“The Value of a Statistical Life under Ambiguity Aversion”

Nicolas Treich &
Toulouse School of Economics (LERNA, INRA), France *
December 2007

Abstract:
We show that ambiguity aversion increases the value of a statistical life as soon as the marginal utility of wealth is higher if alive than dead. The intuition is that ambiguity aversion has a similar effect as an increase in the perceived baseline mortality risk, and thus operates as the “dead anyway” effect. We suggest, however, that ambiguity aversion should usually have a modest effect on the prevention of ambiguous mortality risks within benefit-cost analysis, and can hardly justify the substantial “ambiguity premium” apparently embodied in environmental policy-making.


JEL: D81, Q51, I18.

& I thank David Alary, Hippolyte d’Albis, Jérôme Foncel, Christian Gollier, Jim Hammitt, Kevin Haninger, Andreas Lange, Jean-Marc Tallon and Richard Watt for useful discussions as well as participants to the workshop “Theories and Applications of Ambiguity Aversion” at the Toulouse School of Economics in December 2007, and the PRC "sites et sols pollués" of the Conseil Régional Nord-Pas de Calais.

* Full address: Toulouse School of Economics (LERNA, INRA), Manufacture des Tabacs, Aile J.-J. Laffont, 21 all. de Brienne, 31000 Toulouse, France. Tel: +33 (0)561 128 514. Email address: ntreich@toulouse.inra.fr
1. Introduction

It is sometimes difficult to assess with precision the risks to health and life that we face. For instance, there is often conflicting information about the likelihood of dying from new environmental or technological risks. Remember the debates about the risks related to the mad cow disease or to the avian flu. Due to the scientific uncertainty over the channels of transmission of these diseases to human beings, it was difficult to characterize to predict the number of fatalities. Some experts predicted a few fatalities while other experts predicted several thousands fatalities.

How do we react to the divergent information on the probability of dying from a specific disease? In particular, do we behave as if we average the probabilities given by different experts? Or do we tend for instance to place excessive weight on the most pessimistic experts? The former is consistent with the standard (subjective) expected utility approach, while the latter is more consistent with an approach that allows for an ambiguity aversion effect.

Since the seminal Ellsberg (1961)’s experiment, it is a robust finding in experiments that individuals are averse to ambiguity over probabilities. The development of theories of ambiguity aversion is more recent. Some influential contributions include Gilboa (1987), Segal (1987), Schmeidler (1989), Gilboa and Schmeidler (1989), Klibanoff (2001), Epstein and Schneider (2003) and Klibanoff et al. (2005). These theories have been mainly applied to financial risks so far. For example, Chen and Epstein (2002) suggest that ambiguity aversion might explain the equity premium puzzle.

There exist a few empirical analyses on ambiguity aversion when ambiguity concerns risks to life and health. An example of such an analysis is Viscusi et al. (1991). They designed a survey in which participants were asked to choose between two living areas, A and B, where there is a risk of nerve disease due to environmental pollution. In the Area A, risks are ambiguous: one study indicated a risk level 150 cases per 1 million population, and another study indicated a risk level of 200 per 1 million population. Participants then were asked what risk in the Area B they would view as equivalent to the risk posed in the Area A.

The answers are reported in the Table 1 below. Notice that there was another treatment (treatment 2) in which the ambiguity concerning the risk in the Area A is increased: one study indicated 110 cases per 1 million population and another study indicated 240 cases per 1 million population (thus still with a mean risk of 175 cases per 1 million). Interestingly, Viscusi et al. (1991) shows that survey participants do not simply average baseline mortality risks. In treatment 1, the mean of the answers about the equivalent risk is a bit greater than the mean risk and equal to 178.35, while in the more ambiguous treatment 2 the mean of the

---

1 Some recent exceptions include the theoretical papers by Lange (2003) on climate change policy, Chambers and Melkonyan (2007) on the trade of toxic products and Albis and Thibault (2007) on savings behaviour in face of ambiguous longevity.

2 Ritov and Baron (1990) develop a hypothetical experiment in which they show a reluctance to vaccination under missing information about side effects of the vaccine. Also, Riddle and Show (2006) show, using a survey of Nevada residents, a substantial effect of ambiguity concerning risks from nuclear-waste transport. Shogren (2005) reports a survey study about a food-borne illness posing ambiguous risks; see section 7 for more details.
answers is equal to 191.08. Hence, this survey study seems to indicate that participants suffer from greater ambiguity.³

<table>
<thead>
<tr>
<th>Treatment 1</th>
<th>Risk levels in Area A:</th>
<th>Mean of answers (sample size 65):</th>
</tr>
</thead>
<tbody>
<tr>
<td>[150, 200]</td>
<td>178.35</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Treatment 2</th>
<th>Risk levels in Area A:</th>
<th>Mean of answers (sample size 58):</th>
</tr>
</thead>
<tbody>
<tr>
<td>[110, 240]</td>
<td>191.08</td>
<td></td>
</tr>
</tbody>
</table>

Table 1 – Survey data from Viscusi, Magat and Huber (1991)

This survey study raises another (unanswered) question: What is the willingness to pay to avoid the ambiguous risk? Indeed, ambiguity aversion is expected to have an effect on individuals’ monetary-equivalents in face of change in ambiguous mortality risks. Consistent with benefit-cost analysis, ambiguity aversion may in turn have an effect on the choice of prevention policies concerning risks that are ambiguous. Our objective in this paper is to understand this effect theoretically. More precisely, we study the theoretic impact of ambiguity aversion using the standard value of a statistical life (VSL) model.

We consider the recent Klibanoff et al. (2005)’s theory of ambiguity aversion that encompasses most common theories of ambiguity cited above. Also, this theory introduces a simple and interpretable measure of ambiguity aversion.⁴ We show that the existence of ambiguity over baseline mortality risks always increases the VSL when the decision maker is averse to ambiguity. This result holds as soon as the marginal utility of wealth of the decision maker is higher when he is alive than when he is dead, a standard assumption in mortality risk models. The intuition for the result is that the ambiguity aversion effect operates as the “dead anyway” effect (Pratt and Zeckhauser, 1996). Namely, the effect of ambiguity aversion on the VSL is similar to that of a perceived increase in the baseline mortality risk. Before turning to the presentation of the model and to the derivation of the results, we briefly discuss risk policy-making in the presence of ambiguity.

2. Ambiguous Risks and Policy-making

Some policy analysts have suggested that decision-makers tend to put more effort into reducing ambiguous risks compared with familiar risks. A strand of the risk policy literature has shown that risk policy-making is plagued with a conservatism bias. This tendency has often been presented as an “irrational” response of policy-makers.

Viscusi (1998) for instance argues that policy-makers err on the side of being too stringent when they face ambiguous risks, as exemplified by the higher regulation of synthetic risks compared to more familiar but often more severe carcinogens. Viscusi (1998) also explains how US Environmental Protection Agency (EPA) inflates risk cut-off values for

³ A related effect is documented in Viscusi (1997). He presents to participants conflicting information about an environmental risk, and shows that participants treat the “high risk” information as being more informative.

⁴ As we will see, this theory achieves a separation between ambiguity and ambiguity attitude. Besides, this theory is fairly tractable because preferences are “smooth” (and not kinked), and can easily be extended to state-dependent preferences (Nau, 2006).
individual risk-exposure by computing a theoretical “maximally exposed” individual (combining maximal ingestion rates, maximal exposure duration and minimal body weights). US EPA also typically uses upper bound values (like the 95% percentile) of probability distributions, and routinely applies rule-of-thumb margins of safety.\(^5\) Obviously these practices do not reflect the mean tendency of the risk but instead bias the risk cut-off toward conservatism. Moreover, when several parameters are uncertain, risk assessment can be severely distorted due to the combination of several upper bound values. Belzer (1991) computed for instance that the excess mean risk due to dioxin was overestimated by about 5000 times by US EPA. As Viscusi and Hamilton (1999) suggest: “(t)hese biases, in effect, institutionalize ambiguity aversion biases” (Viscusi and Hamilton, 1999, p. 1013). Along similar lines, Sunstein (2000) argues that, in the presence of divergent risk scenarios, policymakers focus too much on the worst-case scenario, and do not account enough for the low probabilities involved. More generally, Sunstein (2005) argues that risk regulatory decisions based on a precautionary principle approach are usually inconsistent with basic principles of economic efficiency.

Interestingly, most regulations issued by US EPA have a high implicit cost per life saved: it is usually higher than $10 million (Viscusi, 1998), and often a much higher figure in the range of hundreds of millions, or even billions dollars as was the case for the Superfund program (Viscusi and Hamilton, 1999). In contrast, revealed and stated preferences studies in developed countries in general obtain a VSL ranging from $1 to $10 million (Viscusi and Aldy, 2003). These observations suggest that environmental risks may have been over-regulated. In addition, some empirical analyses have shown that environmental risks are far more regulated than health, occupational and transportation risks (Tengs at al., 1995; Hahn, 1996). As many environmental risks may be more ambiguous risks than other risks, the “ambiguity premium” apparently embodied in policy-making is a good candidate to explain part of the observed over-regulation of environmental risks.

Benefit-cost analysis is sometimes presented as a possible “corrector” for inconsistencies in risk regulation. It may in particular help insulate risk policies from too much ambiguity aversion (Viscusi, 1998; Sunstein, 2002). Nevertheless, benefit-cost analysis is based on individuals’ VSL. Hence, if individuals’ VSL embody ambiguity aversion, policy choices should somehow reflect individuals’ ambiguity aversion as well. This raises the following important questions for policy-making. Are the observed individual VSL estimates (usually ranging from $1 to $10 million) reflective of any form of ambiguity aversion? Should we use these VSL estimates to compute the social benefits of reducing ambiguous risks?

There is little rationale to answer positively to these questions. In effect, we observe that VSL estimates are usually obtained either from revealed preferences studies, most often using wage risk differential studies or road safety studies, or from stated preferences, most often using contingent valuation studies (Viscusi and Aldy, 2003; Dionne and Lanoie, 2004; Andersson and Treich, 2007). However, occupational or road safety risks may arguably involve in average less ambiguity compared to most regulated environmental risks. Moreover, contingent valuation studies usually present objective probabilities to respondents, and thus

\(^5\) Relatedly, Adler (2007) discusses for instance what he calls the “de minimis” risk. A “de minimis” risk is a risk cut-off, such as the incremental $1 \times 10^{-6}$ lifetime cancer risk for air pollutants, or the 100-year-flood or the 475-year-earthquake for natural hazards. Adler argues that risk cut-offs are usually instrumental for defining policy objectives. Moreover, a recurrent problem is that risk cut-offs is usually extremely low. As a result, it may lead to target extremely high safety standards, without a careful consideration of the economic costs of these standards.
do not account for ambiguity either. Consequently, estimated VSL usually do not seem to capture an “ambiguity premium”. This may lead to under-estimate the values of the VSL that are applied to the reduction of ambiguous risks. The objective of the paper is, in a sense, to study this last argument.

3. The Value of a Statistical Life Model

Let us first introduce the VSL concept through an example. Consider a society composed of 100,000 identical individuals. They each face a (non-ambiguous) annual mortality risk of 100 in a 100,000. A public prevention program can reduce this risk from 100 to 80 expected fatalities. Moreover, it is known that each individual is willing to pay $500 for benefiting from this risk reduction program. In this example, the VSL would be equal to $2.5 million. Indeed one could collect $50 million in this society to save 20 statistical lives, hence a $2.5 million per statistical life. Observe also the VSL is equal to the individual change in wealth ($500) divided by the individual change in risk (20/100,000). Hence the VSL captures the trade-off between a change in wealth and a change in mortality risks.

We now introduce the standard static VSL model. An individual maximizes a (state-dependent) subjective expected utility given by

\[ V = (1 - p_0)u(w) + p_0 v(w) \]  \hspace{1cm} (1)

where \( p_0 \geq 0 \) is the initial subjective probability of dying, or the baseline mortality risk, \( u(\cdot) \) is the utility of wealth if the individual survives the period, and \( v(\cdot) \) is the utility of wealth if the individual dies, that is, the utility of a bequest. We assume that \( u > v \), with \( u \) (resp. \( v \)) strictly increasing (resp. weakly increasing). This model was introduced by Drèze (1962), Jones-Lee (1974) and Weinstein et al. (1980) and has been commonly used in the literature on the economic valuation of risks to health to health and life (e.g., Viscusi and Aldy, 2003).

Theoretically, the VSL is defined by the marginal rate of substitution between wealth and death probability. It thus captures this tradeoff between a change in wealth and a change in mortality risks. Assuming that \( u \) and \( v \) are differentiable, we thus get the VSL by a total differentiation of (1):\(^6\)

\[ \text{VSL}_0 = \frac{dV}{dp_0} = \frac{u(w) - v(w)}{(1 - p_0)u'(w) + p_0 v'(w)}. \]  \hspace{1cm} (2)

Observe that the VSL may vary across individuals since it depends on \( w, p_0 \) and also on the shape of the utility functions through \( u \) and \( v \).

In this model, the VSL increases with an increase in the baseline mortality risk (see, e.g., Weinstein et al., 1980). Indeed in (2) an increase in \( p_0 \) reduces the value of the

\(^6\) The VSL may also be viewed as the first-order approximation of the willingness to pay for a mortality risk reduction. Indeed, let the willingness to pay \( C_0(z) \) is defined by

\[ (1 - p_0 + z)u(w - C_0(z)) + (p_0 - z)v(w - C_0(z)) = (1 - p_0)u(w) + p_0 v(w) \] in which \( z \) is the risk reduction. We obviously get \( \text{VSL} = C_0'(0) \).
denominator (since $u' \geq v'$), and thus increases the VSL. This effect has been coined the “dead-anyway effect” (Pratt and Zeckhauser, 1996; see also Breyer and Felder, 2005). Notice that the dead-anyway effect can be potentially important in magnitude for large baseline mortality risk $p_0$. Indeed, assuming there is no bequest motive ($v = 0$), the VSL in (2) tends to infinity when $p_0$ tends to one. Intuitively, an individual facing a large total probability of death has little incentive to limit his spending on mortality risk reduction since he is unlikely to survive and thus to have other opportunities for consumption.

4. The Effect of Ambiguity Aversion

In the example above the annual baseline mortality risk was unambiguous and equal to 100 in 100,000. Suppose now that the baseline mortality risk is ambiguous (but of the same magnitude). Suppose for instance that, with equal probability, either 50 individuals or 150 individuals are expected to die out of the 100,000 individuals in this society. How does this ambiguity over the baseline mortality risk affect the VSL? Clearly, under standard subjective expected utility, the VSL is not affected by ambiguity as the (expected) baseline mortality risk equals 100 in 100,000 in both situations. But what does happen if the individual is ambiguity averse? Does it change the VSL, and consequently does it change the social benefits that should be imputed to this prevention program?

To study analytically this question, we consider the Klibanoff et al. (2005)’s model of ambiguity attitude. Formally, and adapting the model above, the decision maker’s expected utility is now written

$$W = \phi^{-1}\{E\{ (1 - \tilde{p})u(w) + \tilde{p}v(w) \}\}$$

in which $\tilde{p} = p_0 + \tilde{\epsilon}$ is a positive random variable that represents the ambiguity over the baseline mortality risk, and $E$ denotes the expectation operator over the random variable $\tilde{\epsilon}$, with $E\tilde{\epsilon} = 0$.

The novelty in model (3) compared to model (1) is the introduction of the increasing function $\phi$ which captures the attitude towards ambiguity. More precisely, the decision-maker has ambiguity averse (seeking) preferences if and only if $\phi$ is concave (convex), as shown by Klibanoff et al. (2005). In this framework, similar to the usual financial risk aversion which is captured by the concavity of the utility function $u$, $\phi$ captures the attitude towards ambiguity. Assuming differentiability, $\phi'' < 0$ thus represents strict ambiguity aversion. Two extreme special cases of this model are constant ambiguity aversion, $\phi(x) = (1 - \exp(-\alpha x))/\alpha$, and ambiguity neutrality, $\phi(x) = x$. Klibanoff et al. (2005)’s model yields Gilboa and Schmeidler (1989)’s well-known maxmin ambiguity model as a limiting case for infinitely ambiguity averse decision makers ($\alpha \to \infty$). But, in contrast to the Gilboa and Schmeidler’s framework, the Klibanoff et al.’s model distinguishes ambiguity (over a set of probability distributions) and ambiguity attitude.

Klibanoff et al. (2005) thus assumes a unique second order probability over a set of given first order probabilities, but relaxes the reduction axiom and weights the probabilities nonlinearly (see also Segal, 1987).
Observe that under a concave $\phi$, utility $W$ is reduced in the presence of ambiguity over baseline mortality risks. An implication of this observation is that the willingness to pay $C$ to eliminate the mortality risk, defined by $u(w-C) = W$, is always higher under ambiguity aversion than under ambiguity neutrality. However, this result does not permit to conclude as to the effect of ambiguity for infinitesimally small mortality risk changes, as usually considered in the VSL literature.

The natural extension of the VSL under ambiguity and ambiguity aversion is obtained by a total differentiation of (3):\(^8\)

\[
\text{VSL}_1 = \frac{dw}{dp_0} = \frac{(u(w) - v(w))E\phi'(1 - \tilde{p})u(w) + \tilde{p}v(w)}{E((1 - \tilde{p})u'(w) + \tilde{p}v'(w))}\phi'(1 - \tilde{p})u(w) + \tilde{p}v(w)}
\]

(4)

Notice that, although we assume that there is ambiguity over baseline risks, there is no ambiguity about the (infinitely small) risk change faced by the individual.\(^9\) Notice also that without ambiguity aversion (or under expected utility), that is under $\phi'$ constant, we would get

\[
\frac{u(w) - v(w)}{E((1 - \tilde{p})u'(w) + \tilde{p}v'(w))}
\]

(5)

which is strictly equal to the expression in (2) since $E\tilde{p} = p_0$.

Our objective is to use this framework to examine the effect of ambiguity aversion on the VSL. Comparing (4) and (5), it is immediate that the VSL is higher with ambiguity aversion than with ambiguity neutrality if and only if

\[
E((1 - \tilde{p})u'(w) + \tilde{p}v'(w)) \times E\phi'(1 - \tilde{p})u(w) + \tilde{p}v(w)) \geq E((1 - \tilde{p})u'(w) + \tilde{p}v'(w))(\phi'(1 - \tilde{p})u(w) + \tilde{p}v(w))
\]

Straightforward computations then show that this inequality holds true if and only if $COV((1 - \tilde{p})u'(w) + \tilde{p}v'(w), \phi'(1 - \tilde{p})u(w) + \tilde{p}v(w)) \leq 0$. That is, using the well-known covariance rule,\(^10\) if and only if $\phi'$ is decreasing, assuming $u' \geq v'$. Hence, provided death reduces the marginal utility of wealth, ambiguity aversion always reduces the VSL.\(^11\) This is

---

\(^8\) Defining again the willingness to pay $C_1(z)$ for a risk-reduction $z$ by $E\phi((1 - \tilde{p} + z)u(w - C_1(z)) + (\tilde{p} - z)v(w - C_1(z))) = E\phi((1 - \tilde{p})u(w) + \tilde{p}v(w))$, it is easy to retrieve $\text{VSL}_1 = C_1'(0)$.

\(^9\) Alternatively, one could attempt to introduce ambiguity over the risk change (as opposed to over the baseline risk). Consider for instance the willingness to pay $C(z)$ defined by the following equality: $\phi^{-1}[E\phi((1 - \tilde{p} + \tilde{k}z)u(w - C(z)) + (\tilde{p} - \tilde{k}z)v(w - C(z)))] = (1 - \tilde{p})u(w) + \tilde{p}v(w)$. Notice that ambiguity (and the expectation operator) is over the random variable $\tilde{k}$. There is thus ambiguity over the risk change $\tilde{k}$, and not about the baseline mortality risk. In that case, it is easy to see that ambiguity aversion reduces, and not increases, the willingness to pay $C(z)$. However, ambiguity aversion has no effect on the approximated willingness to pay for a small risk change, that is $C'(0)$.

\(^10\) Assume that $f(p)$ is increasing in $p$. The covariance rule states that $COV(f(\tilde{p}), g(\tilde{p})) \leq 0$, that is $Ef(\tilde{p})Eg(\bar{p}) \geq Ef(\bar{p})g(\tilde{p})$, for all $\tilde{p}$ if and only if $g(p)$ is decreasing. See, e.g., Kimball (1951).

\(^11\) Notice that we compared two different individuals, an ambiguity averse individual and an ambiguity neutral individual, keeping the level of ambiguity the same. Consider now only an ambiguity averse individual, but
the main result of the paper. Notice that this result directly extends to an increase in ambiguity aversion in the sense defined by Klibanoff et al. (2005): a “more concave” $\phi$ always leads to increase VSL (see the appendix for the derivation of this result).

5. An Intuition based on the Dead-anyway Effect

As we said in the introduction, the intuition for the result is based on the dead-anyway effect. To see this, let us consider for simplicity a discrete distribution of baseline mortality risks. Assume so that the random baseline mortality risk $\tilde{p}$ takes a value $p_i$ with a probability $q_i$, with $\sum_{i=1}^n q_i = 1$ (remember also that $E\tilde{p} = \sum_{i=1}^n q_i p_i = p_0$). Let us then rewrite the VSL under ambiguity aversion

$$\text{VSL}_i = \frac{(u(w) - v(w))}{E((1 - \tilde{p})u'(w) + \tilde{p}v'(w))}$$

(6)

where the operator $\hat{E}$ is taken with respect to the new probability distribution of the baseline mortality risks given by:

$$\hat{q}_i = \frac{q_i \phi'(1 - p_i)u(w) + p_i v(w)}{E\phi'(1 - \tilde{p})u(w) + \tilde{p}v(w))}$$

for all $i = 1, ..., n$ (7)

Then, using the covariance rule again for discrete random variables, it is straightforward to show that $\hat{E}\tilde{p} \geq E\tilde{p}$ if and only if $\phi$ is concave.

Ambiguity over baseline mortality risks thus leads the ambiguity averse decision maker to behave in a way that is consistent with a perceived increase of a baseline mortality risk, from $E\tilde{p}$ to $\hat{E}\tilde{p}$. In other words, $\hat{E}\tilde{p}$ is the certain baseline mortality risk so that the decision maker has the same VSL as in the ambiguous situation. Technically, a new probability $\hat{q}_i$ is associated to each baseline mortality risk $p_i$, $i = 1, ..., n$. Compared to the initial probability $q_i$, observe that the new probability $\hat{q}_i$ is actually “weighted” by the quantity $\omega_i = \phi'(1 - p_i)u(w) + p_i v(w)) / E\phi'(1 - \tilde{p})u(w) + \tilde{p}v(w))$. Importantly, notice that this weight is larger the larger $\phi'(1 - p_i)u(w) + p_i v(w)$ is, that is, the larger $p_i$ is. In other words, the new probability $\hat{q}_i$ attributes respectively more weight to larger baseline mortality risks.

This intuition helps understand why the condition $u' \geq v'$ is instrumental for ambiguity aversion to have a positive effect on VSL. Indeed the “dead anyway” effect rests on the assumption that wealth has a higher marginal value if alive than dead. This assumption seems sensible, and is usually accepted without much discussion. Nevertheless, this suggests that the same result cannot usually be obtained for pure financial risks. Indeed, under risk-
averse preferences, the marginal utility is higher in the bad state than in the good state. This would basically reverse the result we obtained. Along these lines, the effect of ambiguity over baseline risks to health (nonfatal risks) would also depend on how health status affects the marginal utility of consumption. Indeed if health status increases the marginal utility of wealth (see, e.g., Viscusi and Evans, 1994), then our result carries over.

6. Equivalent Certain Baseline Mortality Risks

In the introduction, we mentioned the survey study developed by Viscusi et al. (1991). This study asked respondents to state the certain baseline mortality risk they would judge as equivalent to the ambiguous risk they face. We coin it the “utility-equivalent” certain baseline mortality risk. Formally, let us denote $\bar{p}$ this utility-equivalent baseline mortality risk; it is given by the following equation:

$$ (1 - \bar{p})u(w) + \bar{p}v(w) = \phi^{-1}\{E\phi\{(1 - \bar{p})u(w) + \bar{p}v(w)\}\} $$ (8)

One may then wonder how $\bar{p}$ compares to the “VSL-equivalent” baseline mortality risk, denoted now $\hat{p} = \hat{E}\hat{p}$, that we derived in the previous section. After simple manipulations, it is easy to see that

$$ (1 - \hat{p})u(w) + \hat{p}v(w) = \frac{E((1 - \hat{p})u(w) + \hat{p}v(w))\phi'\{(1 - \hat{p})u(w) + \hat{p}v(w)\}}{E\phi'\{(1 - \hat{p})u(w) + \hat{p}v(w)\}} $$ (9)

so that we get $\hat{p} \geq \bar{p}$ if and only

$$ \phi^{-1}\{E\phi\{(1 - \bar{p})u(w) + \bar{p}v(w)\}\} \geq \frac{E((1 - \hat{p})u(w) + \hat{p}v(w))\phi'\{(1 - \hat{p})u(w) + \hat{p}v(w)\}}{E\phi'\{(1 - \hat{p})u(w) + \hat{p}v(w)\}} $$ (10)

for all $\bar{p}$, $u$ and $v$. This last inequality is equivalent to

$$ \phi^{-1}\{E\phi\{\bar{x}\}\} \geq \frac{E\hat{x}\phi'\{\bar{x}\}}{E\phi'\{\bar{x}\}} \text{ for all } \bar{x}. $$ (11)

This inequality is in fact always true under $\phi$ concave. To see this, we use a result derived in Watt (2008). Let the following function $g(\lambda) = E\phi\{\lambda\bar{x} + (1 - \lambda)\bar{x}\}$ with $\bar{x}$ being the certainty equivalent of $\bar{x}$, or $\phi\{\bar{x}\} = E\phi\{\bar{x}\}$. Since $g(\lambda)$ is concave in $\lambda$, we have $g(\lambda) \geq \lambda g(0) + (1 - \lambda)g(1) = g(0)$ for any $\lambda \in [0,1]$. Thus we must have $g'(0) \geq 0$, which is equivalent to inequality (11).

We have thus shown that the utility-equivalent certain baseline mortality risk is always lower than the VSL-equivalent certain baseline mortality risk. An illustration of this result is

---

12 Let $v(w) = u(w - L)$ and assume that the financial loss $L$ is positive. Then $v'(w)$ is larger than $u'(w)$ under $u$ concave. Our result then tells us that the willingness to pay for a small reduction in the (ambiguous) probability of loss is reduced, and not increased, under ambiguity aversion. Our intuition for this (maybe surprising) result is that a higher willingness to pay would further decrease the final wealth in the case of loss. This effect is disliked by risk-avers, and has precisely more weight under ambiguity aversion.
the following. Suppose for instance that, as in the treatment 2 reported by Viscusi et al. (1991), individuals judge a certain 191 in 1 million risk to be strictly equivalent to an ambiguous risk of either 110 or 240 in 1 million risk. Then, the theoretical result just derived predicts that those individuals are expected to have a VSL in the ambiguous risk case to be at least greater than the VSL they would have for the certain 191 in a million baseline mortality risk, even if the mean of the baseline risk is equal to 175.

We have thus shown that ambiguity aversion increases the VSL. Also, we have obtained a lower bound for the VSL in the ambiguous risk case. This suggests that ambiguity aversion may possibly have an important effect on the VSL. However, in the next section, we will suggest using a numerical illustration that the effect of ambiguity aversion should be in general rather limited.

7. A Numerical Illustration

The few papers about ambiguity over risks to life and health mentioned in the introduction have not elicited monetary equivalents. The only study to do so (that we are aware of) is a survey study mentioned in Shogren (2005) about a food-borne pathogen, Salmonella. This survey study compares monetary equivalents for risk elimination under non-ambiguous and ambiguous probabilities scenarios. Interestingly, Shogren (2005) reports that “mean willingness-to-pay responses were higher for ambiguous versus unambiguous scenarios for all probabilities for food safety, but these differences were not significantly different. This survey has provided evidence that people prefer unambiguous risks for food safety, but not enough to generate a significant difference” (Shogren, 2005, p. 125-26).

We will now briefly illustrate our theoretic result using some of the figures reported in Shogren (2005). We also use some arbitrary utility functions and parameters. We first assume a constant relative risk aversion utility function \( u(w) = w^{1-\gamma} (1-\gamma)^{-1} \) with \( \gamma \in [0,1] \), \(^{13}\) with no bequest motive, that is \( v = 0 \). In that case, the VSL is simply equal to \( w(1-\gamma)^{-1}(1-p_0)^{-1} \). To illustrate, if we take a square root utility function \( u(w) = \sqrt{w} \) and a lifetime wealth of $1 million, the VSL equals $2 million for a zero baseline mortality risk. Notice also that the VSL increases nonlinearly with this baseline mortality risk. As we said above, the VSL tends to infinity when \( p_0 \) tends to 1, truly the “dead anyway” effect. We thus expect the effect of ambiguity aversion to strongly depend on where the baseline mortality risk is located.\(^{14}\) Finally, we will assume a constant ambiguity aversion, that is \( \phi(x) = (1-\exp(-\alpha x))/\alpha \) for \( \alpha > 0 \) (and \( \phi(x) = x \) for \( \alpha = 0 \)).

We further assume, as in one treatment of the study reported in Shogren (2005), that the baseline mortality risk is either equal to \( p_1 = 1/666 \) or to \( p_2 = 1/2000 \) with equal probability. We thus have \( \hat{E}p = 1/1000 \). With these values, we obtain for instance \( \hat{E}p = 1.23/1000 \) for \( \alpha = 0.5 \). Moreover, \( \hat{E}p \) tends asymptotically to 1/666 for \( \alpha \) large;

---

\(^{13}\) Parameter \( \gamma \) less than 1 guarantees \( u > v \).

\(^{14}\) Using the expression \( w(1-\gamma)^{-1}(1-p_0)^{-1} \) for the VSL, notice that the elasticity of the VSL with respect to the baseline mortality risk is equal to \( p_0/(1-p_0) \). The VSL is thus quite sensitive to a change in \( p_0 \) only for high values of \( p_0 \).
namely, under extreme ambiguity aversion the decision maker will behave as if he would face the “worst-case” baseline mortality risk. Interestingly, in this example the impact of ambiguity aversion on the VSL is always modest, even under extreme ambiguity aversion. Indeed, the change from $E\tilde{p} = 1/1000$ to $E\hat{p} = 1/666$ leads to an increase of the VSL (of about $2$ million) by a mere $1000$. Notice that this numerical illustration is somehow consistent with the results of a modest effect of ambiguous probabilities reported in Shogren (2005).

In contrast, assume that the baseline mortality risk is either very high and equal to $p_1 = 1/2$ (50% chance of dying) with a small $1/999$ probability, or equal to $p_2 = 1/2000$. Notice that we still have $E\tilde{p} = 1/1000$. For these new values, we obtain that $E\hat{p}$ is (almost) equal to $1/2$ for $\alpha = 0.5$, and so is the case for higher values of $\alpha$. Hence, the effect is similar to an increase from $E\tilde{p} = 1/1000$ to $E\hat{p} = 1/2$. The VSL thus almost doubles due to an ambiguity aversion effect. Consequently, the possibility of a high baseline mortality risk, even if this possibility is very unlikely, may significantly increase the VSL.

This numerical exercise based on arbitrary functional forms and arbitrary parameters suggests that the effect of ambiguity aversion is usually modest, unless there is a possibility of an extremely high baseline mortality risk. We believe that this insight should carry over in most “regular” numerical exercises. Indeed, the usual modest effect of ambiguity aversion on the VSL is due to the limited impact of the baseline mortality risk on the VSL in general. In other words, ambiguity aversion is expected to have usually a modest impact on the VSL because the dead-anyway effect is usually small (unless the probability of death is extraordinary high). We are thus tempted to conclude that ambiguity aversion can hardly justify a priori the very high implicit cost per life saved of some public environmental programs (e.g., several hundreds of millions dollars), even accounting for the fact that these public programs can reduce ambiguous risks.

8. Differentiated Risk Changes

Let us assume now that there are $n$ equally likely baseline mortality risks $p_i$, $i=1,...,n$, with $p_1 \geq p_2 \geq ... \geq p_n$. The decision maker’s utility is then given by

$$
\phi^{-1}\left\{ \frac{1}{n} \sum_{i=1}^{n} \phi\{(1-p_i)u(w) + p_i v(w)\} \right\}
$$

(12)

The previous analysis assumed that a (infinitesimal) risk change applies uniformly to all $p_i$s. This may be viewed as somehow restrictive, as risk changes might possibly affect differently each baseline mortality risk $p_i$. We call a “differentiated risk change”, a change in risk that is different across the $p_i$s. An example of a differentiated risk change is a prevention measure such that the risk reduction is strictly positive when $\tilde{p}$ is equal to $p_1$, and 0 otherwise. Namely, the prevention measure is only efficient in the worst-case scenario leading to the maximal baseline mortality risk.\footnote{Another example is when an intervention reduces the risk by a constant fraction (e.g., if it reduces exposure to a risk by half).}
Notice first that, within the expected utility framework, only the change in the expected baseline mortality risk matters. Hence, a similar risk change applied differently to a low or to a large baseline mortality risk would have exactly the same monetary-equivalent value to the decision-maker. However, this need not be the case under ambiguity aversion. Technically, this is due to the fact that the objective in (12) is non-linear in the \( p_i \)s under ambiguity aversion.

To further study this problem, we denote \( VSL_i \) the marginal rate of substitution between wealth and a change in the baseline mortality risk \( p_i \), to get:

\[
VSL_i = \frac{\partial w}{\partial p_i} = \frac{(u(w) - v(w))\phi'(1-p_i)u(w) + p_i v(w)}{\sum_i((1-p_i)u'(w) + p_i v'(w))(\phi'(1-p_i)u(w) + p_i v(w))} \quad \text{for } i = 1,\ldots,n \tag{13}
\]

The quantity \( VSL_i \) should be interpreted as the monetary-equivalent value associated to an infinitesimal change in risk contingent to the baseline mortality risk \( p_i \). The expression of \( VSL_i \) is useful to understand how the decision maker would want the risk reduction to be differentiated, and to which extent this differentiation depends on ambiguity aversion. Indeed, it is immediate to obtain that the difference \( VSL_j - VSL_i \) has the sign of \( \phi'(1-p_j)u(w) + p_j v(w) - \phi'(1-p_i)u(w) + p_i v(w) \), and is thus positive when \( p_j \geq p_i \) under ambiguity aversion.

Consequently, ambiguity aversion unsurprisingly leads the decision maker to value more a risk reduction contingent on the highest baseline mortality risk \( p_1 \), rather than any similar risk reduction contingent to another baseline mortality risk. Ambiguity aversion may thus rationalize a focus on the worst-case scenario in public prevention programs, even assuming a mild ambiguity aversion of the decision-maker. Notice, however, that we have only discussed the benefit side of prevention measures. A full examination of how the decision maker would optimally select the “differentiation” of risk changes must account for the relative cost of these measures.

9. A Self-protection Model

We have studied so far how ambiguity aversion affects the monetary-equivalent value of some prevention measures. The objective of this section is to study how ambiguity aversion could affect individual prevention choices. To do so, we need to be more general about how prevention efforts lead to a risk reduction of the baseline mortality risks. We capture this by introducing the function \( p_i(e) \), that represents how the baseline mortality risk varies with the prevention effort. We will assume that more prevention effort decreases the baseline mortality risk \( p_i'(e) < 0 \), at a decreasing rate \( p_i''(e) \geq 0 \). The objective of the decision maker is to choose the prevention effort \( e \) to maximize

---

16 The notations are a bit loose here. Differentiating with respect to \( p_i \) means differentiating with respect to \( p_0 \) in \( p_i = p_0 + \epsilon_i \), keeping all \( p_0s \) constant in all \( p_j, j \neq i \).
This a simple state-dependent self-protection model with ambiguity aversion. It is easy to see that \( g(e) = (1 - p_i(e))u(w - e) + p_i(e)v(w - e) \) is concave in \( e \) under the assumption that \( u \) and \( v \) are concave, which implies that the program in (14) is concave as well under this assumption. The first order condition characterizing the optimal \( e \) can be written as follows

\[
\frac{(u(w-e) - v(w-e))\sum_{i=1}^{n} q_i (-p_i'(e))\phi'(g_i(e))}{\sum_{i=1}^{n} q_i ((1 - p_i(e))u'(w-e) + p_i(e)v'(w-e))\phi'(g_i(e))} = 1
\]

(15)

It is easy to see then that ambiguity aversion raises the optimal prevention effort compared to ambiguity neutrality if and only if the left hand side of (15) is higher than the same expression assuming \( \phi' \) constant.

In order to compare with the results obtained before, assume that the baseline mortality risk has an additive form, given by \( p_i(e) = p_i - r(e) \) (with \( 0 \leq r(e) \leq p_i \) for \( i = 1, ..., n \)). Then, it is straightforward (using similar manipulations as above including the use of the covariance rule) to prove that ambiguity aversion always increases the prevention effort, under \( u' \geq v' \). This result is not a surprise. If an individual who is ambiguity averse does always value more a marginal unit of prevention (implying a higher VSL) than an ambiguity neutral individual, she should also select a higher level of prevention.

However, the result that ambiguity aversion always increases the prevention effort is not general. Indeed, the comparative static analysis depends on the form of \( p_i(e) \). To see this, assume for instance that the baseline mortality risk has a specific multiplicative form, given by \( p_i(e) = 1 - p_i s(e) \) (with \( 0 \leq p_i \leq 1/s(e) \) for \( i = 1, ..., n \)), and assume also that there is no bequest motive \( v = 0 \). Then, it is easy to show that (12) simply reduces to

\[
\frac{u(w-e)s'(e)}{u'(w-e)s(e)} = 1
\]

(16)

Consequently, ambiguity aversion has strictly no effect on the prevention effort in this case. Our intuition for this result is that an increase in the prevention effort, although reducing the expected baseline mortality risk, also increases the level of ambiguity faced by the agent. Formally, an increase in \( e \) increases the survival probability \( p_i s(e) \) (remember that \( s'(e) > 0 \) by assumption), which is itself ambiguous. An increase in \( e \) thus operates as an increase in the spread of the ambiguity support in this case. In contrast, when the baseline mortality risk is additive a change in \( e \) does not affect the level of ambiguity.

The analysis of this self-protection model thus indicates that the VSL model that we have studied before captures solely an aspect of the relationship between prevention motives and ambiguity aversion. In particular, we have considered a specific model in which the initial situation is ambiguous but the effect of the actions of prevention is not ambiguous (see also the discussion in the footnote 8). It is thus important to keep in mind that ambiguity aversion
may not systematically increase the value of a public prevention program that decreases an ambiguous risk if the effects of this program are themselves ambiguous.

10. An Alternative Characterization of Ambiguity Aversion

The above analysis has considered the Klibanoff et al. (2005)’s theory of ambiguity aversion that encompasses most existing ambiguity theories, introduces a measure of ambiguity aversion, and achieves a separation between ambiguity and ambiguity attitude. The recent Gadjos et al. (2007)’s theory of ambiguity also shares these fine properties, but is based on another axiomatics. The purpose of this section is to show that our results extend to this alternative theory of ambiguity.

Under Gadjos et al. (2007)’s theory, the decision maker’s utility can be written

\[ \alpha \left( (1 - \alpha)(E((1 - \hat{p})u(w) + \hat{p}v(w))) + \alpha((1 - p_1)u(w) + p_1v(w)) \right) \]

in which \( \alpha \) is the parameter of ambiguity aversion (or “imprecision aversion”) and \( p_1 \) still denotes the highest baseline mortality risk. This framework, already suggested by Ellsberg (1961), thus considers a linear combination of the minimum of expected utility and of the expected utility.

Using this formulation, it is then easy to show that (remember that \( \hat{p} \equiv p_0 + \tilde{e} \))

\[ VSL_2 = \frac{dw}{dp_0} = \frac{u(w) - v(w)}{(1 - \alpha)(E((1 - \hat{p})u'(w) + \hat{p}v'(w))) + \alpha((1 - p_1)u'(w) + p_1v'(w))} \]

which is also equal to

\[ VSL_2 = \frac{u(w) - v(w)}{(1 - p^*)u'(w) - p^*v'(w)} \]

with \( p^* = (1 - \alpha)E\tilde{p} + \alpha p_1 \). Notice first that the equivalent certain baseline mortality risk \( p^* \) does not depend on individual characteristics (utility, wealth), except on the level of ambiguity aversion \( \alpha \).

Most notably, it is then immediate that \( VSL_2 \) increases with the parameter of ambiguity aversion \( \alpha \), provided \( u' \geq v' \). Consequently, in this framework as well, ambiguity aversion raises the VSL as soon as the marginal utility of wealth is higher if alive than dead. Moreover, the intuition is similar since ambiguity aversion effect also operates as the “dead anyway” effect, through a perceived change of the baseline risk from \( E\tilde{p} (= p_0) \) to \( p^* \).

Finally, it is immediate to see that there is no difference between the utility-equivalent baseline mortality risk and the VSL-equivalent baseline mortality risk, unlike in the previous framework based on Klibanoff et al. (2005)’s theory of ambiguity aversion. This observation may suggest a simple way to discriminate empirically between the two theories of ambiguity that we have considered in this paper in the context of risks to life and health.
11. Risk Preferences and the VSL

This paper is not the first to analyze the theoretic effect of risk preferences on the VSL. Eeckhoudt and Hammitt (2004) study the effect of financial risk aversion and Bleichrodt and Eeckhoudt (2006) that of decision weights consistent with rank-dependent expected utility model. Interestingly, these papers show that financial risk aversion and decision weights usually have an ambiguous effect on the VSL. In contrast, we have shown that ambiguity aversion always increases the VSL. A simple implication of this observation is that effect of ambiguity aversion need not reinforce that of risk aversion (Gollier, 2006) to understand the tradeoff between money and mortality risks.17

Studying the effect of risk preferences on the VSL is relevant both for revealed and stated preferences approach. It may permit for instance to better understand the self-selection bias induced by the revealed preferences approach when individuals make risk-exposure choices (e.g., living in a specific polluted area) posing ambiguous risks to life and health. Also, the VSL obtained from survey studies may be sensitive to the pieces of information that are delivered to participants in surveys. As shown by Viscusi et al. (1991), the communication of ambiguous risk information may have an effect on participants’ responses. As a result, it may be interesting to estimate the effect of ambiguity aversion on the VSL obtained in future survey studies. More generally, it is important to better understand the economic consequences related to the behavioral responses of information policies about ambiguous risks.

A more fundamental question arises, however, when one compares the effect of risk aversion and that of ambiguity aversion. While risk aversion has long been considered as a part of the welfare of individuals, the same is not necessarily true for ambiguity aversion. It is known that some theories of ambiguity aversion may lead to time-inconsistent choices, to some conceptual difficulties in beliefs’ updating and to a negative value of information. This may not be a problem for the descriptive power of ambiguity aversion theories, but this raises some legitimate concerns for benefit-cost analysis and more generally for welfare analysis.

A related policy argument is that the primary objective should be the reduction of the expected number of deaths. Yet, policy-makers could certainly save more lives by targeting familiar risks compared to ambiguous risks. Hence allowing for an ambiguity premium in policy-making may lead to a “statistical murder” (Graham, 1995). A classical counter-argument however is that what should matter is the additional welfare gain associated with the policy, even if this policy does not maximize the total number of lives saved. It is indeed perfectly reasonable to argue that reducing the fear associated with ambiguous risks has a value for individuals, and that this value should be reflected in policy-making. Interestingly, Camerer et al. (2007), using the technique of fMRI (functional magnetic resonance imaging), show that some subjects’ brain areas like the amygdala are more active under the ambiguity conditions in their experiment. They go on to notice that the “amygdala has been specifically implicated in processing information related to fear” (Camerer et al., 2007, p. 131).

12. Conclusion

17 We must add, however, that the classical Pratt (1964)’s concept of comparative risk aversion is not clear-cut under state-dependent preferences (Karni, 1983).
Many mortality risks are ambiguous. The sources of ambiguity are multiple. They may include scientific uncertainty, problems of communication about and credibility, or lack of information about individual heterogeneous risk exposures and differences in susceptibility (e.g., genetics). There is a need to better understand the economic implications of ambiguity aversion. In particular, ambiguity aversion may affect the valuation of health and mortality risks changes. In turn, it may have an effect on applied benefit-cost analysis, and on the welfare analysis of public risk policies.

We have demonstrated that ambiguity aversion increases the value of a statistical life. Ambiguity aversion may thus possibly justify the observed “over-regulation” of some environmental risks. But how much “over-regulation” is justified? We urge to obtain better estimates of ambiguity-related effects to better answer this question. Interestingly, our first numerical analysis suggests that the effect of ambiguity aversion should be small. This paper thus does not justify much over-regulation of ambiguous risks. It rather supports the view that we should “debias” risk regulatory decisions from too much ambiguity aversion. But, clearly, more theoretical and empirical research is needed on this topic in order to be more confident in this policy recommendation. More fundamentally, the welfare implications of the effects of ambiguity aversion should be discussed with great caution.
Appendix: Comparative Ambiguity Aversion

The main result derived in the paper compares an ambiguity averse to an ambiguity neutral individual (or an expected utility maximizer). This appendix shows that this result extends to the notion of a comparison with a “more ambiguity averse” decision maker.

Consider two agents, 1 and 2. Assume that they share the same underlying state-dependent utility functions, the same wealth and the same subjective beliefs. Following Klibanoff et al. (2005), agent 2 is said to be more ambiguity averse than an agent 1 if and only there exists an increasing and concave function $T$ so that $T(\phi_1) = \phi_2$ in the relevant domain.

Assuming $T$ twice differentiable and concave, and using (4), we are done if we show that

$$VSL_1 = \frac{(u(w) - v(w))E\phi_1'}{E((1 - \tilde{p})u'(w) + \tilde{p}v'(w))(\phi_1')}$$

is lower than

$$VSL_2 = \frac{(u(w) - v(w))E\phi_1' T'\phi_1}{E((1 - \tilde{p})u'(w) + \tilde{p}v'(w))(\phi_1' T'\phi_1)}$$

where $\phi_1'$ stands for $(1 - \tilde{p})u'(w) + \tilde{p}v'(w)$. But, using (7), notice that we can write

$$VSL_1 = \frac{(u(w) - v(w))}{E((1 - \tilde{p})u'(w) + \tilde{p}v'(w))}$$

and

$$VSL_2 = \frac{(u(w) - v(w))\hat{E}T'\phi_1}{E((1 - \tilde{p})u'(w) + \tilde{p}v'(w))(T'\phi_1)}$$

Moreover, notice then that comparing eqn. (A3) and eqn. (A4) is similar to comparing eqn. (5) and eqn. (4) where $E$ is replaced by $\hat{E}$ and $\phi'$ is replaced by $T'(\phi_1')$. We are thus done if and only if $T'(\phi_1')$ is decreasing in its argument provided $u' \geq v'$. And this is precisely the case if and only if $T$ is concave. This shows that $T$ concave is sufficient for the result. The necessity part of the proof is straightforward.
References

d’Albis, Hippolyte and Emmanuel Thibault. 2007. Ambiguity aversion and uncertain
longevity. Mimeo, GREMAQ, Toulouse School of Economics.
Andersson, Henrik and Nicolas Treich. 2007. The value of a statistical life. The Handbook of
Transport Economics. Edite by André de Palma, Robin Lindsay, Emile Quinet and
Roger Vickerman. Forthcoming.
Bleichrodt, Han and Louis Eeckhoudt. 2006. Willingness to pay for reductions in health risks
when probabilities are distorted. Health Economics 15, 211-14.
dead-anyway effect revis(it)ed. The Geneva Papers on Risk and Insurance Theory 30,
41-55.
Camerer, Colin F., Bhatt, Meghana and Ming Hsu. 2007. Neuroeconomics: Illustrated by the
study of ambiguity aversion. Economics and Psychology. A Promising New Cross-
Disciplinary Field. Edited by Bruno S. Frey and Alois Slutzer. CESifo seminar series.
The MIT Press.
Chambers, Robert and Tigran Melkonyan. 2007. Pareto optimal trade in an uncertain world:
GMOs and the Precautionary Principle. American Journal of Agricultural Economics
89, 520-32.
time. Econometrica 70, 1403-1443.
Opérationnelle 6, 93-118.
Eeckhoudt Louis and James K. Hammitt. 2004, Does risk aversion increase the value of
75, 643–669.
Gajdos, Thibault, Takashi Hayashi, Jean-Marc Tallon and Jean-Christophe Vergnaud. 2007.
Journal of Mathematical Economics 18, 141–53.
Gollier, Christian. 2006. Does ambiguity aversion reinforce risk aversion ? Applications to
portfolio choices and asset pricing. Mimeo, LERNA, Toulouse School of Economics.
Graham, John D. 1995. Comparing opportunities to reduce health risks: Toxin control,
medicine, and injury prevention. National Center for Public Administration Policy
Report No. 192, Dallas.
Jones-Lee, Michael W. 1974. The value of changes in the probability of death or injury.
Karni, Edi. 1983. Risk aversion for state-dependent utility functions: Measurement and


