Environmental Product Differentiation and Taxation

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Abstract
We examine the impact of a commodity tax associated with an emission tax in a green market characterized by environmental sensitivity of consumers and by competition between firms for environmental quality and prices of products. The model originality comes from the assumption that the quality improvement cost is a fixed one. We show the opposite effects of both taxes on qualities and prices at equilibrium and on total welfare. We deduce then that the ad valorem tax and the emission tax are not complementary. The emission tax is the only one that allows a welfare improvement, thanks to a decline in pollution and despite an accentuation of product differentiation.

Keywords: ad valorem tax, emission tax, ecological awareness, environmental quality, vertical differentiation

JEL classification: D62, H21, L13, Q58
1. Introduction

This article is at the confluence of literature dealing with optimal taxation initiated by Suits and Musgrave (1953) and developed by Delipalla and Keen (1992) in an imperfect competition framework and literature dealing with environmental tax advocated by Pigou (1920) and analysed in accordance with market structure by Barnett (1980), Levin (1985), Ebert (1992) and Requate (1993a, 1993b, 1997).

The first research field focus on government’s choice between an ad valorem tax and a specific tax when he wants to levy public funds. The main underscored result is the domination of the ad valorem tax over the equal-yield unit tax when the market is served by a monopolist or an oligolist and the products are homogeneous (Delipalla and Keen, 1992, Keen, 1998, Boldron, 2003). When products are differentiated, the choice of an ad valorem tax is not always optimal. Anderson et al. (2001a, 2001b) emphasize in a spatial competition framework that an ad valorem tax dominates for a sufficiently low revenue requirement. Keen (1998) underlines that when consumers’ preferences are represented by a Dixit and Stiglitz utility function, the choice between an ad valorem tax and a unit tax depends on the importance consumers attach to diversity. When the utility function includes a quality index, Delipalla and Keen (2006) show that the share of the ad valorem tax in total taxation has to be equal to price elasticity of quality-adjusted compensated demand (i.e. the consumer demand function when product quality rises in order to balance price growth and keep the utility level constant). In a vertical differentiation model framework, Cremer and Thisse (1994) explain that a uniform tax is equivalent to a lump-sum tax and that an ad valorem tax is then better than a unit tax. Their conclusion is however valid if three conditions hold: a consumer buys only one unit of the good or none, the market is fully covered and quality increases marginal production cost. To the best of our knowledge, no article tries to extend this result to the case of quality involving fixed costs of research and development (R&D).

The second research field this article refers to is the one of optimal taxation of pollutant emissions, emissions that are responsible for negative externalities that government must correct. Whereas under perfect competition, the optimal taxation is equal to the marginal environmental damage, imperfect competition leads the regulator to reduce the tax below this threshold. This result is due to the necessity to take into account the second market failure, imperfect competition, besides the first one, pollution externality. Barnett (1980) shows the link between the optimal tax and the price elasticity of demand in the monopoly case. Levin (1985), Ebert (1992), Requate (1993) and Simpson (1995) extend this result to a Cournot oligopoly in a market with homogeneous products. Katsoulacos and Xepapadeas (1995) prove on the contrary that the tax rate is higher than the marginal damage when firm number corresponds to the second best optimum. A few of articles deal with this question in a framework of vertical product differentiation. Lombardini-Riipinen (2005) introduces an emission tax besides an ad valorem tax and shows that the environmental tax is then equal to marginal environmental damage. This result is due to the three assumptions set out previously that are the same in this model and the one of Cremer and Thisse (1994). Poyogo-Theotoky and Teerasuwannajac (2002) assume a tax on emission coefficient by unit of production in a model with fixed quality costs. Therefore, this tax is not a pollution tax.
This literature also leaves in abeyance the question of the optimal tax structure when imperfect competition arises from quality competition and the product quality is defined by the more or less pollutant nature of products. Pirilä (2002) deals with this question in a framework of an oligopoly with homogeneous product. He shows that faced with a negative externality, the second best optimum, given a fixed-revenue requirement, is reached thanks to a mere ad valorem tax (unit tax) if the distortion due to this externality if lower (higher) than the one due to imperfect competition. Arora and Gangopadhyay (1995), Cremer and Thisse (1999), Moraga-González and Padrón-Fumero (2002), Bansal and Gangopadhay (2003), Lombardini-Riipinen (2005) and Brécard (2008) use a framework of differentiation by environmental quality of products in order to analyse effects of an ad valorem tax. In particular, Cremer and Thisse (1999) justify this choice by the following arguments: “Emissions taxes are not widely used in the real world. Much more common are commodity (particularly ad valorem) taxes which are levied for non-environmental reasons.” However, OECD (2007) details about 375 environmentally related taxes in OECD countries, whose revenue is in the order of 2 to 2.5 % of GDP. It is also necessary to examine how implementation of an emission tax may complete or substitute for an ad valorem tax. Our framework allows us to take into account the environmental consciousness of consumers, this awareness translating into a willingness to pay more for green products than for standard ones.

We underline opposite effects of the ad valorem tax and the emission tax on price and quality equilibrium: The first one plays in favour of a fall of prices and qualities via a lesser product differentiation, whereas the second one encourages higher product differentiation and a rise in prices. In the same way, consequences of the taxes on welfare seem to be contradictory: Although the ad valorem tax depresses all welfare components, except redistributed tax revenue, and the global welfare, the emission tax leads generally to improve the welfare thanks to its beneficial effects on pollution and to the revenue that it allows to redistribute. Whereas Lombardini-Riipinen [2005] shows that the ad valorem tax and the emission tax are complement when costs for quality improvement are variable, we show that only the emission tax is welfare improving when costs for quality improvement are fixed.

The rest of the paper is organized as follows. In section 2, we introduce the model. In section 3, we study the game equilibrium and impact of the ad valorem tax and the emission tax on equilibrium qualities and prices. In section 4, we analyse tax effects on welfare components in order to deduce their global effect. Section 5 brings this paper to a conclusion.

2. The framework

Consumers view less pollution due to product production and consumption as an environmental characteristic of products, that increases product quality (all other features of the products being assume equal). All consumers prefer the green product but they are different in their willingness to pay for it. Products are thus differentiated by their environmental quality. According to models of vertical product differentiation developed by Mussa and Rosen (1978), Shaked and Sutton (1982) and Motta (1993), each firm produces one variant of the product and decides on its price. Each consumer buys one unit of the product or none.

Consumer preferences are represented by a standard utility function \( u(\theta) \) defined by:
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\[ u(\theta) = r + \theta q_i - p_i \]  

(1)

with \( r \) the consumer’s income (that we normalize to zero for simplification), \( \theta \) the ecological consciousness parameter which is uniformly distributed over \([\theta, \bar{\theta}]\) with a unit density function, \(\theta_q\), his willingness-to-pay the quality \( q_i \) and \( p_i \) the price of the product \( i \).

Facing both qualities \( q_h \) and \( q_l \) proposed by the duopoly \((q_h > q_l)\), the consumer with a parameter \( \hat{\theta} = p_l / q_l \) is indifferent between consuming the standard product at price \( p_l \) or none product and the consumer \( \tilde{\theta} = (p_h - p_l) / (q_h - q_l) \) is indifferent between consuming the standard product \( q_l \) at price \( p_l \) or the green product \( q_h \) at price \( p_h \). We assume that the market is not covered \((\theta < \hat{\theta})\), so that the demand for standard product is \( d_l = \tilde{\theta} - \hat{\theta} \) and the demand for green product is \( d_h = \bar{\theta} - \tilde{\theta} \).

The production cost of firms is assumed independent of quantity, strictly increasing and convex in quality, with the quadratic form \( c(q_i) = \frac{1}{2} c q_i^2 \). The ecological quality of the product \( i \) is defined by abatement \( q_i = \bar{e} - e_i \), where \( \bar{e} \) is the maximal pollution of firms and \( e_i \) pollution of the firm \( i \). Normalizing \( \bar{e} \) to 1, we identify the environmental quality \( q_i \) as the abatement effort, in percentage, made by the firm \( i \). Quality is then supposed to be defined over the interval \([0,1]\). Moreover, we assume, in line with Arora and Gangopadhyay (1995), Bansal and Gangopadhyay (2003), Moraga-González and Padrón-Fumero (2002), Motta and Thisse (1993) and Poyago-Theotoky and Teerasuwannajac (2002), that abatement is achieved thanks to an R&D investment, so that the quality improvement requires a fixed cost. Our model thus distinguishes itself by the Lombardini-Riipinen’s one (2005). We consider that firms bear an \( ad \ valorem \) tax \( t_v \) and an emission tax \( t_e \). Their profits are thus defined by:

\[ \pi_i = \left( (1 - t_v) p_i - t_e e_i \right) d_i - c(q_i) = \frac{1}{\tau_v} \left( p_i - \tau_v (1 - q_i) \right) d_i - \frac{1}{2} c q_i^2 \]  

\[ i = l, h \]  

(2)

with \( \tau_v = 1 / (1 - t_v) \) and \( \tau_e = t_e / (1 - t_e) \). Let us note that the parameter \( \tau_e \), higher than 1, reflects the \( ad \ valorem \) tax rate whereas the parameter \( \tau_e \), positive, depends at once on the emission tax level \( t_e \) and on the product tax \( t_v \) (\( \tau_e \geq t_e > 0 \)). We can establish the link between the emission tax and the unit tax assumed by Delipalla and Keen (1992) and Pirtillà (1997): the emission tax can be decomposed into a unit tax \( t_e \) and an abatement subsidy \( t_q d_i \).

We assume that firms compete in a two-stage game. They compete in qualities in the first stage of the game and in prices in the second stage. We examine in the next section the effects of an \( ad \ valorem \) tax and an emission tax on the game equilibrium.

3. The effects of taxes on equilibrium

3.1 The price subgame

The game is solved by backward induction in order to provide the subgame perfect equilibrium. In the second stage, firms choose their price knowing the qualities with \( r \) the consumer’s income (that we normalize to zero for simplification), \( \theta \) the ecological consciousness parameter which is uniformly distributed over \([\theta, \bar{\theta}]\) with a unit density function, \(\theta_q\), his willingness-to-pay the quality \( q_i \) and \( p_i \) the price of the product \( i \).

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1 This assumption is usual in duopoly model of vertical product differentiation with fixed cost for quality improvement. It supposes that the space between \( \theta \) and \( \tilde{\theta} \)'s sufficiently large.
determined in the first stage and regarding the price of their competitor as given. The maximization of the profit (2) with respect to price induces the following prices:

\[
\begin{align*}
    p_h^* &= \frac{2(\bar{\theta} q_h(q_h - q_l) + \tau_e q_h(3 - 2q_h - q_l))}{4q_h - q_l} \\
    p_l^* &= \frac{\bar{\theta} q_l(q_h - q_l) + \tau_e(2q_h + q_l - 3q_h q_l)}{4q_h - q_l}
\end{align*}
\]  

(3)

The prices increase with product differentiation, measured by the difference between both quality \(q_h - q_l\), and the maximal ecological awareness \(\bar{\theta}\). The \textit{ad valorem} tax and the emission tax tend to raise prices through a rise in \(\tau_e\) (qualities remaining constant).

The study of direct effects of taxes on demand clarifies a little more the operating mechanisms. Demand functions are written:

\[
\begin{align*}
    d_h^* &= \frac{2(\bar{\theta} + \tau_e)q_h - \tau_e}{(4q_h - q_l)} \\
    d_l^* &= \frac{q_l((\bar{\theta} + \tau_e)q_l - 2\tau_e)}{q_l(4q_h - q_l)}
\end{align*}
\]  

(4)

All other things being equal, the emission tax parameter \(\tau_e\) has two contradictory effects on both product demand: by its abatement subsidy component, it plays as a rise in the maximal ecological consciousness from \(\bar{\theta}\) to \(\bar{\theta} + \tau_e\); by its maximal pollution taxation effect, it tends to depress demand for each product.\(^2\) The demand for the standard product is positive under the following condition:

\[
y_e < \frac{1}{2} q_l \text{ with } y_e = \frac{\tau_e}{\bar{\theta} + \tau_e}
\]  

(5)

This condition is fulfilled when the environmental tax or the product tax is sufficiently low and when the maximal environmental consciousness is sufficiently high. Let us remark that without emission tax, the \textit{ad valorem} tax has no direct impact on demand in such a way that the demand for standard product remains positive whatever the \textit{ad valorem} tax may be.

### 3.2 The quality subgame

In the first stage, firms determine qualities anticipating prices of the second stage and regarding the quality of the other firm as given. They also maximize the following profits, that are rewritten using equations (2), (3) and (4), with respect to qualities:

\[
\begin{align*}
    \pi_h &= \frac{(q_h - q_l)^2}{\tau_v} d_h^2 - \frac{c}{2} q_h^2 \\
    \pi_l &= \frac{q_l(q_h - q_l)}{q_l \tau_v} d_l^2 - \frac{c}{2} q_l^2
\end{align*}
\]  

(6)

The maximization conditions are:

\(^2\) We can verify that introduction of a unit tax instead of an emission tax triggers only the second effect.
\[
\begin{align*}
\frac{\partial \pi_h}{\partial q_h} &= \frac{(2q_h - \gamma_e)(4q_h^2 - 7q_h)\gamma_e + (8q_h^2 - 6q_hq_i + 4q_i^3)}{\tau_v(1 - \gamma_e)^2(4q_h - q_i)^3} - cq_h = 0 \\
\frac{\partial \pi_i}{\partial q_i} &= \frac{q_i(q_i - 2\gamma_e)(4q_h^2 - 7q_h)q_hq_i + (8q_h^2 - 6q_hq_i + 4q_i^3)\gamma_e}{\tau_v(1 - \gamma_e)^2(4q_h - q_i)^3} - cq_i = 0
\end{align*}
\]

Assuming \( \lambda = q_h / q_i \) (with \( \lambda > 1 \)), these two first order conditions induce the following equality:

\[
(2\lambda q_i - \gamma_e)((4\lambda - 7)\gamma_e + 2(4\lambda^2 - 3\lambda + 2)q_i)\tau_v^2 = (q_i - 2\gamma_e)((4\lambda - 7)\lambda + 2(4\lambda^2 - 3\lambda + 2)\gamma_e)\lambda^2
\]

In order to simplify the analysis, we study firstly the case with a mere \textit{ad valorem} tax and secondly a equilibrium research methodology with an environmental tax.

\textbf{Case 1. Product tax}

Without emission tax, the condition (8) comes down to the polynomial function of degree three: \( 4\lambda^3 - 23\lambda^2 + 12\lambda - 8 = 0 \) that has a unique real root above one \( \lambda_0 = 5.2512 \). Replacing \( \lambda \) by \( \lambda_0 \) in one of the first order conditions (7), we deduce the standard quality at the equilibrium \( q_i^* \) and hence the green quality \( q_h^* = \lambda_0q_i^* \). The game equilibrium is then characterized by:

\[
\begin{align*}
q_h^* = 0.2533 \frac{\tau_v}{c} & & p_h^* = 0.1077 \frac{\tau_v^3}{c} & & d_h^* = 0.525\tau_v & & \pi_h^* = 0.0244 \frac{\tau_v^4}{c} \\
q_i^* = 0.0482 \frac{\tau_v}{c} & & p_i^* = 0.0110 \frac{\tau_v^3}{c} & & d_i^* = 0.262\tau_v & & \pi_i^* = 0.0015 \frac{\tau_v^4}{c}
\end{align*}
\]

The qualities sold to consumers are differentiated, rise with the maximal ecological sensitivity \( \bar{\theta} \), and decrease with the slope of the marginal cost curve \( c \) and the \textit{ad valorem} tax \( \tau_v \). The equilibrium prices grow also with \( \bar{\theta} \) and fall with \( c \) and \( \tau_v \). The demand addressed to firms increases with \( \bar{\theta} \) (because of a rise of \( \hat{\theta} = 0.475\bar{\theta} \)) faster than \( \bar{\theta} = 0.212\bar{\theta} \), but slower than \( \bar{\theta} \).

\textbf{Case 2. Taxes on products and pollution}

In case of emission tax, only a variable substitution allows us to solve the quality game. The condition (5) of positivity of demand and examination of the condition (8) let us assume that the parameter \( \gamma_e \) is proportional to the standard quality:

\[
\gamma_e = \rho q_i \text{ with } \rho \in [0, \frac{1}{2}]
\]

Firms consider \( \gamma_e \) as given when they choose optimal qualities, in such a way that the effect of standard quality choice on the parameter \( \gamma_e \) doesn’t intervene in their decision-making (it corresponds to a variable substitution and not an endogenization of \( \gamma_e \)). A

\footnote{The market is indeed partly covered if \( \bar{\theta} \leq \hat{\theta} \), so that \( \bar{\theta}/\hat{\theta} > 4.706 \).}
rise in the parameter $\rho$ and/or in the low quality $q_l$ also tend to increase the parameter $\gamma_e$ and hence emission tax and/or product tax.\textsuperscript{4}

\begin{equation}
\begin{aligned}
P(\lambda; \rho) &= 4(1 - 4\rho^2)\lambda^4 + (12\rho^2 + 8\rho - 23)\lambda^3 + 4(1 + \rho)(3 - 2\rho)\lambda^2 \\
&\quad + (\rho^2 + 2\rho - 2)\lambda + 4\rho - 7\rho^2 = 0
\end{aligned}
\end{equation}

The condition (8) can be rewritten as a polynomial function of degree 4 in $\lambda$, defined by:

\begin{equation}
P(\lambda; \rho) = (1 - 4\rho^2)\lambda^4 + (12\rho^2 + 8\rho - 23)\lambda^3 + 4(1 + \rho)(3 - 2\rho)\lambda^2 \\
\quad + (\rho^2 + 2\rho - 2)\lambda + 4\rho - 7\rho^2 = 0
\end{equation}

The calculation of polynomial roots in accordance with $\rho$’s value underlines that $P(\lambda; \rho)$ has only one real root $\lambda(\rho)$ greater than unity. The solution $\lambda(\rho)$ decreases with $\rho$ and reaches its minimum, equal to 5.174, for $\rho=0.07$, before increasing and then exceeding its initial value for $\rho=0.13$, and tending towards 101.5 when $\rho$ tends towards 1/2 (cf. graph 1a).\textsuperscript{5}

For a given value of $\rho$ and an associated value $\lambda(\rho)$, the optimality conditions (7) come down to the following condition:\textsuperscript{6}

\begin{equation}
G(q^*_i; \rho) = (1 - \rho q^*_i)^2 q^*_i - \phi(\rho) \frac{\partial^2}{c \tau_v} = 0
\end{equation}

with

\begin{equation}
\phi(\rho) = \frac{\lambda(\rho)(1 - 2\rho)(8\rho + 4)\lambda(\rho)^2 - (6\rho + 7)\lambda(\rho) + 4\rho}{(4\lambda(\rho) - 1)^3}
\end{equation}

Polynomial function of degree 3 in $q^*_i$ (12) has only one real root $q^*_i(\rho)$ lower than unity, of which expression is given in appendix A1 (cf. graph 1b). Consequently, the

\textsuperscript{4} We can rewrite this parameter $\gamma_e = \tau_e / (\bar{\theta} + \tau_e) = t_e / (\bar{\theta} + t_e - \bar{t}_e)$. It is straightforward to show that it rises with $t_e$ and $t_e$.

\textsuperscript{5} Calculations and simulations were made with the software Mathematica. We cannot express straightforwardly the form of this root.

\textsuperscript{6} If we assume a unit tax instead of an emission tax, the equation (12) becomes

\begin{equation}
G(q^*_i; \rho) = q^*_i - \phi(\rho) \frac{\partial^2}{c \tau_v} = 0
\end{equation}

and then $q^*_i = \phi(\rho) \frac{\partial^2}{c \tau_v}$ (Brécard, 2008b). Because $\rho \in [0, \frac{1}{3}]$, $q^*_i \in [0, q^*_h]$ and $q^*_h \in [0, 1]$, the term $(1 - \rho q^*_i)^2$ is close to 1, in such a way that the equilibrium quality $q^*_i$ with an emission tax may be very near the one obtained with a common unit tax.
high quality is also defined by \( q_h^* = \lambda(\rho)q_l^* \). We show in appendix A1 that these qualities are the only stable solution of the quality game. Moreover, a simulation of both first order conditions, presented in appendix A2, shows that our methodology not only leads to the same results but also selects the one stable equilibrium. Our results are summarized in the proposition 1.

**Proposition 1.** Under the assumption of proportionality between \( \gamma_e \) and \( q_l \): \( \gamma_e = \rho q_l \) with \( \rho \in [0, \frac{1}{2}] \), the parameter \( \lambda(\rho) \) and the equilibrium qualities \( q_l^* \), \( q_h^* \) are solutions of the following equation system:

\[
\begin{align*}
P(\lambda(\rho)) &= 0 \quad \text{defined by (11)} \\
G(q_l^*, \rho) &= 0 \quad \text{defined by (12) and (13)} \\
q_h^* &= \lambda(\rho)q_l^*
\end{align*}
\]

When \( \rho=0 \), the system solution corresponds to the quality equilibrium without emission tax (9) such as \( \lambda(0) = \lambda_0 = 5,2512 \) and \( \phi(0) = 0,0482 \).

Only simulations of parameters \( \rho \) and \( \tau_v \), arising from regulator’s tax choice, allow us to study game equilibrium in the following sub-section.

### 3.3 Game equilibrium

In order to clarify the analysis of the game equilibrium, we successively study the effects of a variation in \( \tau_v \), all other things being equal, and then the effects of a variation in \( \rho \), all other things being equal. In this way, we distinguish the game mechanisms due to the product tax and those due to the emission tax.

**The effects of a variation in \( \tau_v \)**

A rise in the ad valorem tax \( \tau_v \) leads, \( \rho \) remaining constant, to deterioration in the product qualities and drop in differentiation (cf. graph 2a). We also find the expected effects of a product tax that succeeds in correcting excessive differentiation wanted by firms in order to relax price competition and to enhance profits. The product tax leads therefore to decrease quality of both products, differentiation and prices. It lessens demand addressed to both firms. Firms’ profits, defined by equations (6) at equilibrium, are then reduced (graph 2b and 2c).
Graph 2. The effects of a rise in the *ad valorem* tax
for \( \rho = 0.1 \) (such as \( \lambda(0.1) = 5.2015 \) and \( \rho(0.1) = 0.0486 \)) and \( \overline{\sigma}^2 / c = 1 \)

The effects of a variation in \( \rho \)

The analysis of the effects of a variation in \( \rho \) on equilibrium qualities is similar, all other things being equal, to a study of the impact of the emission tax variation. For more clarity, we carry out the following analysis assuming that the *ad valorem* tax is constant. Moreover, we restrict the study to the duopolist case, in such a way that definition space of \( \rho \) becomes \( [0, \overline{\rho}] \), with \( \overline{\rho} = 0.18 \) whatever the other model parameters may be.\(^7\)

A rise in the emission tax leads to an improvement of the low quality \( q_l^* \) when \( \rho \) remains lower than the threshold 0.07 and to a deterioration of it beyond this threshold (graph 3a). By contrast, it tends to enhance the high quality \( q_h^* \) (graph 3b). As a consequence, differentiation slightly decreases when \( \rho \) is low but noticeably increases with \( \rho \) for values above 0.07 (graph 3c).

\(^7\) A simulation example is provided in appendix A2 with \( \overline{\theta} = 1, \tau_v = 1 \) and \( c = 1 \).

\(^8\) When \( \rho \) increases, the parameter \( \tau_e \) follows a bell-shaped curve, where the maximum is reached for a \( \rho \) value near 0.3, greater than the threshold \( \overline{\rho} \). A rise in \( \rho \) leads thus to an increase in \( \tau_e \) (cf. graph A1 in appendix A3). Moreover, some sensitivity tests enable us to show that the threshold \( \overline{\rho} \) doesn’t vary with the parameters \( \overline{\theta}, \tau_v \) and \( c \). These simulations can be obtained on request.
Graph 3. The effects of a rise in $\rho$ (for $\tau_v = 1$ and $\bar{\sigma}^2/c = 1$)

Graph 3a

Graph 3b

Graph 3c

Graph 3d

Graph 3e

Graph 3f

Graph 3g

Graph 3h

Graph 3i
A rise in the emission tax increases prices, through its direct effect on production cost and its indirect impact on product differentiation. The deterioration of the quality $q_i$, beyond a threshold 0.07, plays however in opposite direction. For both products, the first effect prevails over the second one, in such a way that prices are all the greater since $\rho$ is large (graph 3d and 3e).

The combined effect of a higher emission taxation on qualities and prices leads to a loss in the product demand (graph 3f and 3g). As a consequence, the profit of the standard product firm decreases with the emission tax and becomes equal to zero for the threshold $\bar{\rho}$ (graph 3h). By contrast, the profit of the green product firm follow a U-shaped curve, where the minimum is reached for a value of $\rho$ equal to 0.14, thanks to the higher and higher price of this product (graph 3i). Whatever the scenario may be, the profit of the standard product firm is much lower than the one of the green product firm: without emission tax, more than half of demand concerns green product whereas only the quarter of demand falls on standard product. A rise in $\rho$ decreases demand for standard product towards zero whereas demand for green product remains around half of consumers. As a result, a high pollution tax leads to a green product monopoly, for which the analysis is relegated in appendix A4. These results are synthesized in the proposition below.

**Proposition 2.** *A rise in the ad valorem tax tends to decrease environmental qualities and prices at the equilibrium, via a fall in product differentiation.*

The emission tax induces, all other things being equal: An improvement in the standard quality if $\rho$ is lower than 0.07 and a degradation otherwise; an improvement in the green quality; a rise in differentiation; an increase in prices. Beyond the threshold $\bar{\rho}$ equal to 0.18, the standard product firm is ousted.

Whereas a low emission tax enhances quality of both products, and hence the quality of the environment, and increases product differentiation, an *ad valorem* tax plays in opposite direction. Correction of both market failures thanks to these two taxes implements contradictory mechanisms. The question of the effects of the taxes on the social welfare has thus to be studied.

### 4. The effects of taxes on welfare

Besides consumers’ surplus and firms’ profits regulator takes pollution externality into account in his research of optimum. According to the literature on environmental economics, we assume that the global pollution generates environmental damage, which corresponds to the monetary valuation of the impact of pollution on the society as a whole (Baumol and Oates, 1988). The welfare function is also defined in the following way:

$$W = CS_h + CS_l + \pi_h + \pi_l + GR - D$$  \hspace{1cm} (14)

with $CS_i$ the surplus of consumers purchasing the quality $q_i$, $\pi_i$ the profit of the firm $i$ ($i=h,l$), $GR$ the tax revenue and $D$ the environmental damage function. The damage is assumed to be linear with respect to total emissions $E$. Let $\delta$ be the marginal environmental damage, the environmental damage is then defined by $D = \delta E$. In the rest of our paper, we analyse the effects of both taxes on each component of welfare before inferring its global impact.

The net surplus of consumers of green and standard products are defined by:
\[ CS_i = \int_{q_i}^{\theta_i} (\theta q_i - p_i) d\theta = \frac{1}{2} q_i d_i (\theta_i + \theta_i) - p_i d_i \quad i = h, l \]  

(15)

with \( \theta_i = \tilde{\theta}, \theta_h = \theta \) and \( \theta_l = \tilde{\theta} \). A rise in the product tax tends to decrease the quality of both products, their prices and demand. The final effect on consumers’ surplus is negative (graph 4a and 4b). In particular, we can verify that without emission tax, surplus is defined by \( CS_h = 0.0416 \bar{\theta}^4 / c \tau_v \) and \( CS_i = 0.0017 \bar{\theta}^4 / c \tau_v \). An increase in the emission tax, through a rise in \( \rho \) (all other things being equal), leads to an improvement of the standard quality (below \( \bar{\rho} = 0.07 \) without product tax) and the green quality as well as to higher prices. Surplus of both types of consumers is reduced because the price effect outweighs the quality effect (graphs 4c and 4d).

Graph 4. The effects of taxes on consumers’ surplus (for \( \bar{\theta}^2 / c = 1 \))

Graph 4a (\( \rho = 0.1 \))

Graph 4b (\( \rho = 0.1 \))

Graph 4c (\( \tau_v = 1 \))

Graph 4c (\( \tau_v = 1 \))

Tax revenue is defined by:

\[ GR = \tau_v (p_h d_h + p_l d_l) + \tau_v E = \left( 1 - \frac{1}{\tau_v} \right) (p_h d_h + p_l d_l) + \frac{\tau_v}{\tau_v} E \]  

(17)

It grows with the parameter \( \rho \) in the scenario without product tax and \( \bar{\theta}^2 / c = 1 \) (graph 6b). A rise in \textit{ad valorem} tax tends to increase the revenue. In the case without emission tax, tax revenue is defined by \( GR = 0.0592 (\tau_v - 1) \bar{\theta}^4 / (c \tau_v^2) \) and is maximal when the tax rate is 50% \( (\tau_v = 2) \).
Pollution is defined by:

\[
E = (1 - q^*_h) \xi^*_h + (1 - q^*_l) \xi^*_l = \frac{\theta}{1 - \rho} \left[ \frac{3 \lambda - \rho (1 + 2 \lambda) - \lambda (2 \lambda + 1 - 3 \rho) q^*_l}{(4 \lambda - 1) \tau_v} \right] \tag{18}
\]

An increase in product tax raises pollution emitted by firms, because of degradation of both qualities and despite demand decline (graph 5a). Without emission tax, global pollution is indeed characterised by \( E = 0,7875 \theta - 0,1456 \theta^3/c \tau_v \). By contrast, a rise in the emission tax through the parameter \( \rho \) tends to reduce pollution thanks to an improvement in the ecological quality of products and a drop in the total demand (graph 5b).

The effects of taxes on the global welfare are not a priori obvious. Without emission tax, product tax depresses all the welfare components, except the tax revenue. The welfare is written as:

\[
W = \frac{\theta^3}{c \tau_v^2} \left[ -0,0332 \theta + (0,1456 \delta + 0,1024 \theta) \tau_v \right] - 0,7875 \delta \theta \tag{19}
\]

It decreases with the ad valorem tax:

\[
\frac{\partial W}{\partial \tau_v} = \frac{\theta^3}{c \tau_v^2} \left[ 0,0665 \theta - 0,1024 \theta \tau_v - 0,1456 \delta \tau_v \right] \tag{20}
\]

The difference between the first two terms in brackets being necessarily negative in case of tax (\( \tau_v > 1 \)), the welfare is reduced by the product tax. This result is identical with
those of Bansal and Gangopadhyay (2002) and Brécard (2008a). In all scenarios, the ad valorem tax plays negatively on welfare (graph 7a). By contrast, without product tax, welfare tends to grow with $\rho$ thanks to a profit supplement of the green product firm and pollution reduction. For a very low marginal damage, welfare follows a concave curve where the maximum is reached for a parameter $\rho$ near its maximal value $\overline{\rho}$ (graph 7b). If the marginal environmental damage is higher, welfare is a growing function of $\rho$. The welfare is then maximal for the value $\overline{\rho}$ (for which $\tau_e=0.008$ when $\overline{\theta}$, $c$ and $\tau$, are equal to 1, see appendix A5).

**Proposition 3.** A rise in the ad valorem tax tends, all other things being equal, to depress the consumers’ surplus and the firms’ profits, to increase pollution and to raise the tax revenue until a given threshold. The global effect of the tax is always negative on total welfare.

A rise in the emission tax tends, all other things being equal to depress the consumers’ surplus, the firms’ profits and pollution and to raise, until a given tax threshold, the tax revenue. When the marginal environmental damage is low, the global effect of the tax is positive on total welfare for small values of $\rho$. For higher values of marginal damage, a rise in $\rho$ leads to increase welfare.

The emission tax seems thus more adapted than the ad valorem tax in order to move the game equilibrium closer to the Pareto’s optimum. However, It runs counter to a higher competition between firms by favouring product differentiation and green product firm. In the case of high marginal environmental damage, the optimal emission tax even leads to evict the standard product firm and hence to advantage the green product firm. A second-best optimum is also reached under the taxation conditions summarized in the next proposition.

**Proposition 4.** In the case of environmental product differentiation, the second-best optimal structure of taxes is such that no ad valorem tax is imposed to duopoly whereas a maximal emission tax is levied: $\tau_c^* = 0$ and $\tau_e^* = \gamma_e^* / (\overline{\theta} + \gamma_e^*) = \overline{\rho} q_1^*$.

The most pollutant firm is also ejected of the market and the less pollutant firm becomes a green product monopolist.

---

9 In the scenario in which $\overline{\theta}$, $c$ and $\tau$, are equal to 1, the welfare follows a concave curve if $\delta \leq 0.025$ and the maximum is reached for $\rho = 0.04$ and $\tau_e=0.002$ if $\delta = 0.01$, $\rho = 0.14$ and $\tau_e=0.007$ if $\delta = 0.02$ and $\rho = 0.18$ and $\tau_e=0.008$ if $\delta = 0.025$. 
In consequence, it may be advisable to reduce the inefficiency source due to monopoly power of the green firm using an economical policy other than an *ad valorem* tax. Without such a tax, the emission tax is lower than the marginal environmental damage when the most environmental consciousness \( \bar{\theta} \) and the marginal damage are not too high. For instance, when \( \bar{\theta} \) is normalized to 1, the optimal environmental tax is always lesser than the marginal damage, whatever \( \delta \) may be (see the footnote 9). By contrast, when \( \bar{\theta} \) is higher than 1, the optimal environmental tax may be above the marginal damage when \( \delta \) is sufficiently high.\(^{10}\)

5. Conclusion

While many articles deal with optimality of an *ad valorem* tax in a green market, only a few of them associate it with an emission tax. Merely Lombardini-Riipinen (2005) studies an optimal combination of these two taxes in a framework of vertical product differentiation. She shows that the *ad valorem* tax has to be coupled up with a pigouvian tax, equal to marginal environmental damage, in order to improve welfare. This result comes from two major assumptions in her model: market is fully covered and production costs of a quality are variable. We extend the analysis by relaxing these assumptions: market is assumed not covered and production costs of a quality are fixed.

In our framework, the *ad valorem* tax succeeds in strengthening price competition between both firms thanks to a reduction in the environmental quality of both products. As a consequence, it decreases the quality of the environment. On the contrary, the emission tax achieves reduction in pollution thanks to an improvement of the environmental quality of both products. At the same time, it relaxes price competition between firms. However, even if both taxes play correctly their part in correction of the market failures, the product tax degrades the social welfare through a fall in all its components except the government revenue, whereas the emission tax enhances welfare thanks to its beneficial effect on pollution and government revenue. As a result, the optimal product tax is zero whereas the optimal environmental tax is positive. Furthermore, the optimal emission tax may lead to a green monopoly when the marginal environmental damage is not too low. A second policy is thus necessary in order to draw the game equilibrium to the first-best optimum.

This paper also completes results from Bansal and Gangopadhay (2002) who show that an *ad valorem* tax worsens environmental quality and Brécard (2008a) who underlines the negative effect of such a tax on all welfare components except tax revenue whatever the nature of cost induced by environmental quality improvement (variable or fixed) may be. In order to extend our analysis, it may be useful to study thoroughly the game equilibrium in order to find analytical results and hence provide some comparative statics of environmental quality, prices and welfare. However, it requires solving difficulty coming from the introduction of a variable cost in a product differentiation model with fixed cost for quality.

\(^{10}\) Let us illustrate these results with simulations in which \( c \) and \( \tau_v \) are equal to 1 and \( \delta \) is sufficiently high to lead to an optimal emission tax equal to \( \tau^*_e = \bar{\delta} q^*_v \): if \( \bar{\delta} = 1 \), \( \tau^*_e = 0.008 \); if \( \bar{\delta} = 1.2 \), \( \tau^*_e = 0.015 \); if \( \bar{\delta} = 1.4 \), \( \tau^*_e = 0.024 \); if \( \bar{\delta} = 1.6 \), \( \tau^*_e = 0.036 \); and if \( \bar{\delta} = 1.8 \), \( \tau^*_e = 0.052 \). When \( \delta = 0.03 \), the optimal emission tax is then higher than \( \delta \) for \( \bar{\delta} \) smaller than 1.6 but higher than \( \delta \) for a larger \( \bar{\delta} \).
6. References


Requate (1997), “Green Taxes in Oligopoly if the Number of Firms is Endogenous”, *Finanzarchiv* 54(2), 261-280.


### 7. Appendix

**A1. Conditions for existence of an equilibrium**

We study here the first order conditions (7) of the quality subgame. The first equation of conditions (7) is decreasing with \( q_i \): \[
\frac{\partial^2 \pi_i}{\partial q_i^2} = -\frac{8(q_i - 2\gamma_e)\gamma_e(q_i - q_i) + (q_i - 2\gamma_e)q_i + 5(q_i - \gamma_e)q_i}{\tau_e(1 - \gamma_e)(4q_i - q_i)^2} < 0
\]

because the first term in numerator is positive and the term in brackets is positive. Moreover,
\[
\frac{\partial \pi_i}{\partial q_i} \bigg|_{q_i=0} = \tau_e(4\bar{q}_i - 7\tau_e)/\tau_eq_i^3 > 0
\]

when the condition (5) is met. There is thus a solution \( \bar{q}_i(q_i) \) for the first equation (7).

The second equation of conditions (7) is increasing for low values of \( q_i \) and then decreasing for large values of \( q_i \) (graph A1). The second derivative of firm \( i \)’s profit is:
\[
\frac{\partial^2 \pi_i}{\partial q_i^2} = -\frac{2q_i^2(16q_i^3 - 16q_i^2q_i + 6q_iq_i^2 - 3q_i^2)\gamma_e^2 - 4(5q_i + q_i)q_i\gamma_e - (8q_i + 7q_i)q_i\gamma_e}{\tau_e(1 - \gamma_e)(4q_i - q_i)^2q_i^3} < 0
\]

A solution exists if there is \( \sigma > 0 \) such as \( \frac{\partial^2 \pi_i(q)}{\partial q_i^2} = 0 \) and \( \frac{\partial \pi_i(q)}{\partial q_i} > 0 \). Some numerical simulations allow us to show that two equilibria are possible. However, the first one, with the lowest quality, is not stable because the firm \( i \)’s profit is increasing for a quality slightly higher than this point. Only the second one is then a stable solution of the second condition (7). Nevertheless, the existence of an equilibrium requires a tax parameter lower than a threshold \( \bar{p}_e \). This threshold is all the more high as the maximal marginal willingness to pay is high. For \( \bar{p} = 1 \), \( c = 1 \) and \( \tau_e = 1 \), the threshold \( \bar{p}_e \) is equal 0.011.

Simulations in subsection A2 show that our methodology for game resolution enables us to select the stable equilibrium. The expression of the low quality at equilibrium is given by:
\[
q_i = \frac{2}{3p} + \left( \frac{2}{3p} \right)^{\frac{1}{3}} \left[ \frac{-2 + 27\beta + 3^{\frac{1}{3}}\beta^2(-4 + 27\beta)^{\frac{1}{3}}} {3^{\frac{1}{3}}\rho} \right]^{\frac{1}{3}} \text{ with } \beta = \rho\phi(\rho)\frac{\bar{p}^2}{c\tau_e}
\]
Environmental Product Differentiation and Taxation

\[ (a) \bar{\theta} = 1 \]

Graph A1. Simulation of \( \frac{\partial \pi_l}{\partial q_l} \) for \( q_h = 0.25 \), \( \tau_v = 1 \)
\( \gamma_e = 0 \) (clear grey curve), 0.005 (dark grey curve) and 0.01 (black curve)

A2. Simulations of the game equilibrium

The assumption on the value of \( \bar{\theta} \) affects the scale of the equilibrium qualities, prices and profits: the more \( \bar{\theta} \) is high, the more the qualities, the prices and the profits are great. The assumptions on the values of \( c \) and \( \tau_v \) play in an opposite way: the more \( c \) and \( \tau_v \) are high, the more the qualities, the prices and the profits are low. Without loss of generality in the analysis of the effects of the taxes, all the model simulations are then made with the value 1 for \( \bar{\theta} \) and \( c \). The analysis of the impact of the emission tax is made with \( \tau_v = 1 \) (i.e. no product tax).

Tab. A1 Direct simulations of first order conditions

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<th>( \gamma_e )</th>
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<td>0.0490</td>
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<tr>
<td>( d_l^* )</td>
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<td>( d_h^* )</td>
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Tab. A2 Simulations of first order conditions with the assumption \( \gamma_e = \rho q_l \)

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<td>0.2563</td>
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Environmental Product Differentiation and Taxation

A3. The variation of $\tau_e$ with respect to the parameter $\rho$.

Graph A2. $\tau_e$

A4. The monopoly case

The emission tax can exclude the standard product firm from green market. In that case, the green product monopoly faces demand $d_m = \overline{\theta} - p_m / q_m$ and determines price and quality that maximizes its profit:

$$\pi_m = \frac{1}{\tau_v} \left( p_m - \tau_e (1 - q_m) \right) \left( \overline{\theta} - \frac{p_m}{q_m} \right) - \frac{1}{2} cq_m^2$$

The optimal price is

$$p_m = \frac{1}{2} \left( \frac{\tau_e}{\tau_v} + \left( \overline{\theta} - \tau_e \right) q_m \right).$$

It’s an increasing function of quality if $\tau_e < \overline{\theta}$.

Substituting the optimal price into the profit expression leads to the following profit equation:

$$\pi_m = \frac{\tau_e^2 (1 - q_m)^2}{4 \tau_v q_m} - \frac{\theta - \tau_e q_m}{2 c q_m^2}$$

The optimal quality is then solution of the first order condition of profit maximization:

$$4 \tau_v q_m^3 - (\overline{\theta} + \tau_e)^2 q_m^2 + \tau_e^2 = 0 \text{ or } 4 \tau_v q_m^3 (1 - \gamma_e)^2 - \overline{\theta}^2 q_m^2 + \tau_e^2 \gamma_e = 0.$$ Without emission tax, the monopoly produces the quality $q_m^0 = \overline{\theta}^2 / 4 c \tau_v$. With a positive emission tax, the optimal green quality of the monopoly is then defined by:

$$q_m^* = \beta_m + \frac{2 \sqrt{\beta_m^2} \gamma_e}{3 \left( -2 \beta_m + 27 \beta_m \gamma_e + 3 \gamma_e \left( -4 \beta_m + 27 \gamma_e \right)^{\frac{1}{3}} \right)^{\frac{1}{3}}} \left[ -2 \beta_m^3 + 27 \beta_m \gamma_e + 3 \gamma_e \left( -4 \beta_m + 27 \gamma_e \right)^{\frac{1}{3}} \right]^{\frac{1}{3}}$$

with $\beta_m = \frac{\overline{\theta}^2}{4 c \tau_v (1 - \gamma_e)^2}$.

Graph A3. Monopoly’s quality and profit for $\overline{\theta} / c = 1$ and $\tau_v = 1$. 
Some numerical simulations allow us to show that \( q_m^* \) increases with \( \gamma_e \) until a given threshold and then decreases, the demand for green product and the monopoly’s profit decrease with \( \gamma_e \). As a consequence, all components of welfare except government revenue tend to decrease with \( \gamma_e \) in such a way that the emission tax is welfare decreasing.

**A5. Simulations of the social welfare**

A comparison between welfare levels in the duopoly case and in the monopoly case highlights a higher welfare with a green monopoly than with a duopoly. The optimal emission tax seems also to be equal to the one that ousts the firm with the lowest quality of the product when marginal damage is sufficiently high.

![Graph A4. Welfare for \( \delta = 0.1 \)](image)

**Tab. A2 Welfare results in the duopoly case (for \( \theta/c =1 \) and \( \tau_v =1 \))**

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>0</th>
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<th>0.1</th>
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<td>( \gamma_e )</td>
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<td>( \pi^*_h )</td>
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<td>0.0238</td>
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</tr>
<tr>
<td>( E )</td>
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<td>( GR )</td>
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</tr>
<tr>
<td>( W )</td>
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<td>0.0686</td>
<td>0.0681</td>
<td>0.0678</td>
</tr>
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</table>

**Tab. A3 Welfare results in the monopoly case (for \( \theta/c =1 \) and \( \tau_v =1 \))**

| \( \gamma_e \) | 0.0084 | 0.02 | 0.04 | 0.06 | 0.08 |
| \( \tau_e \) | 0.0084 | 0.0204 | 0.0417 | 0.0638 | 0.0869 |
| \( CS_m \) | 0.0302 | 0.0287 | 0.0265 | 0.0229 | 0.0196 |
| \( \pi^*_m \) | 0.0281 | 0.0239 | 0.0167 | 0.0098 | 0.0030 |
| \( E \) | 0.3637 | 0.3489 | 0.3250 | 0.3021 | 0.2791 |
| \( GR \) | 0.0031 | 0.0071 | 0.0135 | 0.0193 | 0.0243 |
| \( W \) | 0.0691 | 0.0597 | 0.0562 | 0.0520 | 0.0678 |

\( -0.3637\delta \) \quad -0.3489\delta \quad -0.3250\delta \quad -0.3021\delta \quad -0.2791\delta \)