Double irreversibility and environmental policy design

Aude Pommeret* Fabien Prieur†

Abstract

The design of environmental policy typically takes place in a framework in which uncertainty over the future impact of pollution and two different kinds of irreversibilities interact. The first kind of irreversibility concerns the sunk cost of environmental degradation. The second kind of irreversibility is related to the sunk cost of environmental policy. Clearly, the two irreversibilities play in opposite direction: policy irreversibility leads to more pollution and less/later policy while environmental irreversibility generates less pollution and more/sooner policy. Using a real option approach and an infinite time horizon model, this paper simultaneously considers both irreversibilities. First, the model is developed by paying particular attention to the option values related to pollution and policy adoption. Next, solving the model in closed form provides solutions for both the optimal pollution level and the optimal environmental policy timing. Finally, the model is "calibrated" with the purpose of appraising which irreversibility has the prevailing effects and what is the total effect of both irreversibilities on pollution and policy design.

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*Corresponding and presenting author. University of Lausanne and IREGE-University of Savoie. Postal address: IMUS, Université de Savoie, Chemin de Bellevue, 74940 Annecy-le-Vieux, France. E-mail address: aude.pommeret@univ-savoie.fr.

†IREGE-University of Savoie. Postal address: IMUS, Université de Savoie, Chemin de Bellevue, 74940 Annecy-le-Vieux, France. E-mail address: fabien.prieur@univ-savoie.fr.
1 Introduction

Irreversibility has often been credited with inducing an immediate reduction in emissions of greenhouse gases (Chichilnisky and Heal (1993)). The intuition has been provided in the literature by Arrow and Fisher (1974), Henry (1974) and Freixas and Laffont (1984) amongst others. The kind of irreversibility this literature considers is the sunk cost of environmental degradation. For instance, some evidence suggests that if the global economy maintains the same modes of consumption and production then economic growth will be accompanied by global warming. In turn, the economy will bear the costs of irreversible degradation of natural processes, such as the thermohaline circulation (see Keller et al. (2004)).

The existing literature then shows that the possibility of getting better information in the future about the future benefits and/or costs of current actions should lead to current decisions which involve a lower level of irreversible commitment than without any possibility of getting better information. This would imply a higher current abatement of pollution.

Nevertheless, the design of environmental policy typically takes place in a framework in which uncertainty over the future impact of pollution and two different kinds of irreversibilities interact. Not only the pollution irreversibility matters. The second kind of irreversibility is related to the sunk cost of environmental policy. One may expect that these two irreversibilities do not play in the same direction. Indeed, due to uncertainty about environmental damages, an economy may prefer under-invest in pollution abatement, therefore avoiding the cost of acting. Alternatively, the same uncertainty may be an incentive, for the economy, to under-emittoday in order to prevent potential irreversible damages.

The analysis on how these effects interact is largely absent from the literature. The exceptions are Kolstad (1996) and Ulph and Ulph (1997). Both rather consider the irreversibility in the decision making process. In the presence of stock externalities, the basic idea behind this approach is the following. If one emits too much and finds that environmental damages are too high, she cannot instantaneously undo the damages by reducing the stock (with negative emissions). Theses studies appraise the consequences of both irreversibilities on the pollution level or on the design of environmental policy. But, they restrict the analysis to simple two periods models. Another interesting paper is Pindyck (2000) who concentrates on the policy irreversibility but leaves aside the other one.
This paper attempts to fill this gap by studying more precisely the effect of uncertainty and both irreversibilities on the optimal level of pollution in a particular economy. The decision to emit involves the following trade-off. On the one hand, emissions are viewed as a source of benefit since they enter the production process. On the other, they contribute to the accumulation of pollution which is costly for the economy. Uncertainty deals with the economic costs of pollution. If we refer to the issue of global warming, there exists uncertainty regarding the exact impact of a given concentration of GHG on temperature. In addition, for a given rise in temperature, it is difficult to give precise estimations of the related economic cost. Using a real option approach and an infinite time horizon model, this paper first focus on the environmental irreversibility. It is modelled as in Kolstad (1996) or Ulph and Ulph (1997) and consists of a condition which prevents from having negative emissions. We show that the irreversibility of pollution is associated with an option value to pollute and induces a lower optimal level of pollution. The more uncertain the evolution of pollution costs borne by the economy, the lower the desired pollution stock since polluting more now rather than waiting involves an higher opportunity cost. In the spirit of Pindyck (2000), the next part of the analysis deals with the second irreversibility, which refers to the costs of environmental policy. Actually, we simultaneously consider the two kinds of irreversibilities. The aim is to point out the two distinct option values related to pollution and to policy adoption. Once the policy is adopted, the study reduces to the optimal pollution problem. What is more interesting is the situation where the two irreversibilities play a role, provided that they work in opposite direction. The integration of both irreversibilities makes the model more complicated. However, solving the model numerically allows us to appraise which irreversibility has the prevailing effect and what is overall impact of both irreversibilities on pollution and policy design. The analysis notably stresses the importance of the sunk cost of environmental policy. If abatement occurs at least cost, then the optimal pollution stock is higher than the one that would prevail in the absence of irreversibilities. In fact, even if the irreversibility associated with the pollution decision forces the economy to reduce its emissions, this effect is more than compensated by the opportunity to adopt a policy. To the contrary, when environmental policy costs are high enough, the option to adopt the policy becomes less attractive. In other words, the impact of the environmental irreversibility exceeds the one of the irreversibility related to the policy. In this case, the economy reduces the optimal pollution stock when both irreversibilities are taken into account.
The next section sets out the model. The optimization problem is solved in the presence of the environmental irreversibility only. The policy irreversibility is added in section 3. We provide the model calibration and conclude with respect to the total effect of both irreversibilities in section 4. Finally, section 5 concludes.

2 Optimal pollution under environmental irreversibility

2.1 The basic set-up

Pollution $M_t$ accumulates according to the following process:

$$dM_t = (E_t - \delta M_t)dt$$

with $E_t$ the amount of emissions at time $t$, and $\delta > 0$ the natural rate of assimilation. The stock of pollution may be seen as the concentration of greenhouse gases in the atmosphere (GHG, such as carbon dioxide). Accordingly, emissions $E_t$ refer to the anthropogenic emissions of GHG. Pollution is damaging for the economy. Through the greenhouse effect, emissions accelerate the rise in global temperature. According to the IPCC (2007), the expected rise in temperature will be comprised in the range 1.1-6.4°C at the horizon of 2100. Among the most serious economic impacts of global warming is the multiplication of extreme weather events or the submersion of coastal regions (because of the raising oceans level).

Economic models of climate change (grouped under the appellation "integrated assessment models" such as the DICE model developed by Nordhaus (1994)) usually define damages as a quadratic function of the temperature increase, provided that this increase is mainly due to the increasing concentration of GHG. In this paper, as in Pindyck (2000) the relationship between economic damages and pollution is simplified by assuming that the formers directly express as a function of the latter:

$$C(M_t, \theta_t) = -\theta_t M_t^2$$

where $\theta_t$, that measures the intensity (or scale) of the impact of pollution, evolves according to a geometric Brownian motion:

$$d\theta_t = \alpha \theta_t dt + \sigma \theta_t dz_t$$ (2)
Through the formalization of $\theta_t$, we are aiming at recognizing that many uncertainties bear on the economic cost of pollution. For instance, uncertainty remains regarding the exact impact of a given concentration of GHG on temperature. In addition, for a given rise in temperature, it is difficult to give precise estimations of the economic cost.\footnote{In fact, there are many studies that attempt to estimate these costs (see Tol (2005) for a survey) but the results sensitively differ from one to the other.} One may also mention the possibility of catastrophic consequences, such as the complete shutdown of the thermohaline circulation, that should drastically modify the appraisal of the impact of climate change.

Choosing the emission level involves the following trade-off. If emissions contribute to the accumulation of the pollutant, they are also used as an input in the production sector. The economic benefits of emissions are represented in the simplest way by considering a linear function production function, with $B > 0$,

$$\pi(E_t) = BE_t$$

thus, the economy’s instantaneous net payoff writes as follows:

$$W(E_t, M_t) = \pi(E_t) + C(M_t, \theta_t) \leftrightarrow W(E_t, M_t) = BE_t - \theta_t M_t^2$$

Finally, we assume pollution is irreversible, in the sense of Kostad (1996) and Ulph and Ulph (1997), meaning that the stock of pollution cannot be reduced overnight through negative emissions. More formally:

$$E_t \geqslant 0 \quad (3)$$

The irreversibility of pollution should be understood as an irreversibility in the decision process. It has nothing to do neither with the irreversibility of pollution accumulation (see notably Tavhonen and Withagen (1996) or Prieur (2008)) nor with the irreversibility of environmental damages according to which if a catastrophic phenomenon occurs, there is no means of reversing.\footnote{Think again of the shutdown of the thermohaline circulation, Keller et al. (2004).}

### 2.2 The central planner program

The central planner objective is to determine the emission policy that maximizes the expected present value of net payoffs. Her value function writes:

$$V(\theta_t, M_t) \equiv \max_{E_{t+s} \geqslant 0} E \left\{ \int_0^{+\infty} e^{-\rho s} \left[ C(\theta_{t+s}, M_{t+s}) + \pi(E_{t+s}) \right] ds \right\} \quad (4)$$
This optimization problem looks like traditional irreversible decision problem (Pindyck (1988), Abel and Eberly (1994)). Using standard techniques, it is possible to show that the Hamilton-Jacobi-Bellman equation associated with the optimization problem (4) is:

$$\rho V(\theta_t, M_t) = C(\theta_t, M_t) + \frac{E(dV)}{dt}$$

Applying Itô’s lemma:

$$\rho V(\theta_t, M_t) = C(\theta_t, M_t) - \delta M_t V_M(\theta_t, M_t) + \alpha \theta_t V_\theta(\theta_t, M_t) + \frac{1}{2} \sigma^2 \theta^2 V_{\theta\theta}(\theta_t, M_t)$$

where the Kuhn-Tucker conditions for the maximization are $V_M(\theta_t, M_t) \leq 1$, $E_t \geq 0$, $[V_M(\theta_t, M_t) + B] E_t = 0, \forall t \geq 0$. The left-hand side (LHS) of equation (5) reflects the required rate of return. The right-hand side (RHS) is the expected change in the value in the region for the state variable $\theta$ where there are no emissions i.e. in the continuation region. Equation (5) holds identically in $M$. Therefore, the partial derivative of the LHS with respect to $M$ equals the partial derivative of the RHS with respect to $M$. Performing this differentiation yields:

$$\rho V_M(\theta_t, M_t) = \left\{ C_M(\theta_t, M_t) - \delta [V_M(\theta_t, M_t) + M_t V_{MM}(\theta_t, M_t)] + + \alpha \theta_t V_{\theta M}(\theta_t, M_t) + \frac{1}{2} \sigma^2 \theta^2 V_{\theta\theta M}(\theta_t, M_t) \right\}$$

We define $\phi_t = 1/\theta_t$ for the ease of the presentation. It implies that the higher the intensity of pollution damages, the smaller $\phi_t$. As a result of this new notation, the instantaneous profit function becomes homogeneous of degree one in $\phi$ and $M$. Moreover, we can deduce using Itô’s lemma that $\phi$ follows a geometric Brownian motion:

$$d\phi_t = (\sigma^2 - \alpha) \phi_t dt - \sigma \phi_t dz_t$$

To solve equation (6), we use the fact that since the instantaneous payoffs are homogeneous of degree one in $\phi$ and $M$, the value function $V$ is homogeneous of degree one in $\phi$ and $M$. This property of the value function implies that the marginal valuation of pollution $V_M$ is homogeneous of degree zero in $\phi$ and $M$ and, hence, can be written simply as a function of $y_t$; the ratio of $M_t$ to $\phi_t$. This ratio increases with both the pollution stock $M_t$ and the pollution intensity $\theta_t$. Define the marginal valuation of pollution by:

$$q(y_t) = V_M(\phi_t, M_t)$$
Differentiating this equation and using the definition of $y$ yields expressions for the partial derivatives of the value-function. Substituting the definition of $q(y)$ and its partial derivatives in equation ((6)) yields the following differential equation for the marginal valuation of pollution $q(y)$:

$$(\delta + \rho)q(y_t) = -2y_t + q'(y_t)y_t(\mu + \delta - \sigma^2) + \frac{1}{2}q''(y_t)\sigma^2 y_t^2$$

The optimization problem (4) can be solved using this differential equation and appropriate boundary conditions. In particular, as pollution sensitivity increases, the marginal valuation of pollution remains finite:

$$\lim_{y \to +\infty} q(y) < \infty$$

The marginal value of pollution is then:

$$q(y) = \frac{-2}{\rho + 2\delta + \mu - \sigma^2}y + Ay^{-\beta_2} \leftrightarrow q(y) = \frac{-2}{\rho + 2\delta - \alpha}y + Ay^{-\beta_2}$$

with $\beta_2$ the positive root of the fundamental quadratic equation. $Ay^{-\beta_2}$ is the option to pollute more in the future where $A$ is a constant to be determined.

The central planner exercises the emission option when the state variable $y$ becomes low enough so as to reach a trigger value $y^*$. Let $E$ be the amount of emission used. Pollution then increases by a marginal amount $dM$ and the economy benefits from $BE$. This generates the following condition:

$$V(\phi, M) = V(\phi, M + dM) + BE$$

Dividing by the increment $dM$, this condition can be written as follows:

$$q(y^*) = -B$$

To ensure that investment occurs along the optimal path, we also require the smooth pasting condition to be satisfied:

$$q'(y^*) = 0$$

Solving the system composed of the two boundary conditions in the two unknowns \{y^*, A\} yields:

$$y^* = \frac{1}{2}(\rho + 2\delta - \alpha) B \left( \frac{\beta_2}{1 + \beta_2} \right)$$

$$\Leftrightarrow M^* = \phi \frac{(\rho + 2\delta - \alpha) B}{2} \left( \frac{\beta_2}{1 + \beta_2} \right)$$
and, the expression for the value of the option to pollute is:

$$Ay^{-\beta_2} = \frac{-B}{(\beta_2 + 1)} \left[ \frac{2}{(\rho + 2\delta - \alpha)B} \left( \frac{1 + \beta_2}{\beta_2} \right)^{\frac{2}{\beta_2}} \right]^{-\beta_2} y^{-\beta_2}$$

The larger the natural assimilation rate $\delta$, the higher the desired level of pollution. Obviously, the larger the return on emissions ($B$) or the lower the intensity of pollution ($1/\phi$), the larger the desired level of pollution.

Ignoring the joined effect of uncertainty and irreversibility by simply applying a standard NPV rule, the desired pollution would be:

$$M_{NI}^* = \phi \left( \frac{\rho + 2\delta - \alpha}{2} B \right)$$

$$\Leftrightarrow y_{NI}^* = \frac{1}{2} (\rho + 2\delta - \alpha) B$$

Since $\left( \frac{\beta_2}{\beta_2 + 1} \right) < 1$, the pollution irreversibility leads to a smaller desired level of pollution. Note that the optimal level of pollution does not correspond to the desired one when the realizations of the stochastic variable $\phi$ (resp. $\theta$) happen to decrease (resp. increase) in time. This means that the economy may be stuck with too much pollution given the current realization of the stochastic variable.

Uncertainty affects $y^*$ and $M^*_t = \phi_t / y^*$ through the ratio involving $\beta_2$. The effect highlights the impact of the irreversibility of pollution on optimal decisions. It plays through the ratio involving $\beta_2$. For a given $\phi$, an increase in $\sigma$, by reducing $\beta_2$, is accompanied by a decrease in $M^*$: the more uncertain the evolution of pollution costs bearing by the economy, the lower the desired pollution stock. Polluting more now rather than waiting involves an higher opportunity cost. In other words, due to the irreversibility and the increasing uncertainty, the economy will pollute less.

### 3 Policy design under environmental and policy irreversibilities

In this section, we will be considering the pollution irreversibility together with a policy irreversibility. By dealing with the two irreversibilities at the same time, the aim is on the one hand to stress on their respective impact on decision making and, on the other, to understand which one prevails provided that they play in opposite direction. Following an adapting Pindyck (2000)'s approach, the policy
intervention consists in a once-and-for-all reduction in $M_t$. The policy is the opportunity to reduce pollution from its current level to its desired level and involves a sunk cost for society. Such a policy becomes of interest when the sensitivity to pollution becomes too high given the stock of pollution. The policy cost, $K M_t$, is assumed to be increasing in the stock of pollution at the moment of the adoption. The value of the central planner program is now:

$$V^D(\theta_t, M_t) \equiv \max_{E_{t+s} \geq 0, \tilde{T}} \mathbb{E}_t \left\{ \int_{0}^{\tilde{T}} e^{-\rho s} \left[ C(\theta_{t+s}, M_{t+s}) + \pi(E_{t+s}) \right] ds \right. $$

$$+ \left[ C(\theta_{\tilde{T}}, M_{\tilde{T}}) + \pi(E_{\tilde{T}}) - K M_{\tilde{T}} \right] e^{-\rho \tilde{T}} $$

$$+ \left. \int_{\tilde{T}+dt}^{\infty} e^{-\rho s} \left[ C(\theta_{t+s}, M_{t+s} - D) + \pi(E_{t+s}) \right] ds \right\} , $$

subject to (1), (2) with $\tilde{T}$ the time when the adoption occurs. $V^D(\theta_t, M_t)$ denotes the value function in the "dirty" region, i.e. before adoption; $M^*_t$ is the desired level of pollution at the adoption time.

This program combines:

- the optimal investment problem, exposed in section 1, that gives the desired pollution stock, and

- an optimal stopping problem that determines when it is optimal to spend $K M_t$ to reduce $M_t$ to its desired level

We note $M^{D*}_t$ the desired level of pollution in the "dirty" economy i.e. before adoption and $M^{C*}_t$ the desired level in the "clean" economy i.e. after adoption. The resolution still mobilizes the tools of dynamic programming and requires to decompose the problem into two parts depending on whether or not the policy has been adopted.

### 3.1 Optimal pollution when the policy is adopted

The problem is formally similar to the one studied in section 1. Indeed, let us define $V^C(\theta_t, M_t)$ as the value function for the "clean" region where $M^{C*}_t$ is the desired stock of pollution after the adoption. The value function $V^C(\theta_t, M_t)$ must satisfy the Bellman equation:

$$\rho V^C(\theta_t, M_t) = C(\theta_t, M_t) - \delta M_t V^C_{M_t}(\theta_t, M_t) + \alpha \theta_t V^C_{\theta_t}(\theta_t, M_t) + \frac{1}{2} \sigma^2 \theta^2 \theta V^C_{\theta \theta}(\theta_t, M_t) $$

(10)
Recall that \( y_t = M_t \theta_t \) and \( q^C(y) = V^C_M(\phi_t, M_t) \). Similarity with section 1 problem allows us to deduce:

\[
q^C(y) = \frac{-2}{\rho + 2\delta - \alpha} y + A'_2(y)^{\beta_2}, \tag{11}
\]

where the term \( A'_2(y)^{\beta_2} \) still corresponds to the option to pollute. Moreover:

\[
y^*_C = \frac{1}{2} (\rho + 2\delta - \alpha) B \left( \frac{\beta_2}{1 + \beta_2} \right)
\]

and

\[
A'_2 = \frac{-B}{(\beta_2 + 1)} \left[ \frac{2}{(\rho + 2\delta - \alpha) B \left( \frac{1 + \beta_2}{\beta_2} \right)} \right]^{-\beta_2}
\]

For the remainder of the analysis, it is useful to derive from (11) the expression of the value function \( V^C(\phi_t, M_t) \). Integrating \( V^C_M(\phi_t, M_t) \) between the appropriate bounds provides the solution:

\[
V^C(\phi_t, M_t) = -\frac{1}{\rho + 2\delta - \alpha} \frac{(M_t)^2}{\phi_t} + \frac{A'_2 \phi_t^{\beta_2}}{1 - \beta_2 (M_t)^{1-\beta_2}}. \tag{12}
\]

The next section deals with the case where both irreversible decisions have to be taken.

### 3.2 Optimal pollution and optimal adoption

Let us denote \( V^D(M_t, \theta_t) \) and \( q^D(y_t) \) the value function and the marginal valuation of pollution in the "dirty" region i.e. before the adoption. Then, by making use of the same techniques as before, it is easy to show that \( q^D(y_t) \) admits the following general form:

\[
q(y) = \frac{-2}{\rho + 2\delta - \alpha} y + A_1(y)^{-\beta_1} + A_2(y)^{-\beta_2}
\]

with \( \beta_1 < 0 \) and \( \beta_2 > 1 \), \( A_1, A_2 > 0 \).

The marginal valuation of pollution exhibits an additional term \( A_1(y)^{-\beta_1} \) that is the second option value related to policy adoption. \( A_2(y)^{-\beta_2} \) is the option value related to pollution, which also exists after policy adoption. Assume \( \theta \to 0 \), which means that the cost of pollution is almost nil, then the option to adopt the policy becomes negligible. This is precisely the basic property conveyed by the term \( A_1(y)^{-\beta_1} \) since it satisfies:

\[
\lim_{y \to 0} A_1(y)^{-\beta_1} = 0.
\]
The associated value function writes:

\[ V^D(\phi_t, M_t) = -\frac{1}{\rho + 2\delta - \alpha} \phi_t^2 + A_1 \phi_t^{\beta_1} + A_2 \phi_t^{\beta_2}. \]  

(13)

Four optimality conditions must be considered:

- the two conditions for the irreversible investment decision:

\[
\begin{align*}
V_{MD}^D(\phi, M^D) &= -B \\
\phi(M^D) &= 0
\end{align*}
\]

that provide \( M^D(\phi) \) and \( A_2 \)

- the two conditions for the optimal stopping problem:

\[
\begin{align*}
V_{\phi}^D(\phi^*, M^D) &= V_C^C(\phi^*, M^C) - KM^D \\
V_{\phi}^D(\phi^*, M^D) &= V_C^C(\phi^*, M^C)
\end{align*}
\]

that provide \( \phi^*(M^D) \) and \( A_1 \)

Thus, we are left with a system of four equations, expressed in terms of \( \theta = \phi^{-1} \) (with \( \chi = (\rho + 2\delta - \alpha)^{-1} \)):

\[
\begin{align*}
-2\chi \theta M^D + A_1 (\theta M^D)^{-\beta_1} + A_2 (\theta M^D)^{-\beta_2} &= -B \\
-2\chi (\theta M^D)^2 + A_1 \beta_1 (\theta M^D)^{1-\beta_1} + A_2 \beta_2 (\theta M^D)^{1-\beta_2} &= 0 \\
M \left[ -\chi \theta^* M + A_1 \beta_1 (\theta^* M)^{-\beta_1} + A_2 \beta_2 (\theta^* M)^{-\beta_2} \right] &= M^C^* \left[ -\chi \theta^* M^C^* + A_1 \beta_1 (\theta^* M^C^*)^{-\beta_1} + A_2 \beta_2 (\theta^* M^C^*)^{-\beta_2} \right] - KM^D \\
\chi (\theta^* M)^2 + A_1 \beta_1 (\theta^* M)^{1-\beta_1} + A_2 \beta_2 (\theta^* M)^{1-\beta_2} &= \chi (\theta^* M^C^*)^2 + A_1 \beta_1 (\theta^* M^C^*)^{1-\beta_1} + A_2 \beta_2 (\theta^* M^C^*)^{1-\beta_2}
\end{align*}
\]

with four unknowns: \( M^C^*, A_2, \theta^* \) and \( A_1 \). These optimality conditions become more tractable when rewritten in terms of \( y^D \) et \( y^C \). In fact, the following logic applies.

- Let \( y^{**} \) be the trigger value at which the policy option is exercised. The value \( y^{**} \) is the first unknown (corresponding to the ratio of \( M^D \) to \( \phi^* \))

- The adoption allows the economy to "jump" to the post-adoption desired and known level \( y^{C^*} \)

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3Thereafter, a variable with a superscript D (resp. C) refers to the dirty (resp. clean) region.
• As for the emission decision, let us denote \( y^{D^*} = \frac{M^{D^*}}{\phi} \) the trigger level at which the option to pollute is exercised (ie the desired level before policy adoption). \( y^{D^*} \) is the second unknown to be determined.\(^4\)

The last two unknowns remain \( A_1 \) and \( A_2 \) that provide the option values (i) to adopt the environmental policy and (ii) to pollute more.

An intuitive representation of the behavior of \( y = M\theta \) is provided by figure 1:

![Figure 1](image.png)

The system of optimality conditions is:

\[
\begin{align*}
\left\{ y^{a*} \left( -\chi y^{a*} + \frac{A_1}{1-\beta_1} (y^{a*})^{-\beta_1} + \frac{A_2}{1-\beta_2} (y^{a*})^{-\beta_2} + K \right) = Z_1 \\
\chi (y^{a*})^2 + \frac{A_1 \beta_1}{1-\beta_1} (y^{a*})^{1-\beta_1} + \frac{A_2 \beta_2}{1-\beta_2} (y^{a*})^{1-\beta_2} = Z_2 \\
-2\chi y^{D^*} + A_1 (y^{D^*})^{-\beta_1} + A_2 (y^{D^*})^{-\beta_2} = -B \\
-2\chi - A_1 \beta_1 (y^{D^*})^{-\beta_1-1} - A_2 \beta_2 (y^{D^*})^{-\beta_2-1} = 0
\end{align*}
\]

where \( Z_1 \) and \( Z_2 \) are constant:

\[
\begin{align*}
Z_1 &= -\chi (y^{C*})^2 + \frac{A_1^*}{1-\beta_1} (y^{C*})^{1-\beta_2} \\
Z_2 &= \chi (y^{C*})^2 + \frac{A_2^* \beta_2}{1-\beta_2} (y^{C*})^{1-\beta_2}
\end{align*}
\]

Due to the non linearities, this system cannot be solved analytically.

\(^4\)Accordingly, \( y^{C*} \) writes \( y^{C*} = \frac{M^{C*}}{\phi^*} \).
3.3 Discussion

Based on numerical simulations, we assess the impact of the striking parameters on optimal decisions. Simulations are performed by making use of the following set of baseline parameters:

\[ \{\alpha, \delta, B, K, \rho, \sigma\} = \{0.1, 0.03, 1, 1, 0.05, 0.3\} \]

- Effect of uncertainty

Both thresholds \( y_{D}^{*} = \theta M_{D}^{*} \) and \( y_{C}^{*} = \theta M_{C}^{*} \) providing the desired pollution levels before and after policy adoption are monotonically decreasing in \( \sigma \), as shown by Figure 2. This means that the higher the uncertainty, the smaller the pollution for a given value of the stochastic variable \( \theta \). To the contrary, the threshold \( y_{a}^{*} \) governing the adoption monotonically increases with uncertainty. Therefore, for a given amount of pollution, the larger the uncertainty, the later adoption. Since the value before adoption \( V_{D} \) and the value after adoption \( V_{C} \) both increase with uncertainty, the total effect on policy adoption is \textit{a priori} ambiguous.

![Figure 2](image-url)
• In order to have insights on the respective effects of both irreversibilities, we compare the respective evolution of the $y^{D*}$, that encompasses the two options, and $y^{*}_{NI}$, defined as the trigger value when ignoring the joined effect of irreversibility and uncertainty (see equation (8)). For low levels of environmental policy cost, we observe $y^{D*} > y^{*}_{NI}$, or equivalently, $M^{D*} > M^{*}_{NI}$ for a given realization of $\theta$. The effect of the opportunity to adopt a policy prevails on the effect of the irreversibility associated with the pollution, leading to more pollution than when ignoring irreversibility.

To the contrary, it appears that, for high levels of environmental policy cost, $y^{D*} < y^{*}_{NI}$, or equivalently, $M^{D*} < M^{*}_{NI}$. This means that the impact of the pollution irreversibility exceeds the one of the policy irreversibility. In this case, the economy pollutes less when both irreversibilities are taken into account. For an infinite level of $K$ the economy never adopts the policy, thus $y^{D*}$ tends towards $y^{C*}$.

![Figure 3](image)

**Figure 3**

4 Conclusion

In this paper we simultaneously deal with the two irreversibilities that characterize pollution issues, namely, the sunk cost of environmental policy and the sunk cost of environmental degradation. The first part of the paper only concentrates on pollution irreversibility. We obtain that such an irreversibility generates a lower level of pollution when the damages of pollution are uncertain. Adding the environmental policy irreversibility leads to a situation where the two irreversibilities work in opposite direction. Solving the model provides implicit solutions for
both the optimal pollution level and the optimal environmental policy timing. For a low level of environmental policy cost, the irreversibility associated with the policy adoption prevails and pollution is higher than in the absence of any irreversibility. For sufficiently high policy costs, the contrary holds. Further research will consist in calibrating the model with appropriate data in order to conclude on the total effect of both irreversibilities.

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