R(p_2) = \frac{1}{2}p_2 + \frac{t}{2} + \frac{1 - \delta}{2}k$$  \hfill (16)

and

$$p_2^R(p_1) = \frac{1}{2}p_1 + \frac{t}{2} + \frac{1 - \delta}{2}k.$$  \hfill (17)

The symmetric pure-strategy price equilibrium is then

$$p_1^c = p_2^c = t + (1 - \delta)k$$ \hfill (18)

and equilibrium profits are

$$\Pi_1^c = \Pi_2^c = \frac{t}{2}.$$ \hfill (19)

□

The equilibrium is illustrated in Figure 3 by the intersection point of price reaction functions. As compared to the standard Bertrand-equilibrium premiums $p_1^B = p_2^B = 0$, it is easy to see that equilibrium premiums are higher than fair.

By informing all uninformed consumers about product suitability via insurance intermediaries, the commission system leads to a symmetric pure-strategy
full-information equilibrium. Insurers make positive profits and share market demand equally. Intermediation via the commission system is desirable for both uninformed consumers and insurers because uninformed consumers are optimally matched to product varieties and insurers earn higher profits than without intermediation.\textsuperscript{22} Equilibrium profits in (19) would be the same in a Hotelling-market with full information and no intermediation which suggests that costs of commissions are entirely allocated to policyholders via insurance premiums, a phenomenon which can be empirically confirmed.\textsuperscript{23} The key point of the commission system is therefore that in equilibrium uninformed consumers are subsidized by informed ones.

5.2 Fees

While under the commission system, all uninformed consumers automatically ask for information from an intermediary since it is costless for them, under the fee system, consumers must pay the broker for his service before purchasing insurance from one of the two providers. It seems realistic to assume that consumers differ in their individual willingness to pay for the brokers’ information since they generally differ in expected loss (loss size and/or probability of loss) and thus being mismatched with the wrong provider matters differently. In particular, a mismatch matters more for ”high risk” consumer types than for

\textsuperscript{22} This follows easily from the proof of proposition 2 in the next section.

“low risk” types. Especially low-probability/high-severity events result in very few claims per contract. To describe consumer heterogeneity, we introduce a parameter \( \theta \in [\underline{\theta}, \overline{\theta}], \theta > 0 \). \( \theta \) is distributed with \( F(\theta) \).\(^{24}\) The broker market is again competitive and marginal cost pricing leads to the brokers’ fee \( m = k \). We conclude that uninformed consumers decide to ask for information and become informed if the individual value of the information from the broker exceeds the cost of providing it, i.e.\(^{25}\)

\[
\theta - k - \left(\frac{1}{4}\right)^2 t \geq - \left(\frac{1}{2}\right)^2 t \iff \frac{3}{16} t + \theta \geq k.
\]

Making the reasonable assumption that \( \overline{\theta} > k > \frac{3}{16} t \), consumers with \( \theta \geq \hat{\theta} := k - \frac{3}{16} t \) ask for product suitability analysis from the broker and consumers with \( \theta < \hat{\theta} \) do not. Since \( \hat{\theta} > 0 \) but \( \theta \in [0, \overline{\theta}] \) there always exist some consumers who do not buy from the broker. Some consumers remain uninformed about their best product match and no pure-strategy insurance market equilibrium can result under the fee system.

Hence, only a part of previously uninformed consumers \( 1 - \delta \) decide to become informed by a broker. This leads to a new proportion of informed consumers under the fee system to read

\[
\tilde{\delta} = \delta + (1 - \delta)(1 - F(\hat{\theta})).
\]

**Proposition 2** Under a fee system, some consumers remain uninformed and no pure-strategy equilibrium exists. The equilibrium is then in mixed strategies and insurers’ profits are lower than under commissions.

**Proof:** The first part has already been shown. The equilibrium under fees is in mixed strategies due to \( \tilde{\delta} \in (0, 1) \). Polo (1991) shows that there is a symmetric equilibrium in mixed strategies in the original Hotelling model with quadratic

\(^{24}\) While transport cost \( t \) represents disutility of mismatch resulting from not having an ideal insurance product, \( \theta \) measures the impact of this mismatch. Uninformed consumers are indifferent between insurance providers. However, they value the impact of mismatch differently due to their different underlying risks. A higher risk makes a claim more likely which results in stronger manifestation of mismatch.

\(^{25}\) For an uninformed consumer, the mean distance to an insurance provider is 1/2. When he is informed, his location is between 0 and 1/2 or 1/2 and 1. Thus the mean distance for an informed consumer is 1/4.
transportation cost. Equilibrium prices are defined over a finite interval which lies below the full information equilibrium prices and shifts down as the mass of uninformed consumers increases (i.e. $\delta$ decreases). Given the symmetry of the problem, equilibrium profits under fees will be lower than under commissions.

5.3 Social Welfare

Let us now look at social welfare (or social cost) under both remuneration systems. In order to compare both systems, social cost may be represented by the sum of disutilities of mismatch over all consumers and the information cost of the brokers. In a welfare optimum social cost is minimized. From a social welfare viewpoint, the fee system seems superior since it induces only those consumers to buy the brokers’ information for whom it is indeed worthwhile to do so. Hence, the fee system leads to the social optimum: Social costs are minimized. Under the commission system, in contrast, costs of becoming informed are zero (at stage three of the game) so that all uninformed consumers ask for information, even those for whom the value of information is smaller than the cost of providing it. Hence, uninformed consumers are subsidized by informed consumers. The commission system misses the social optimum. We summarize this in

**Proposition 3** Social welfare under fees is first-best efficient. Social welfare under commissions leads to 'overinformation' of consumers compared to the social optimum.

*Proof:* Under a commission system, all uninformed consumers ask for a risk analysis. Social cost is then,

$$S^c = \left(\frac{1}{4}\right)^2 t + (1 - \delta)k,$$  \hspace{1cm} (22)

26 See Polo (1991), proposition 4, p. 708 and proposition 5, p. 711. Note that full information prices in Hotelling’s model would be $p^*_1 = p^*_2 = t$ and are thus even smaller than in (18).

27 Overall disutility of informed consumers is $-(1/4)^2 t$. Overall disutility of uninformed consumers is $-(1/2)^2 t$. 

17
where the proportion of previously uninformed consumers is given by $1 - \delta$.

Under a fee-for-advice system, we may write social cost to read

$$S^f = \tilde{\delta} \left( \frac{1}{4} \right)^2 t + (1 - \tilde{\delta}) \left( \frac{1}{2} \right)^2 t + (\tilde{\delta} - \delta) k.$$  \hfill (23)

Comparing (23) to (22), simple calculation reveals

$$S^c - S^f = (1 - \tilde{\delta})(k - \frac{3}{16} t) > 0,$$ \hfill (24)

i.e. social cost under commissions is higher than under fees. This implies that social welfare is indeed higher under fees than under commissions. \hfill \Box

Summarizing our results, we may conclude that the fee system is socially optimal but has no equilibrium in pure strategies. The commission system, in contrast, leads to 'overinformation' of consumers relative to the social optimum, but seems preferable from the view of insurance providers due to the fact that it leads to a stable full-information market equilibrium in pure strategies and higher profits.

6 Concluding Remarks

The introduction of product differentiation and intermediation into insurance demand broadens the range of possible market equilibria compared to the standard Bertrand models. Coexistence of perfectly and imperfectly informed consumers in differentiated insurance markets provokes that competition among insurers leads to discontinuities in the reaction functions so that a (subgame perfect) pure-strategy equilibrium fails to exist. The introduction of intermediation, however, may result in a symmetric pure-strategy equilibrium exhibiting higher profits of insurers and full information of consumers in the insurance market. The rationale for the latter is that increasing market transparency via intermediation increases product differentiation (since previously uninformed consumers become informed ones), and therefore leads to less tight competition. This mechanism ensures higher profits of insurers and full information of consumers in equilibrium. While higher transparency on the consumer side

\footnote{Actually, it is not clear where insurance providers will locate in $[0, 1]$ since there is no pure-strategy equilibrium in the price game. For comparison, however, we assume the same locations.}
is usually thought to promote competition, our framework where consumers become informed about product match via intermediation suggests that increasing consumers’ information makes the market less competitive and thus leads to higher profits. Insurance premiums are higher than actuarially fair in competitive equilibrium.

Interestingly, our results are opposite to those obtained by Schultz (2004) and Focht, Richter, and Schiller (2006). While these authors find that, given prices and product characteristics are unobservable by some consumers, increasing market transparency on the consumer side implies less product differentiation, lower prices and profits, we show that – when prices are observable by consumers – increasing market transparency (for example via intermediation) increases product differentiation (since previously uninformed consumers become informed ones), and therefore leads to less intense competition associated with higher prices and profits. Uninformed consumers are price sensitive since the price is the only criterion which actually matters to them. In the models by Schultz (2004) and Focht, Richter, and Schiller (2006), consumers act stochastically since they do not have any information at all. This suggests that basic assumptions about the information structure of the game seem to play an important role.

Our results also add to those obtained by Cummins and Doherty (2006) who argue that in a perfectly competitive insurance market where insurers cannot observe loss probabilities, brokers might prevent a market failure by informing insurers about loss probabilities of their customers. We show that (in an oligopolistic insurance market) even if loss probabilities are perfectly observable, a failure of (pure strategy) market equilibrium might arise without intermediation due to lack of information about product match. The failure of equilibrium might be solved by introducing information intermediaries.

We explicitly model commissions versus fees. We find that fees are associated with a first-best optimum but yield lower profits for insurers and failure of an equilibrium in pure strategies. The equilibrium under fees is in mixed strategies only. Commissions imply that more consumers are informed than would be socially optimal. In equilibrium, uninformed consumers are subsidized by informed ones. This raises the question whether the commission system is indeed stable. An insurer may have an incentive to opt out of the system and directly target the group of informed consumers without using an intermediary. This might be a profitable strategy and it is indeed common practice in insurance markets to use intermediated and direct distribution channels separately. Co-
existence of distribution channels might lead to separate markets. In this case, informed consumers would use the direct distribution channel and uninformed consumers would use the intermediated market. Denoting prices of the intermediated and direct marketing policy by $p^I$ and $p^D$, respectively, separating conditions are:

$$p^D \leq p^I$$

(25) and

$$p^I \leq p^D + \theta$$

(26) which gives

$$0 \leq p^I - p^D \leq \theta.$$  

(27)

The separating conditions ensure that informed consumers do not buy the intermediated policy (hence the price of the direct marketing policy should not be higher)$^{29}$ and that none of the uninformed consumers purchases the direct marketing policy (hence the price of the intermediated policy should not exceed the price of the direct marketing policy plus minimum value of information). We leave a more detailed determination of the separating equilibrium prices for future research since it seems not very interesting from our viewpoint. However, given the separating equilibrium, we obtain borderline cases of our analysis. In particular, the commission market leads to a full-information pure strategy equilibrium, higher profits of insurers and lower social welfare.

All in all, the commission system seems preferable from the viewpoint of insurance providers since it yields both a stable full-information market equilibrium and higher profits. This might explain why intermediaries are generally compensated by insurance providers in practice. The superiority of the commission system over the fee system from the insurers’ viewpoint has not been shown yet. Existing models focus on zero profits equilibria (or even find lower profits after introducing intermediation) and thus cannot explain the usefulness of information intermediaries – and particularly of the commission system – for insurers in practice.$^{30}$ Predictions of our model might thus help to explain phenomena currently observed in insurance markets. Finally, we suggest that – when insurance products are differentiated by insurers in order to soften price competition – information intermediaries may play an important role due to their capacity

$^{29}$ This condition is, however, not binding.

of increasing market transparency and leading to higher equilibrium profits of insurers compared to a situation without intermediation.

Of course, our theoretical model abstracts from other dimensions of the contractual relationship between policyholders and brokers which might also be relevant in insurance markets with intermediation. For instance, we abstain from the brokers’ advice quality and their incentives to inform consumers perfectly about product match. While the fee system seems to offer brokers a high incentive to inform consumers truthfully about their ’best match’, the commission system might offer lower incentives. Sass and Gisser (1989), for instance, argue that turning a broker who acts under the commission system into an exclusive agent (like one acting under the fee system for one particular insurer) will induce the broker to use a higher effort level for this particular insurer than before. This is because the broker has no possibility to sell rival products any more which lowers his opportunity cost. However, even if we assume these incentive problems away, our analysis suggests that the commission system still misses the social optimum. Even in a ’perfect world’ where all intermediaries act non-strategically and honestly reveal the best product for their customers, the commission system – which is inferior from a social welfare viewpoint – will be established in the industry due to its property of being associated with higher profits and a full-information pure-strategy market equilibrium.
Appendix

Throughout our analysis, we have supposed 'locations' 0 and 1 of insurers, respectively. Our concern here is to verify these equilibrium locations resulting from the game when there is a 'stage zero', i.e. when providers can at an initial zero-stage freely choose locations on the Hotelling line. Product characteristics may be represented by a location $q_i$ of provider $i$ on the unit line. Without loss of generality, we assume that provider 1 is to the left of provider 2: $0 \leq q_1 \leq q_2 \leq 1$. Of course, at $q_1 = q_2$, we have no product differentiation: Products are identical and Bertrand competition yields the well-known Bertrand result. In our relevant case where $\delta = 1$, the structure of the game can be reduced as follows: In the first stage, firms choose their product characteristics, and then, in the second stage, firms compete in prices. D’Aspremont, Gabszewicz, and Thisse (1979) have shown that in case of a two-stage game where firms choose a location at an initial stage, and then compete in prices at the second stage, there will be 'maximum differentiation', i.e. the firms select locations as far apart from each other as possible. We show that maximum differentiation also holds in our framework under the commission system. Given some positive transportation cost $t > 0$, a consumer located at $x \in [0, 1]$ faces disutility $t(q_i - x)^2$ from purchasing a product from firm $i$. By choosing different locations, however, firms can ensure themselves positive profits. A consumer purchases from provider 1 if his net utility is higher, so

$$r - p_1 - t(x - q_1)^2 \geq r - p_2 - t(q_2 - x)^2$$

which, together with $q_2 - q_1 := \Delta q$ and $(q_1 + q_2)/2 := \bar{q} \in [0, 1]$, is equivalent to

$$x \leq \frac{p_2 - p_1}{2t\Delta q} + \bar{q}. \quad (29)$$

A consumer is indifferent between purchasing from provider 1 or 2 at

$$\hat{x} = \frac{p_2 - p_1}{2t\Delta q} + \bar{q}. \quad (30)$$

Market shares for providers 1 and 2 are then $\hat{x}$ and $1 - \hat{x}$, respectively. Profits of insurers are

$$\Pi_1 = (p_1 - (1 - \delta)k) \cdot \left(\frac{p_2 - p_1}{t\Delta q} + \bar{q}\right) \quad (31)$$
and
\[ \Pi_2 = (p_2 - (1 - \delta)k) \cdot (1 - \frac{p_2 - p_1}{t\Delta q} - \bar{q}). \] (32)

Price reaction functions are then given by
\[ p_1^R(p_2) = \frac{1}{2}p_2 + \frac{(1 - \delta)k}{2} + \bar{q} \cdot t\Delta q \] (33)
and
\[ p_2^R(p_1) = \frac{1}{2}p_2 + \frac{(1 - \delta)k}{2} + (1 - \bar{q}) \cdot t\Delta q. \] (34)

Equilibrium prices are
\[ p_1^*(q_1, q_2) = (1 - \delta)k + t\Delta q \cdot \frac{2}{3}(1 + \bar{q}) \] (35)
and
\[ p_2^*(q_1, q_2) = (1 - \delta)k + t\Delta q \cdot \frac{2}{3}(2 - \bar{q}). \] (36)

We can now establish reduced profit functions by substituting equilibrium prices in (31) and (32). We then have
\[ \Pi_{1,red}^*(q_1, q_2) = \frac{1}{18}t(2 + q_1 + q_2)^2(q_2 - q_1) \] (37)
and
\[ \Pi_{2,red}^*(q_1, q_2) = \frac{1}{18}t(4 - q_1 + q_2)^2(q_2 - q_1). \] (38)

It is now easy to see that insurance providers choose so called 'maximum differentiation'. Due to
\[ \frac{\partial \Pi_{1,red}^*}{\partial q_1} < 0 \] (39)
and
\[ \frac{\partial \Pi_{2,red}^*}{\partial q_2} > 0, \] (40)
providers will optimally select locations as far apart from each other as possible. Therefore, when providers are restricted to locate in \([0, 1]\), they will locate in \(q_1^* = 0\) and \(q_2^* = 1\), respectively.
References


