Fiscal Policy in an Estimated Model of the European Monetary Union

Aurélien Eyquem

First version: November 2006
This version: November 2007

Abstract
We explore the welfare implications of several fiscal policies in an estimated two-country model of the European Monetary Union with an incomplete financial market and home bias in consumption. In addition to the optimal provision of public goods, the second order approximation of the optimal policy involves active fiscal policies. Since this optimal solution cannot be decentralized, we also investigate the welfare implications of simple public spending rules. We find that (i) welfare maximizing public spending rules imply significant welfare losses with respect to the optimal policy - equivalent to an average 7.3% drop in permanent consumption and (ii) estimated public spending rules imply low welfare losses with respect to welfare maximizing rules.

Keywords: Monetary Union, Fiscal Stabilization, Optimal Monetary and Fiscal Policy, Welfare Analysis, Fiscal Rules.


*I am particularly grateful to Stéphane Auray, Hafedh Bouakez, Nicolas Coeurdacier, Enrique Martinez-Garcia, Daniel Mirza, Sébastien Pommier, Jean-Christophe Poutineau, Fabien Rondeau and participants of the 2nd Open Macro and Development Meeting in Aix-en-Provence for helpful comments.

CREM, Faculté des Sciences Economiques, 7, place Hoche, 35065 Rennes Cedex, France. Email: aurelien.eyquem@univ-rennes1.fr.
1 Introduction

In a recent contribution, Lane [2006] shows that the European Monetary Union (EMU) is still characterized by strong inflation and output differentials. The theory of optimal currency areas (OCA) suggests these asymmetries might be related to the imperfect diffusion of asymmetric shocks in a context of imperfectly integrated financial, input and goods markets. According to this literature, reducing the costs related to asymmetric shocks in the EMU can be achieved thanks to structural improvements in goods or financial markets. However, since this processes may take time, if ever, to produce its expected benefits, the literature suggests that fiscal policy is the only instrument left to smooth the consequences of asymmetric shocks in a monetary union. In a recent contribution, Cooper and Kempf [2004] show that adequate fiscal policies can perfectly offset theses negative consequences. However, in the case of EMU, the Stability and Growth Pact (SGP) constrains fiscal policies and limits the possibilities to address asymmetric shocks adequately.\(^1\)

In this paper, we address the question of fiscal policies in an estimated two-country model of the EMU where financial markets are incomplete and imperfectly integrated, and where constraints bearing on fiscal policies (such as the SGP) are binding. We develop a framework \textit{à la} Beetsma and Jensen [2005] and introduce public spending in the utility function of agents, which creates a room for an optimal (first order) provision of public goods and implies a (second order) motivation for fiscal stabilization. However, we extend the analysis (\(i\)) by assuming home bias in consumption and an incomplete financial market, (\(ii\)) by estimating the model with the simulated method of moments (SMM), and (\(iii\)) by analyzing the welfare implications of both welfare maximizing and estimated public spending rules with respect to the optimal stabilization path.

First, the model of the EMU developed and estimated features an incomplete financial market, home bias in final goods and Calvo staggered prices. These assumptions imply that asymmetric shocks diffuse asymmetrically, which is costly for agents in terms of welfare and international risk-sharing. A clear motivation for active fiscal policies thus arises in our framework since adequately designed active fiscal policies may reduce these asymmetries and generate welfare gains.

Second, we express our model of the EMU in deviation to the undistorted natural equilibrium and derive the quadratic welfare-loss function to determine the optimal
plan. In line with the recent literature, we find that the optimal monetary policy is a standard targeting rule consisting in the stabilization of both the union-wide inflation rate and the union-wide output gap (see Benigno [2004]). On the other hand, we show that optimal fiscal targeting rules react proportionally to the final terms of trade gaps. As in Beetsma and Jensen [2005], the optimal plan significantly reduces inflation differentials, thus improving the welfare in the monetary union.

Third, we show that optimal targeting rules can not be decentralized, mostly because the optimal plan involves too many variables for the set of policy instruments that authorities may actually use to stabilize the economy. In order to evaluate the welfare properties of fiscal policies in the EMU, we focus on a situation where (i) monetary policy is conducted through the control of the interest rate and consists of closing the aggregate inflation rate and the aggregate output gap and where (ii) restrictions such as the SGP are binding, leaving authorities with the control of public spending as only policy instrument. We study the welfare distance implied by two types of instrument (public spending) rules with respect to the optimal situation: (i) welfare maximizing rules and (ii) estimated rules. Welfare maximizing rules yield the best outcome in terms of welfare that authorities may reach in practice while estimated rules evaluate the actual outcome in terms of welfare.

First, we show that welfare maximizing spending rules are countercyclical with respect to the national output gap and result in a stabilization of national inflation rates. However, they imply welfare losses with respect to the optimal targeting rules - equivalent to an average 7.3% drop in permanent consumption. This result relies on the assumption of an incomplete financial market, the assumption of home bias in final consumption and on the restriction of policy instruments available. We show that these losses may be reduced in the long run by implementing structural reforms fostering smoother external adjustment conditions in the EMU, such as a reduction in the home bias in the trade of final goods.

Second, we examine the characteristics of estimated public spending rules within the EMU. Once again, these policies are countercyclical and stabilizing. The welfare distance resulting of these estimated rules with respect to the situation of welfare maximizing rules is negligible (less than 1% of permanent consumption), i.e. the design of estimated public spending rules is close to its best possible design. Conclusions related
to means of improving the design of estimated fiscal rules are similar to the conclusions reached in the case of welfare maximizing rules.

The paper is organized as follows: Section 2 describes a two-country model of a suboptimal monetary union. Section 3 solves and estimates the model. Section 4 describes the linear-quadratic framework. Section 5 exposes the optimal (commitment and centralized) policy. Assuming that authorities are left with the interest rate and the level of public spending in each country to stabilize the economy, Section 6 studies the implications of welfare maximizing and estimated public spending rules. Section 7 concludes.

2 A two-country model of a suboptimal monetary union

We model a two-country monetary union with a common central bank controlling the nominal interest rate. Each country is populated by continuum of infinitely-living households, an infinite number of firms that are specialized in the production of differentiated goods and a government. Agents have access to a single financial asset—a one period composite bond, so that the financial market between the two countries is incomplete. The financial market is also imperfectly integrated since agents trading financial assets have to pay a portfolio management cost, depending on the quantity of assets they trade. Final goods market is characterized by home bias in consumption bundles and Calvo staggered adjustment of goods prices. Final goods are produced with a simple capital-free technology. These assumptions imply that the diffusion of asymmetric shocks is imperfect across the monetary union, which may affect the design of monetary and fiscal policies.

2.1 Households

In each country the number of infinitely-living households is normalized to one. The representative household \( j \in [0, 1] \) of country \( i \in \{h, f\} \) maximizes a welfare index,

\[
\omega^i_t = \sum_{t=0}^{\infty} \beta^t E_0 \left\{ \frac{C^i_t(j)^{1-\rho}}{1-\rho} + \frac{G^i_t^{1-\rho_g}}{1-\rho_g} - \frac{N^i_t(j)^{1+\psi}}{1+\psi} \right\}, \tag{1}
\]
subject to the budget constraint, 

\[ B_{t+1}^i(j) - R_t B_t^i(j) = W_t^i N_t^i(j) + \Pi_t^i(j) - P_t^i C_t^i(j) - P_{t,t} AC_t^i(j) - T_t^i(j), \]  

(2)

and the transversality condition, \( \lim_{T \to \infty} \Pi_T R_t^{-1} E_t \{ B_{T+1}^i(j) \} = 0. \) In (1), the parameter \( \beta = (1 + \delta)^{-1} \) is the subjective discount factor, \( N_t^i(j) \) is the consumption bundle chosen by the representative agent, \( C_t^i(j) \) is the consumption bundle chosen by the representative agent, \( N_t^i(j) \) is the quantity of labour competitively supplied to the firms of country \( i \), \( G_t^i \) is the level of public spending in country \( i \), \( \rho \) is the index of risk aversion and \( \psi^{-1} \) determines the Frischian elasticity. In (2), \( W_t^i \) is the nominal wage supplied in country \( i \) for period \( t \), \( \Pi_t^i(j) = \int_0^1 \Pi_t^i(k,j)dk \) is the profit paid by national firms to the representative national agent, \( B_t^i(j) \) is the holding of the composite one period nominal bond at the end of period \( (t-1) \) that pays a gross nominal rate of interest \( R_t \) between periods \( (t-1) \) and \( t \), \( T_t^i(j) \) is a lump-sum tax paid by household \( j \) to the national government of country \( i \), \( P_t^i \) is the consumer price index in country \( i \) in period \( t \), \( P_{t,t} \) is the producer price index in country \( i \) in period \( t \) and \( AC_t^i(j) \) represents portfolio adjustment costs.

The financial market of the monetary union is incomplete since (i) households have access to a single one period composite financial asset and imperfect since (ii) trading bonds is costly. Buying (resp. selling) bonds affects negatively (resp. positively) the individualized interest rate, so that (i) agents have a strong incentive to return to their initial position in the long run and (ii) agents belonging to a creditor country face lower nominal interest rates than agents in the debtor country. As underlined by Schmitt-Grohé and Uribe [2003] this assumption is a convenient way to balance the current account in the long run between union members while preserving its short run dynamics. We thus impose a standard quadratic form for portfolio adjustment costs,

\[ AC_t^i(j) = \frac{\chi}{2} [B_{t+1}^i(j) - B_t^i(j)]^2, \]

where \( B_t^i(j) \) is the steady state level of net foreign assets and \( \chi \) is the portfolio adjustment cost. Portfolio adjustment costs affect the Euler condition since,

\[ \beta I_{t+1}^i E_t \left\{ \frac{P_t^i C_t^i(j)^\rho}{P_{t+1}^i C_{t+1}^i(j)^\rho} \right\} = 1, \]  

(3)
with, $P_{t+1}^i(j) = R_{t+1} \left[ 1 + \chi P_{t,t}^i(B_{t+1}^i(j) - B^i(j)) \right]^{-1}$. The value of $\chi$ affects the intertemporal consumption choice in (3): an increase in the cost of bonds trading reduces the sensitivity of wealth’s accumulation to a variation of the interest rate, as it becomes more costly to smooth consumption using the financial market. The labour supply function is standard since it depends on the level of consumption and on the real wage,

$$
N_t^i(j) = \frac{W_t^i}{P_t^i C_t^i(j)^{\psi}}.
$$

Following Galí and Monacelli [2005], we assume home bias in the final consumption bundles. The aggregate consumption of consumer $j$ living in country $i$, $C_t^i(j)$ is,

$$
C_t^i(j) = \left[ (1 - \alpha_i)^{\frac{1}{\mu}} \left( C_{h,t}^i(j) \right)^{\frac{\mu - 1}{\mu}} + \alpha_i^\frac{1}{\mu} \left( C_{f,t}^i(j) \right)^{\frac{\mu - 1}{\mu}} \right]^{\frac{1}{\mu - 1}},
$$

and the companion consumption price index $P_t^i$ is,

$$
P_t^i = \left[ (1 - \alpha_i) \left( P_{h,t}^i \right)^{1-\mu} + \alpha_i \left( P_{f,t}^i \right)^{1-\mu} \right]^{\frac{1}{1-\mu}},
$$

where $\alpha_i \in [0, \frac{1}{2}]$ is the home bias, also measuring the openness of the final goods market in country $i$ (see Corsetti [2006] and Goldberg and Tille [2008]). In these expressions, $\mu$ is the elasticity of substitution between domestic and foreign goods.

Standard Dixit and Stiglitz [1977] consumption subindexes are,

$$
C_{h,t}^i(k,j) = \left[ \int_0^1 C_{h,t}^i(k,j)^{\frac{\theta - 1}{\theta}} dk \right]^{\frac{\theta}{\theta - 1}}, \text{ and } C_{f,t}^i(j) = \left[ \int_0^1 C_{f,t}^i(k,j)^{\frac{\theta - 1}{\theta}} dk \right]^{\frac{\theta}{\theta - 1}},
$$

where $C_{h,t}^i(k,j)$ (resp. $C_{f,t}^i(k,j)$) is the consumption of a typical final good $k$ of country home (resp. foreign) by the representative consumer $j$ of country $i$ and $\theta > \mu$ is the elasticity of substitution between national varieties of final goods. We assume that producers do not discriminate the markets they sell their products on, so that prices of domestic and foreign goods are identical across the monetary union,

$$
P_{h,t}^i = P_{h,t} = \left[ \int_0^1 P_{h,t}(k)^{1-\theta} dk \right]^{\frac{1}{1-\theta}} \text{ and } P_{f,t}^i = P_{f,t} = \left[ \int_0^1 P_{f,t}(k)^{1-\theta} dk \right]^{\frac{1}{1-\theta}}.
$$
Accordingly, optimal variety demands depend on relative prices of goods, of varieties and on the aggregate consumption level in each country,

\[ C_{i,t}(k,j) = (1 - \alpha_i) \left[ \frac{P_{h,t}(k)}{P_i^t} \right]^{-\mu} \left[ \frac{P_{h,t}(k)}{P_{h,t}} \right]^{-\theta} C_i^i(j), \quad C_{i,t}(k,j) = \alpha_i \left[ \frac{P_{f,t}(k)}{P_i^t} \right]^{-\mu} \left[ \frac{P_{f,t}(k)}{P_{f,t}} \right]^{-\theta} C_i^i(j). \]

We impose the mirror assumption, so that \( h = 1 - f \) and \( f = 1 - h \). A bias in favour of national goods in each country thus simply requires that \( \alpha < \frac{1}{2} \). Finally, we define the terms of trade as,

\[ S_t = \frac{P_{f,t}}{P_{h,t}}. \]

### 2.2 Firms

Firms produce varieties \( k \) in country \( i \in \{ h, f \} \) using national labour according to a standard technology,

\[ Y_t^i(k) = A_t^i L_t^i(k), \quad \text{with} \quad A_{t+1}^i = (1 - \rho_a^i) A_i^i + \rho_a A_t^i + \xi_{t+1}^i, \]

where \( \xi_{t+1}^i \) is an I.I.D innovation. The marginal cost of the firm \( k \) in country \( i \), \( MC_t^i(k) \) is,

\[ MC_t^i(k) = MC_t^i = W_t^i / A_t^i. \]

We assume that production prices are governed by Calvo [1983] pricing contracts. In economy \( i \in \{ h, f \} \), only a fraction \( (1 - \eta^i) \) of randomly selected firms is allowed to set new prices each period. Instead of a typical mark-up behavior that would prevail in flexible price settings, firms set a higher price when allowed to, depending on the expected period during which they will be unable to reset their price. The corresponding optimal price is,

\[ P_{i,t}^*(k) = \frac{\theta}{(\theta - 1)(1 - \tau)} \sum_{\nu=0}^{\infty} \left( \eta^i \beta \right)^{\nu} E_t \left\{ \frac{Y_{t+\nu}^i(k)}{P_{i,t+\nu}^i C_{t+\nu}^i(j)^{\nu}} MC_{t+\nu}^i \right\} \sum_{\nu=0}^{\infty} \left( \eta^i \beta \right)^{\nu} E_t \left\{ \frac{Y_{t+\nu}^i(k)}{P_{i,t+\nu}^i C_{t+\nu}^i(j)^{\nu}} \right\}. \]

In this expression, \( \tau \) is a subsidy that compensates the distorting effects of monopolistic competition in the economy. \( Y_t^i(k) \) is the aggregate demand addressed to firm \( k \).
Aggregating among final firms and assuming behavioral symmetry of Calvo producers, the average price of final goods in country $i \in \{h, f\}$ is,

$$P_{i,t} = \left[ (1 - \eta^t) P_{i,t}^x(k)^{1-\theta} + \eta^t P_{i,t-1}^{1-\theta} \right]^{1/\gamma}.$$

### 2.3 National authorities

According to the specification of the utility function, we assume that public spending are useful. Beetsma and Jensen [2005] that the corresponding optimal provision of pubic goods is,

$$\left( G^i_t \right)^{-\rho_g} = \int_0^1 N_i(j)^\psi dj. \quad (4)$$

The optimal provision of public goods thus aims at offsetting the disutility of variations in hours worked at the aggregate level. But fiscal policy is also aimed at closing first order distortions, i.e., national governments compensate for distortions on goods market by taxing households to finance the subvention made to firms. As a consequence, the budget constraint of authorities is simply,

$$\int_0^1 T_i^i(j) dj + \tau \int_0^1 P_{i,t}(k) Y_i^i(k) dk = P_{i,t} G^i_t.$$

In the model, exception made of the subsidy made to firms, we assume lump-sum taxation, implying that budgets are balanced at each period. It may not be a problem since Brück and Zwiener [2006] show that these policies may have preferable stabilization properties with respect to budget deficits policies. Furthermore, the introduction of distorting taxes would lower the real determinacy area and jeopardize the compatibility of monetary and fiscal policy (see Leith and Wren-Lewis [2006]).

### 2.4 Equilibrium

For any sequence of productivity shocks $\left\{ A^h_t, A^f_t \right\}_{t=0}^\infty$, an equilibrium is a sequence of quantities,

$$\left\{ Q_t \right\}_{t=0}^\infty = \left\{ Y_t^h, Y_t^f, C_t^h, C_t^f, N_t^h, N_t^f, B_t^h, B_t^f, AC_t^h, AC_t^f \right\}_{t=0}^\infty.$$
and a fiscal policy \( \{G_t^h, G_t^f\}_{t=0}^{\infty} \) that satisfy households, firms and governments optimality conditions for a given set of prices,

\[
\{\mathcal{P}_t\}_{t=0}^{\infty} = \left\{W_t^h, W_t^f, P_t^h, P_t^f, P_{h,t}, P_{f,t}, P_{h,t}^*(k), P_{f,t}^*(k)\right\}_{t=0}^{\infty}
\]

in a way that is compatible with the clearing of markets described below.

Defining aggregate supply bundles as

\[
Y_t^i = \left[\int_0^1 Y_t^i(k) \frac{\theta-1}{\theta} dk \right]^{\frac{\theta}{\theta-1}},
\]

final goods markets clear according to,

\[
Y_t^h = (1-\alpha) \left[\frac{P_{h,t}}{P_t^h}\right]^{-\mu} C_t^h + \alpha \left[\frac{P_{h,t}}{P_t^f}\right]^{-\mu} C_t^f + G_t^h + AC_t^h,
\]

\[
Y_t^f = (1-\alpha) \left[\frac{P_{f,t}}{P_t^f}\right]^{-\mu} C_t^f + \alpha \left[\frac{P_{f,t}}{P_t^h}\right]^{-\mu} C_t^h + G_t^f + AC_t^f.
\]

Labour is immobile, thus for \( i \in \{h, f\} \),

\[
N_t^i = \int_0^1 N_t^i(j) dj = \int_0^1 L_t^i(k) dk,
\]

and the aggregate production function of country \( i \in \{h, f\} \) is, \( Y_t^i DP_{i,t} = A_t^i N_t^i \), where

\[
DP_{i,t} = \int_0^1 \left[\frac{P_{i,t}(k)}{P_t^i}\right]^{-\theta} dk \geq 1
\]

is the dispersion of production prices in country \( i \).

The equilibrium of the international financial market is,

\[
\int_0^1 B_t^h(j) dj + \int_0^1 B_t^f(j) dj = 0.
\]

Finally, the aggregation of constraints yields the dynamics of net foreign assets in the home country \( h \),

\[
B_{t+1}^h - R_t B_t^h = P_{h,t} Y_t^h - P_t^h C_t^h - P_{h,t} C_t^h,
\]

(5)

with \( B_s^h + B_s^f = 0, \forall s \geq t \). Eq. (5) indicates that a country accumulation of net financial claims with respect to the rest of the monetary union depends on differences in public and private consumption and production time profiles.
3 Estimation

Most NOEM models are simply calibrated using standard values found in the empirical literature. This section estimates the model in logdeviation to the steady state with the simulated method of moments. This procedure provides consistent estimates for the parameters of the model, allowing to run more realistic policy analysis and simulations.

3.1 Linearization and models dynamics

We firstly solve the model in logdeviation with respect to a symmetric steady state. In the symmetric steady state $A^i_t = A$. We assume $G = \kappa Y$. Symmetry imposes $P_h = P_f = P^c = P^* (k) = P$, implying $\frac{w}{P} = \frac{w}{P} = \frac{\left(\frac{\theta - 1}{\theta}\right)}{\theta} A$. Following Benigno and Woodford [2005], we set, $\tau = \frac{1}{1-\theta} \leq 0$, implying $\frac{w}{P} = A$. Portfolio costs imply $I = R = \beta^{-1}$. Price indexes are such that $DP = 1$, final goods markets give $C = (1 - \kappa) Y$, and the production function is $Y = AN$. Combining these relations with the leisure arbitrage, we get, $N = \left[ \frac{A^{1-\rho}}{(1-\kappa)^\rho} \right]^{\frac{1}{1+\rho}}$.

Applying the standard linearization procedure, we consider $x^i_t$ as the logdeviation of $X^i_t$, $\forall t$ for $i \in \{h, f\}$. The model in logdeviation is summarized in Table 1. Notice that price dispersion variations are of second order (see appendix A), implying $dp_{i,t} = 0$. Furthermore, one should remember that the loglinear expression for $I_{t+1}$ is $\frac{dI_{t+1}}{I} = \frac{\delta}{1+\rho} r_{t+1} = \chi C b_{t+1}$. Finally, since $B = 0$, we have to define the loglinear expression of $B^i_t$ as $b^i_t = \frac{b^i_t}{PC}$.

[Insert Table 1 here]

For the sake of the estimation and to close the model, we augment equations in Table 1 with an assumption on the monetary policy implemented by the central bank. As we show further, optimal targeting rules defined by the minimization of a quadratic welfare loss function imply a full stabilization of the aggregate inflation rate and a zero aggregate output gap. As a consequence, the optimal instrument policy for the central bank is to equate the nominal interest rate to its natural equilibrium expression,

$$r_{t+1} = \tilde{r}_{t+1},$$

where $\tilde{r}_{t+1}$ is defined in Table 5.
3.2 Data and results

Following Benigno [2004] and Alvarez et al. [2006], we differentiate countries of the EMU according to their levels of nominal rigidities and aggregate them in two regions, corresponding to the two countries of the model.

Table 2 indicates the percentage of goods prices in the consumer price index changing every month in EMU countries. Setting the limit at 15%, the EMU divides in two groups of high and low nominal rigidities. In the first group, we have Germany, Spain and Italy. In the second group, we have all remaining countries, including France. For simplicity, we assume that these two groups represent a half of EMU’s GDP. The quarterly data we use come from the OECD Economic Outlook database and range from 1970:1 to 2004:4. Data for Austria, Luxembourg and Greece are unavailable or not reliable. Aggregates are converted in the same currency and we focus on the following seasonally adjusted series: GDP (volume), private consumption (volume), GDP deflator, current account balance (relative to GDP), employment and real wages.

We firstly aggregate series for each group of countries. We then correct for the German reunification. \(^7\) We finally compute the variance covariance matrix before selecting interest moments that our model should be able to account for. Posing \(w_{it} = w_t - p_t\) and \(ca_t = b_t - b_{t-1}\), Table 3 describes the moments that we think our model should be able to reproduce.

We calibrate \(\beta = 0.99\), that corresponds to a 4.1% average annual interest rate. This value is consistent with observed average short-term real interest rates over the period in the EMU.

Table 4 collects the results of the estimation. First, one shall note that the adequacy of the model is satisfying since the \(J - \text{stat}\) has a 0.5844% \(p - \text{value}\). Second, most parameters are 99% significant, except \(\rho_g, \kappa\) and \(\chi\). Parameters values are consistent
with most estimates or calibrations in the literature. $\psi = 7.4657$ is on the upper bound of the range put forth by Canzoneri, Cumby and Diba [2006], $\rho = 2.7128$ is close to standard values (see Benigno [2004]). $2\alpha = 0.3001$ features the openness of final goods markets and is consistent with standard openness measures. In the literature, estimates of $\mu$ range from 1 to 15. For example, the estimations of Harrigan [1993] range from 5 to 12. Our estimate is on the lower bound of that interval since $\mu = 4.4441$. The steady state share of public spending over the GDP is $\kappa = 0.1803$, in line with most estimates. The estimation of $\chi = 0.0009$ is very close to the calibration proposed by Schmitt-Grohé and Uribe [2003] and corresponds to an average annual 0.36% interest premium. Our estimates for nominal rigidities are $\eta^h = 0.5865$ and $\eta^f = 0.5070$. These values match the values put forth in Alvarez et al. [2006] and confirm our choice of differenciating EMU countries based on their degree of nominal rigidities. According to our estimations, the first group of countries features higher nominal rigidities. Finally our estimation of the autoregressive parameter $\rho_a = 0.9701$, and of the standard deviation of shocks $\text{std}(\xi^h_t) = 0.012$ and $\text{std}(\xi^f_t) = 0.0085$ are consistent with most values found in the RBC literature (see Backus, Kehoe and Kydland [1992]).

### 4 A linear-quadratic framework

In this section, we define the framework suitable for the definition of the optimal stabilization policy. First, we express the dynamics of the model in deviation to the natural equilibrium. Second, the welfare-based authorities loss function is expressed in a pure quadratic fashion, implying that the solution to the (second order approximated) Ramsey problem can be set in the convenient linear-quadratic form.$^8$

#### 4.1 The model in logdeviation to the natural equilibrium

We consider the natural equilibrium, characterized by flexible prices ($\eta^i = 0$) and a complete financial market ($\chi = 0$) as the benchmark for the definition of the optimal policy. We thus express the model in deviation to the natural equilibrium. Defining $\tilde{x}_t$ as the natural logdeviation of $x_t$, $\tilde{x}_t^v = \frac{1}{2}(\tilde{x}_t^h + \tilde{x}_t^f)$ as its average value and $\tilde{x}_t^r = \frac{1}{2}(\tilde{x}_t^f - \tilde{x}_t^h)$ as its relative value, the natural equilibrium is summarized in Table 5 (see appendix B for details).
Defining the deviation of a variable to its loglinear natural equilibrium as \( \tilde{x}_t = x_t - \bar{x}_t \), the dynamics of the model is,

\[
\tilde{n}_t' = (1 - 2\alpha) (1 - \kappa) \tilde{c}_t^u - 2 (1 - \kappa) \mu \alpha (1 - \alpha) \tilde{s}_t + \kappa \tilde{g}_t^r, \tag{6}
\]

\[
\rho E_t \{ \tilde{c}_{t+1}^u - \tilde{c}_t^u \} = \frac{\delta}{1 + \delta} \tilde{n}_{t+1} - E_t \{ \tilde{\pi}^u_{t+1} \}, \tag{7}
\]

\[
\rho E_t \{ \tilde{c}_{t+1}^r - \tilde{c}_t^r \} = \chi \tilde{C}_{h,t+1}^b - (1 - 2\alpha) E_t \{ \tilde{\pi}^r_{t+1} \}, \tag{8}
\]

\[
\pi_{h,t} = \beta E_t \{ \pi_{h,t+1} \} + k^h [\phi_1 \tilde{c}_t^u + \kappa \psi \tilde{g}_t^u - \psi \tilde{n}_t^r - \rho \tilde{c}_t^r + \alpha \tilde{s}_t], \tag{9}
\]

\[
\pi_{f,t} = \beta E_t \{ \pi_{f,t+1} \} + k^f [\phi_1 \tilde{c}_t^u + \kappa \psi \tilde{g}_t^u + \psi \tilde{n}_t^r + \rho \tilde{c}_t^r - \alpha \tilde{s}_t], \tag{10}
\]

\[
\tilde{s}_t - \tilde{s}_{t-1} = (\pi_{f,t} - \pi_{h,t}) - (\tilde{s}_t - \tilde{s}_{t-1}), \tag{11}
\]

\[
\tilde{b}_{t+1}^h - \tilde{b}_t^h = \delta \tilde{b}_t^h + \alpha [2 \mu (1 - \alpha) - 1] \tilde{s}_t + 2\alpha \tilde{c}_t^r. \tag{12}
\]

In these expressions, \( \phi_1 = \rho + \psi (1 - \kappa) \). Eq. (6) is the contraction of the relative expression of final goods market equilibria (for the right-hand side element) and the relative production functions (for the left-hand side element). Eqs. (7) and (8) summarize the relative and union-wide expressions of Euler equations. Eqs. (9) and (10) are the modified expressions of the Phillips curves, obtained by expressing the marginal cost in terms of variables in deviation to the natural equilibrium. Eq. (11) is the simple dynamic definitions of terms of trade. Finally, Eq. (12) describes the dynamics of net foreign assets where \( \alpha [2 \mu (1 - \alpha) - 1] \tilde{s}_t + 2\alpha \tilde{c}_t^r \) is the trade balance at time \( t \).

Using equation (6), Phillips curve can be expressed in a more standard form,

\[
\pi_{h,t} = \beta E_t \{ \pi_{h,t+1} \} + k^h [\phi_1 \tilde{c}_t^u - \phi_2 \tilde{c}_t^r + \phi_3 \tilde{s}_t + \kappa \psi \tilde{g}_t^h], \tag{9a}
\]

\[
\pi_{f,t} = \beta E_t \{ \pi_{f,t+1} \} + k^f [\phi_1 \tilde{c}_t^u + \phi_2 \tilde{c}_t^r - \phi_3 \tilde{s}_t + \kappa \psi \tilde{g}_t^f], \tag{10a}
\]

where \( \phi_1 = \rho + \psi (1 - \kappa), \phi_2 = \rho + \psi (1 - 2\alpha) (1 - \kappa), \) and \( \phi_3 = [2 \psi (1 - \kappa) \mu \alpha (1 - \alpha) + \alpha] \).
4.2 The authorities loss function

We adopt a welfare measure based upon a second order approximation of the aggregate utility function to rank alternative policies,

\[ \omega_T = \frac{1}{2} \int_0^1 \omega^h_T(j) dj + \frac{1}{2} \int_0^1 \omega^f_T(j) dj, \]

where, after using the symmetry among agents,

\[ \omega^i_T(j) = \omega^j_T = \sum_{t=0}^{t=T} \beta^t E_0 \left\{ \left( C^i_t \right)^{1-\rho} \frac{1}{1-\rho} + \left( G^i_t \right)^{1-\rho_g} \frac{1}{1-\rho_g} - \left( N^i_t \right)^{1+\psi} \right\}. \]

Using a second order approximation of variables to their steady state values and assuming that second order expressions of shocks are equal to zero, this function is expressed as a quadratic function of endogenous variables in deviation to their natural paths. After some algebra the expression of the welfare measure is,

\[ \omega_T = -\frac{q}{2} \sum_{t=0}^{t=T} \beta^t E_0 \{ \ell_t \} + t.i.p + O \left( \| \xi^3 \| \right), \]

where \( \ell_t \) is the welfare loss in period \( t \), defined as,

\[ \ell_t = \frac{\theta}{2k} \pi^2 h_t + \frac{\theta}{2k} \pi^2 f_t + \varphi_c [\hat{\pi}^2 c_t] + (1-\kappa) \varsigma^\alpha [\hat{s}_t] + (1-\kappa) \rho [\hat{c}_t]^2 + \psi [\hat{r}_t]^2 + \varphi_g [\hat{g}_t]^2 + \varphi_g [\hat{g}_t] + \varphi_g c g_t h_t - \varphi g_g h_t \hat{s}_t, \]

with,

\[ q = \frac{C^{1-\rho}}{(1-\kappa)} \geq 0, \varsigma^\alpha = \mu \alpha (1-\alpha), \varphi_c = (1-\kappa) \left[ \rho + \psi (1-\kappa) \right], \]

\[ \varphi_g = \frac{\kappa (\kappa \psi + \rho_g)}{2}, \varphi_g c = 2 \kappa \psi (1-\kappa), \varphi_g s = 2 \kappa \psi (1-\kappa) \varsigma^\alpha. \]

The welfare measure \( (13) \) admits standard arguments, such as national inflation rates and the aggregate consumption gap in the monetary union. The measure also admits original arguments, related to the relative paths of the national consumption gaps, national effort gaps, the gap of terms of trade or public spending gaps.
5 Optimal stabilization policies

In this section, our approach to the problem of optimal monetary and fiscal policies follows Lucas and Stokey [1983] or Chari and Kehoe [1999], and consists in finding the path of the economy that maximizes the welfare. Two major differences with respect to that literature emerge since (i) production prices are sticky, generating second order distortions to the efficient equilibrium and (ii) the optimal policy problem is approximated up to a second order and put in a convenient linear-quadratic form. Benevolent authorities thus commit to a sequence of equilibria of the monetary union defined by the minimization of the loss function based on (13) subject to the model. Since optimal policy problems are known to be time-variant, we adopt the timeless perspective and assume initial conditions away (see Woodford [2003a]) in order to get the time invariant optimal policy.

5.1 The intuition behind stabilizing fiscal policies

In the paper, fiscal policy can be either passive ($\beta^i_t = 0$), i.e. aiming at securing an optimal provision of public goods and strictly stabilizing the level of hours worked or active ($\beta^i_t \neq 0$), i.e. stabilizing the economies beyond this first motivation. The justification of active policies is related to the fact that nominal rigidities introduce quadratic distortions related to the dispersion of price within the economy.

After a domestic productivity shock, the model features two major effects: an effect directly related to nominal rigidities and a wealth effect, related to deflation and indirectly related to nominal rigidities. The first effect restricts variables adjustment with respect to their natural path. This is the case for the production of home goods since the domestic output gap is negative after a domestic productivity shock. Identically, the adjustment of final terms of trade is restricted and the gap in final terms of trade is negative. The domestic deflation generates a significant wealth effect that translates into a positive final consumption gap and into a corresponding negative gap in hours, since hours decrease more than in the flexible price situation.

The combination of these two effects creates the room for welfare improving active fiscal policies. Indeed, fiscal policies are able to bring the system closer to its natural equilibrium path since, in the model in logdeviation to the steady state, a domestic
fiscal expansion translates into an increase in production, in hours worked, in the price level and dampens private consumption through ricardian equivalence. Fiscal adjustments thus reduce the volatility of most variables entering the welfare index of households and bring the monetary union closer to the natural equilibrium.

This argument is the source of welfare improving fiscal policies in our framework, as demonstrated by Beetsma and Jensen, who also report a quantitative assessment of the corresponding welfare gains. However, in this paper, we do not focus on these gains, but rank alternative active fiscal policies in terms of the welfare distance they imply with respect to the optimal plan.

5.2 Optimal targeting rules

The optimal plan is defined by the minimization of the welfare-based loss function subject to (6)-(12). Authorities minimize the following Lagrangian,

\[ \mathcal{L} = \sum_{t=0}^{T} \beta^t E_0 \{ \ell_t \} \]

\[ + 2 \Lambda_{1,t} \left[ \pi_{h,t} - \beta \pi_{h,t+1} - k^h \left[ (\rho + \psi (1 - \kappa)) \bar{c}^u_t + \kappa \psi \bar{g}^u_t - \psi \bar{n}^r_t - \rho \bar{c}^r_t + \alpha \bar{s}_t \right] \right] \]

\[ + 2 \Lambda_{2,t} \left[ \pi_{f,t} - \beta \pi_{f,t+1} - k^f \left[ (\rho + \psi (1 - \kappa)) \bar{c}^u_t + \kappa \psi \bar{g}^u_t + \psi \bar{n}^r_t + \rho \bar{c}^r_t - \alpha \bar{s}_t \right] \right] \]

\[ + 2 \Lambda_{3,t} \left[ (\bar{s}_t - \bar{s}_{t-1}) - (\pi_{f,t} - \pi_{h,t}) + (\bar{s}_t - \bar{s}_{t-1}) \right] \].

After some algebra, first order conditions of the problem define the following optimal targeting rules,

\[ \bar{g}^u_t = 0, \quad (\kappa + \psi^{-1} \rho_g) \bar{g}^r_t = (1 - \kappa) \zeta_\alpha \bar{s}_t, \quad (1 - \kappa) \bar{c}^r_t = \bar{n}^r_t, \]

\[ \frac{\theta}{2} (\pi_{h,t} + \pi_{f,t}) + (1 - \kappa) (\bar{c}^u_t - \bar{c}^u_{t-1}) + \Lambda_{3,t} (k^h - k^f) = 0, \]

\[ \frac{\theta}{2} (\pi_{f,t} - \pi_{h,t}) + (1 - \kappa) (\bar{c}^r_t - \bar{c}^r_{t-1}) - \Lambda_{3,t} (k^h + k^f) = 0, \]

\[ (1 - \kappa) \zeta_\alpha (\bar{s}_{t-1} - \kappa \psi \bar{g}^r_{t-1}) + \alpha (1 - \kappa) \bar{c}^r_{t-1} + (\Lambda_{3,t-1} - \beta \Lambda_{3,t}) = 0, \]
augmented with the following conditions,

\[
\pi_{h,t} = \beta E_t \{\pi_{h,t+1}\} + k^h [\rho + \psi (1 - \kappa)] \bar{c}_t^u + \kappa \psi \bar{g}_t^u - \psi \bar{n}_t^r - \rho \bar{c}_t^r + \alpha \bar{s}_t, \\
\pi_{f,t} = \beta E_t \{\pi_{f,t+1}\} + k^f [\rho + \psi (1 - \kappa)] \bar{c}_t^u + \kappa \psi \bar{g}_t^u + \psi \bar{n}_t^r + \rho \bar{c}_t^r - \alpha \bar{s}_t, \\
\bar{s}_t - \bar{s}_{t-1} = \pi_{f,t} - \pi_{h,t} - (\bar{s}_t - \bar{s}_{t-1}).
\]

The corresponding recursive system requires that \( k^h = k^f = k \) for the equilibrium to be unique and stable. In the equilibrium, combining the optimal targeting rules (17) and (14) with equations (20) and (21) implies that the aggregate inflation rate is fully stabilized (\( \pi^u_t = 0 \)) and that the aggregate output gap is closed (\( \bar{g}_t^u = 0 \)). At the aggregate level, there is no room for active fiscal policy, i.e. the aggregate fiscal policy simply aims at securing the optimal aggregate level of public goods (\( \bar{g}_t^c = 0 \)). One should notice though that national inflation rates are not fully stabilized. However, the solution of the second order Ramsey problem also features some motivation for regional fiscal stabilization. Indeed, as demonstrated by equation (15), the coordinated cross-country allocation of the optimal provision of public good affects national inflation rates and the terms of trade, implying a non-zero path for national fiscal gaps.

[Insert Figure 1 here]

Figure 1 depicts the response of key variables under optimal targeting rules to a unit domestic productivity innovation. By affecting the productivity of labour, the shock generates a drop in the marginal costs of domestic firms translating into a significant deflation. Since the optimal targeting rule implies that the aggregate inflation rate is zero, a deflation at home requires that the central bank lowers its interest rate, boosting the demand in the foreign country and implying a significant inflation in this country (exactly equal to the deflation observed at home). The gap of terms of trade is negative since terms of trade do not increase as much as they would with purely flexible prices. Finally, countercyclical fiscal policies consist in a proportional reaction to final terms of trade. The negative terms of trade gap translates into a positive home public spending gap and a negative foreign public spending gap. Fiscal policies thus aim at stabilizing relative inflation rates and terms of trade quicker, i.e. bringing these variables closer to their natural paths. The shock finally implies an accumulation of net foreign assets for the home country.
6 The welfare consequences of public spending rules

6.1 Motivation

Several reasons might explain why the abovementioned targeting rules are not easy to decentralize. First, authorities are bound to commit for an infinity of periods. They may not dispose of such a commitment technology. Second, because we have assumed lump-sum taxation and balanced budgets, fiscal instruments are restricted to the amount of public spending. As a consequence, optimal targeting rules can not be fully decentralized, since they implies too many variables compared to set of instruments that authorities may actually use to stabilize the economy.

We thus adopt the optimal targeting rules as a benchmark for analyzing alternative ways of conducting stabilization policies within our model of the EMU. Following the seminal contributions of Kydland and Prescott [1977] and Taylor [1993], rules have been considered an efficient and credible mean to implement stabilization policies. Focusing on stabilizing properties of public spending rules, we ask two questions: (i) Can public spending rules replicate the stabilizing properties of the optimal plan? (ii) How close are public spending rules to welfare maximizing rules in the EMU?

First, we define $R_{\text{max}}$ as the regime of welfare maximizing policy rules and $R_{\text{est}}$ as the regime of estimated policy rules. We contrast the welfare implications of both regimes $p \in \{R_{\text{max}}, R_{\text{est}}\}$ in terms of their welfare distance to the optimal scheme $(\omega_{\text{opt},T})$ defined by (14)-(22), with respect to the implied equivalent permanent consumption loss for representative union-wide agents. We express the loss as $c_p$,

$$c_p = 100 \cdot \left[ \frac{(1 - \beta)}{(1 - \kappa) [\rho + \psi (1 - \kappa)]} \right]^{\frac{1}{2}} \left( \omega_{\text{opt},T} - \omega_{p,T} \right).$$

We simulate the model under alternative policies and compute the average welfare distance $c_p$. Fiscal rules considered in this section fall into the following generic class,

$$\hat{g}^h_t = q_h \hat{g}^h_{t-1} + d_{s,h} \hat{s}_t + d_{y,h} \hat{y}^h_t,$$

$$\hat{g}^f_t = q_f \hat{g}^f_{t-1} + d_{s,f} \hat{s}_t + d_{y,f} \hat{y}^f_t.$$
Fiscal rules are assumed to react to national output gaps and to terms of trade gap fluctuations. These rules are augmented with a smoothing component, since, as demonstrated by Woodford [2003b], agents may be averse to an excessive volatility of policy instruments.

### 6.2 Welfare maximizing public spending rules

In this paragraph, we contrast the welfare implications of welfare maximizing rules. Welfare maximizing rules are defined as the vector of parameters $\varphi^*$ that maximizes the function that associates a certain number simulated paths of the monetary union to a given value of $\varphi = \{q_{h}, q_{f}, d_{s,h}, d_{s,f}, d_{y,f}, d_{y,f}\}$ and averages the resulting aggregate welfare levels. Table 6 reports the optimized coefficients of welfare maximizing fiscal rules as well as the welfare losses they entail with respect to the optimal plan, for various parameters combinations.

![Insert Table 6 here]

Table 6 shows that welfare maximizing rules are react countercyclically to national output gaps. This result is insensitive to a wide range of parameters. The optimal response to terms of trade gaps is negative for the home country and positive for the foreign country, which is in line with the results described in the section describing the response of public spending to productivity innovations in the optimal plan.

Figure 2 depicts the IRF of the monetary union to a unit domestic productivity shock under welfare maximizing rules in the baseline case. The adjustment scheme has a lot in common with the one described in the previous section. Differences with respect to the optimal plan are: (i) a much slower stabilization of inflation rates and of terms of trade gaps and (ii) a much larger reaction of public spending gaps. These aspects translate in a greater volatility of inflation rates, that are the most weighted component of the welfare based loss function and in a larger use of the current account to adjust internationally, which is also costly with respect to the natural equilibrium where the financial market is complete.

![Insert Figure 2 here]
We now detail more precisely the welfare consequences reported in Table 6. First, the average welfare loss associated to welfare maximizing public spending rules corresponds to an average 7.3% drop in permanent consumption, much higher the losses documented by Pappa and Vassilatos [2007]. We have underlined that these losses arise because national inflation rates are much more volatile and because agents are more prone to use the financial market to smooth their consumption in case of asymmetric shock. Those losses are also related to the fact that authorities lack of instruments to place the economy closer to its optimal path.

The sensitivity analysis on preference parameters reveals that higher values of lower the welfare losses. The value of the Frischian elasticity affects the response of employment to productivity shocks. When increases, the volatility of hours dampens, thereby reducing the motivation of welfare-based stabilization policies. Welfare losses thus suggest either (i) that higher values of bring welfare maximizing rules closer to the optimal situation and (ii) that agents penalize less the distance between the two regimes, since enters the expression of the consumption equivalent. Inversely, the intertemporal elasticity of substitution impacts positively on the welfare losses.

A reduction in the home bias for final goods implies a lower distance to the optimal scheme and provides significant welfare gains. This is not surprising since a rise in lowers the volatility of relative output gaps and the volatility of the current account. Indeed, the impact of on the volatility of the current account is non-linear. Two competing effects can be contrasted when rises: (i) the volatility of the trade balance increases since households trade more and (ii) the composition of final consumptions and prices tends to become more similar. When , the first effect is stronger and the rise of translates into a higher volatility of the current account, while the second effect dominates when and lowers the volatility of the current account. Our results suggest that trade integration could bring the monetary union closer to its optimal equilibrium with simple fiscal rules. The related gain is equivalent to a 3-4% increase in permanent consumption. Finally, varying values of do not imply important welfare gains.

Summing up, welfare maximizing simple public spending rules imply welfare losses with respect to optimal targeting rules, equivalent to an average 7.3% drop in permanent consumption. However, we show that structural reforms, i.e. reforms affecting
trade openness and financial institutions in the long run, could impact on these losses. For example, an decrease of $\chi$ or an increase of $\alpha$ would be associated either with a lower need to stabilize the consequences of asymmetric shocks in the monetary union or with more effective stabilizing policies. In both cases, the equilibrium under welfare maximizing rules would be closer to the optimal solution.

6.3 Estimated public spending rules

In this subsection, we estimate public spending rules separately with a standard OLS estimator over European data, thereby assuming perfect exogeneity of these rules with respect to the estimation run in section 3. Quarterly data range from 1985:01 to 2004:04 and we choose to focus on 4 large EMU countries: Germany, France, Netherlands and Spain. Data used in the estimation imply the following variables: Share of public consumption (excluding interest payments) over the GDP, real GDP and terms trade (measured as the home CPI inflation rate relative to the aggregate EMU CPI inflation rate). Here again, we extract any exogenous event from the data. All series are then HP-filtered in order to estimate rules over short run components of interest variables. We then estimate the public spending rules over our European dataset. In order to break the functional relation between output and public spending, we estimate rules using the lagged output gap. Results of these estimations are reported in Table 7.

[Insert Table 7 here]

In Table 7, most coefficients are 5% significant and the hypothesis of correlated residuals is systematically rejected. Public spending rules are countercyclical with respect to the national output gap, which is consistent with the results in the two last sections and more generally with the motivation of stabilization policies. Furthermore, the instrument features a lot of inertia, in line with most empirical estimates. After testing panel restrictions (rejected and not reported here), we choose to calibrate the rules in both countries over the German (country $h$) and French (country $f$) estimated rules and get,

$$\hat{g}_t^h = 0.9639 \cdot \hat{g}_{t-1}^h + 0.2835 \cdot \hat{s}_t - 0.1869 \cdot \hat{y}_t^h,$$

$$\hat{g}_t^f = 0.8919 \cdot \hat{g}_{t-1}^f + 0.0059 \cdot \hat{s}_t - 0.1826 \cdot \hat{y}_t^f.$$
Table 8 reports the welfare losses of estimated policy rules with respect to the optimal plan.

First, the welfare losses of estimated fiscal rules correspond to an average 8.3% drop in permanent consumption with respect to the optimal monetary and fiscal policy. This means that additional losses, related to the difference between the estimated design of fiscal rules and the optimal design, are very low. They correspond to an average 1% drop in permanent consumption. Moreover, a glance at the welfare losses of estimated rules when reducing the smoothing component of fiscal instruments (replacing the estimated values of $q_i$ by 0.7) shows that excessive instrument inertia is responsible for most of this 1% average loss. Posing $q_i = 0.7$ lowers the additional losses with respect to welfare maximizing rules to an average 0.3% equivalent drop in permanent consumption. Consequently, estimated public spending rules are very close to their best possible design. Our results support the idea that European public spending rules, even if they feature a bit too much of instrument inertia, remain adequately designed to address asymmetric shocks in a monetary union.

The sensitivity analysis yields results similar to those of Table 6. One shall note that, instrument inertia emphasizes the welfare losses sensitivity to structural parameters variations. More especially, the positive impact of the openness parameter, $\alpha$, on welfare losses suggests that trade integration provides structural insurance against asymmetric shocks and reduces the need for fiscal policies to address these shocks. The welfare costs associated with the actual (estimated) situation could thus be significantly dampened either by reforming the SGP - by allowing budget deficits to vary more than now, or by promoting a deeper long run integration of trade in final goods within the union. The first proposition would increase the number of available instruments to reach welfare improving equilibria, while the second would reduce the need for stabilizing asymmetric shocks.
7 Conclusion

This paper has developed and estimated a two-country NOEM model of a suboptimal monetary union to address the problem of fiscal stabilization in the EMU. Deriving the optimal plan under commitment, it has shown that the active fiscal policy consisted in reacting to final terms of trade variations and implied the stabilization of several variables.

However, the corresponding optimal targeting rules cannot be fully decentralized. We therefore have considered the welfare consequences of simple public spending rules and shown that welfare maximizing rules were associated to significant welfare losses with respect to the optimal targeting rules. Our results suggest that only structural reforms, such as a reduction of home bias in final consumption, could enhance the stabilizing properties of simple instrument rules and reach welfare improving equilibria. Finally, after estimating simple instrument rules over European data, we highlighted the fact that these rules imply very low welfare losses with respect to welfare maximizing rules.

These results support the ideas that EMU members fiscal policies are adequately designed to address asymmetric shocks and suggest that welfare improving changes in the design of coordinated fiscal policies in the EMU could be achieved by relaxing some of the constraints imposed by the SGP on primary balances. Another solution could be the realization of structural reforms, such as the promotion of deeper trade integration in the long run. These reforms would dampen the need asymmetric shocks stabilization in the monetary union.
Notes

2The model features no money holdings, since money is endogenously supplied depending on the level of nominal interest rate.
3Since portfolio adjustment costs are paid in units of goods, \( AC^i_t(j) = \left[ \int_0^1 AC^i_t(k,j) \frac{\partial \Gamma}{\partial k} dk \right]^{\frac{\theta}{\theta - 1}} \), these costs imply simple variety demands, defined as \( AC^i_t(k,j) = \left[ \frac{P_{H,t}(k)}{\bar{P}_{H,t}} \right]^{-\theta} AC^i_t(j) \).
4At this stage, the introduction of a home biased market of intermediate goods could be an interesting way of introducing a substitute to labor mobility through the trade of intermediate goods. However, after estimations, this assumption did not prove to be significant in the data (home bias seemed to be very important on input markets). As a consequence, we do not detail this assumption here.
5Monopolistic competition distorts the first-best allocation through mark-up pricing and a lower output. As shown by Benigno and Woodford [2005], an optimal subsidy policy restores the optimal perfectly competitive allocation.
6Since \( G = \frac{Y}{A} \left( \frac{\pi}{\bar{\pi}} \right) \) because of (4), we simply constrain the value of \( A \) to get \( \frac{G}{Y} = \kappa \), where \( \kappa \) is the "target" level of public spending to GDP.
7This exogenous process can not be explained by the model. We consider the growth rate of raw series and replace any growth rate superior to 3% or inferior to \(-3\%\) by the average growth rate of observation \( t - 2, t - 1, t + 1 \) and \( t + 2 \). Series are then rebuilt in level, logged and HP-filtered.
8Benigno and Woodford [2006] show that the derivation of an optimal policy, requires that the model is solved with a quadratic approximation when the loss function is not purely quadratic. If the loss function is purely quadratic, the linear-quadratic procedure yields an accurate approximation of the optimal policy.
References


Figure 1: IRF to a unit domestic productivity shock under optimal targeting rules.
Figure 2: IRF to a unit domestic productivity shock under welfare maximizing rules
Table 1: The model in log-deviation to the steady state

Households
\[ \rho E_t \{ c_{i+1} - c_i^t \} = \frac{\delta}{1+\gamma} r_{i+1} - E_t [\pi_{i+1}^t] - \chi C b_{i+1}^t, \text{ for } i \in \{ h, f \} \]
\[ \psi n_i^t + \rho c_i^t = w_i^t - p_i^t, \text{ for } i \in \{ h, f \} \]

Firms
\[ \pi_{h,t} = \beta E_t \{ \pi_{h,t+1} \} + \frac{(1-\eta^h)(1-\eta^h)}{\eta^h} (w_t^h - a_t^h - p_{h,t}) \]
\[ \pi_{f,t} = \beta E_t \{ \pi_{f,t+1} \} + \frac{(1-\eta^f)(1-\eta^f)}{\eta^f} (w_t^f - a_t^f - p_{f,t}) \]

Goods markets
\[ a_t^h + n_t^h = (1-\kappa) [(1-\alpha) c_t^h + \alpha c_t^f + 2\mu \alpha (1-\alpha) s_t] + \kappa g_t^h \]
\[ a_t^f + n_t^f = (1-\kappa) [(1-\alpha) c_t^f + \alpha c_t^h - 2\mu \alpha (1-\alpha) s_t] + \kappa g_t^f \]

Exogenous shocks and public spending
\[ a_{i,t+1} = \rho_i a_i^t + \xi_{i,t+1}, \text{ for } i \in \{ h, f \} \]
\[ g_i = \frac{-\psi}{\rho_i} n_i^t, \text{ for } i \in \{ h, f \} \]

Definitions
\[ \pi_{h,t+1} = p_{h,t+1} - p_{h,t} \]
\[ \pi_{f,t+1} = p_{f,t+1} - p_{f,t} \]
\[ p_t^h = (1-\alpha) p_{h,t} + \alpha p_{f,t} \]
\[ p_t^f = (1-\alpha) p_{h,t} + \alpha p_{f,t} \]
\[ s_t = p_{f,t} - p_{h,t} \]
\[ \pi_{i,t+1} = p_{i,t+1}^i - p_i^t, \text{ for } i \in \{ h, f \} \]

Current account
\[ b_{i,t+1}^h - b_t^h = \delta b_t^h + \alpha [c_t^f - c_t^h + [2\mu (1-\alpha) - 1] s_t] \]
\[ b_{i,s}^h = -b_{i,s}^h, \forall s = t, t+1 \]
<table>
<thead>
<tr>
<th>Nominal Rigidities in the EMU</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>13.5</td>
</tr>
<tr>
<td>France</td>
<td>23.9</td>
</tr>
<tr>
<td>Italy</td>
<td>10.0</td>
</tr>
<tr>
<td>Spain</td>
<td>13.3</td>
</tr>
<tr>
<td>Netherlands</td>
<td>16.2</td>
</tr>
<tr>
<td>Belgium</td>
<td>17.6</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>23.0</td>
</tr>
<tr>
<td>Austria</td>
<td>15.4</td>
</tr>
<tr>
<td>Finland</td>
<td>20.3</td>
</tr>
<tr>
<td>Portugal</td>
<td>21.1</td>
</tr>
</tbody>
</table>

Source: Alvarez et al. (2006)

<table>
<thead>
<tr>
<th>41 moments of interest for the SMM estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Variances</strong></td>
</tr>
<tr>
<td>$\text{var}(y^h_t)$, $\text{var}(y^f_t)$, $\text{var}(c^h_t)$, $\text{var}(c^f_t)$, $\text{var}(\pi^h_t)$, $\text{var}(\pi^f_t)$, $\text{var}(n^h_t)$, $\text{var}(n^f_t)$, $\text{var}(ca^h_t)$, $\text{var}(ca^f_t)$, $\text{var}(w^h_{r,t})$, $\text{var}(w^f_{r,t})$</td>
</tr>
<tr>
<td><strong>Autocorrelations</strong></td>
</tr>
<tr>
<td>$\text{cov}(y^h_t, y^h_{t-1})$, $\text{cov}(y^f_t, y^f_{t-1})$, $\text{cov}(c^h_t, c^h_{t-1})$, $\text{cov}(c^f_t, c^f_{t-1})$, $\text{cov}(\pi^h_t, \pi^h_{t-1})$, $\text{cov}(\pi^f_t, \pi^f_{t-1})$, $\text{cov}(n^h_t, n^h_{t-1})$, $\text{cov}(n^f_t, n^f_{t-1})$, $\text{cov}(ca^h_t, ca^h_{t-1})$, $\text{cov}(ca^f_t, ca^f_{t-1})$, $\text{cov}(w^h_{r,t}, w^h_{r,t-1})$, $\text{cov}(w^f_{r,t}, w^f_{r,t-1})$</td>
</tr>
<tr>
<td><strong>Covariances</strong></td>
</tr>
<tr>
<td>$\text{cov}(y^h_t, y^f_t)$, $\text{cov}(y^h_t, c^h_t)$, $\text{cov}(y^h_t, \pi^h_t)$, $\text{cov}(y^f_t, c^f_t)$, $\text{cov}(y^f_t, \pi^f_t)$, $\text{cov}(c^h_t, n^h_t)$, $\text{cov}(c^h_t, ca^h_t)$, $\text{cov}(c^f_t, n^f_t)$, $\text{cov}(c^f_t, ca^f_t)$, $\text{cov}(y^h_t, w^h_{r,t})$, $\text{cov}(n^h_t, w^h_{r,t})$, $\text{cov}(y^f_t, w^f_{r,t})$, $\text{cov}(n^f_t, w^f_{r,t})$, $\text{cov}(y^h_{t-1}, w^h_{r,t-1})$, $\text{cov}(n^h_{t-1}, w^h_{r,t-1})$, $\text{cov}(y^f_{t-1}, w^f_{r,t-1})$, $\text{cov}(n^f_{t-1}, w^f_{r,t-1})$</td>
</tr>
</tbody>
</table>
Table 4: Structural parameters estimation

<table>
<thead>
<tr>
<th></th>
<th>( \psi )</th>
<th>( \mu )</th>
<th>( \eta^u )</th>
<th>( \rho_a )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7.4657</td>
<td>4.4441</td>
<td>0.5865</td>
<td>0.9701</td>
</tr>
<tr>
<td>Boot.</td>
<td>(1.93)</td>
<td>(2.17)</td>
<td>(3.04)</td>
<td>(9.28)</td>
</tr>
<tr>
<td>( \rho )</td>
<td>2.7128</td>
<td>0.1803</td>
<td>0.5070</td>
<td>0.0120</td>
</tr>
<tr>
<td>Boot.</td>
<td>(1.49)</td>
<td>(1.25)</td>
<td>(4.27)</td>
<td>(7.44)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.3001</td>
<td>0.0009</td>
<td>4.5431</td>
<td>0.0085</td>
</tr>
<tr>
<td>Boot.</td>
<td>(2.02)</td>
<td>(0.15)</td>
<td>(0.23)</td>
<td>(8.61)</td>
</tr>
</tbody>
</table>

- \( J - \text{stat} \)
- \( \Psi \left( \hat{\theta}_T \right) \)
- \( p - \text{value} \)

\( \chi^2(29) \)

0.5844

Table 5: The natural equilibrium

\[
\tilde{y}_t^u = \frac{\rho_y (1-\kappa)(1+\psi) + \rho \psi \eta}{\rho_y (\rho + (1-\kappa)) + \rho \psi \eta} a^u_t, \quad \tilde{g}_t^u = \frac{\rho_y (1+\psi) + \kappa \psi}{\rho_y (\rho + (1-\kappa)) + \rho \psi \eta} a^u_t
\]

\[
\tilde{g}_t = \frac{\psi [\psi - (1-\kappa)]}{\kappa \psi + \rho_y (1+2\psi \omega)} a^u_t, \quad \tilde{r}_{t+1} = \frac{\rho (1+\delta)}{\delta} \frac{[\rho_y (1+\psi) + \kappa \psi]}{\rho_y (\rho + (1-\kappa)) + \rho \psi \eta} \Delta \alpha_{t+1}
\]

\[
\tilde{s}_t = \frac{-2 \rho_y (1+\psi) + 2 \kappa \psi}{\kappa \psi + \rho_y (1+2\psi \omega)} a^r_t, \quad \tilde{c}_t^r = \frac{\rho_y (1+\psi) (1-2\alpha) + 2 \kappa \psi (1-2\alpha)}{\rho_y \kappa \psi + \rho_y (1+2\psi \omega)} a^r_t
\]

\[
\tilde{n}_t^r = \frac{\rho_y (2 \omega \alpha - 1)}{\kappa \psi + \rho_y (1+2\psi \omega)} a^r_t, \quad \tilde{g}_t = \frac{-\psi [2 \omega \alpha - 1]}{\kappa \psi + \rho_y (1+2\psi \omega)} a^r_t
\]

\[
\zeta_\alpha = \mu \alpha (1 - \alpha), \quad \omega_\alpha = \frac{(1-\kappa)(1-2\alpha)^2 + 4 \kappa \alpha}{2 \rho}
\]

Table 6: Welfare analysis - welfare maximizing rules

<table>
<thead>
<tr>
<th></th>
<th>( c_{\hat{R}_{max}} )</th>
<th>( q_h )</th>
<th>( q_f )</th>
<th>( d_{s,h} )</th>
<th>( d_{a,h} )</th>
<th>( d_{y,h} )</th>
<th>( d_{y,f} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>7.2921</td>
<td>0.21</td>
<td>0.40</td>
<td>-0.33</td>
<td>0.31</td>
<td>-0.51</td>
<td>-0.31</td>
</tr>
<tr>
<td>( \psi = 10 )</td>
<td>6.6020</td>
<td>0.26</td>
<td>0.45</td>
<td>-0.42</td>
<td>0.35</td>
<td>-0.55</td>
<td>-0.35</td>
</tr>
<tr>
<td>( \psi = 20 )</td>
<td>4.9599</td>
<td>0.44</td>
<td>0.50</td>
<td>-0.75</td>
<td>0.65</td>
<td>-0.10</td>
<td>-0.11</td>
</tr>
<tr>
<td>( \rho = 5 )</td>
<td>7.0922</td>
<td>0.35</td>
<td>0.42</td>
<td>-0.26</td>
<td>0.16</td>
<td>-0.37</td>
<td>-0.42</td>
</tr>
<tr>
<td>( \rho = 10 )</td>
<td>6.5045</td>
<td>0.41</td>
<td>0.08</td>
<td>0.05</td>
<td>0.34</td>
<td>-0.57</td>
<td>-0.50</td>
</tr>
<tr>
<td>( \alpha = 0.35 )</td>
<td>5.2922</td>
<td>0.42</td>
<td>0.11</td>
<td>-0.30</td>
<td>0.45</td>
<td>-0.33</td>
<td>-0.51</td>
</tr>
<tr>
<td>( \alpha = 0.4 )</td>
<td>3.4709</td>
<td>0.01</td>
<td>0.04</td>
<td>0.07</td>
<td>0.93</td>
<td>-1.15</td>
<td>-1.17</td>
</tr>
<tr>
<td>( \mu = 2 )</td>
<td>7.5613</td>
<td>0.00</td>
<td>0.15</td>
<td>-0.39</td>
<td>0.28</td>
<td>-0.05</td>
<td>-0.12</td>
</tr>
<tr>
<td>( \mu = 10 )</td>
<td>7.1861</td>
<td>0.56</td>
<td>0.42</td>
<td>-0.35</td>
<td>0.27</td>
<td>-0.77</td>
<td>-0.88</td>
</tr>
</tbody>
</table>
Table 7: Estimation of fiscal rules in the EMU

<table>
<thead>
<tr>
<th></th>
<th>GER</th>
<th>SPA</th>
<th>fRA</th>
<th>NETh</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$</td>
<td>0.9639</td>
<td>0.6343</td>
<td>0.8919</td>
<td>0.7464</td>
</tr>
<tr>
<td>(t-stat)</td>
<td>(14.73)</td>
<td>(6.66)</td>
<td>(13.22)</td>
<td>(8.24)</td>
</tr>
<tr>
<td>$d_y$</td>
<td>-0.1869</td>
<td>-0.0754</td>
<td>-0.1826</td>
<td>-0.2478</td>
</tr>
<tr>
<td>(t-stat)</td>
<td>(-3.15)</td>
<td>(-2.41)</td>
<td>(-6.95)</td>
<td>(-7.44)</td>
</tr>
<tr>
<td>$d_s$</td>
<td>0.2835</td>
<td>-0.0286</td>
<td>0.0059</td>
<td>-0.0197</td>
</tr>
<tr>
<td>(t-stat)</td>
<td>(6.28)</td>
<td>(-0.92)</td>
<td>(0.32)</td>
<td>(-1.33)</td>
</tr>
<tr>
<td>cste</td>
<td>-0.0017</td>
<td>0.0000</td>
<td>-0.0001</td>
<td>0.0000</td>
</tr>
<tr>
<td>(t-stat)</td>
<td>(-3.08)</td>
<td>(0.14)</td>
<td>(-1.77)</td>
<td>(0.26)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.8015</td>
<td>0.5821</td>
<td>0.9352</td>
<td>0.8207</td>
</tr>
</tbody>
</table>

Ljung-Box P-value

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1 lag</td>
<td>0.0853</td>
<td>0.1551</td>
<td>0.2215</td>
<td>0.3634</td>
</tr>
<tr>
<td>4 lags</td>
<td>0.2766</td>
<td>0.1010</td>
<td>0.0525</td>
<td>0.3439</td>
</tr>
</tbody>
</table>

Table 8: Welfare analysis - estimated rules

<table>
<thead>
<tr>
<th></th>
<th>$c_{R_{\text{max}}}$</th>
<th>$c_{R_{\text{est}}}$</th>
<th>$c_{R_{\text{est}}}$ with $q_i = 0.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>7.2921</td>
<td>8.2675</td>
<td>7.5322</td>
</tr>
<tr>
<td>$\psi = 10$</td>
<td>6.6020</td>
<td>7.6223</td>
<td>6.8795</td>
</tr>
<tr>
<td>$\psi = 20$</td>
<td>4.9599</td>
<td>6.1096</td>
<td>5.3439</td>
</tr>
<tr>
<td>$\rho = 5$</td>
<td>7.0922</td>
<td>7.6834</td>
<td>7.2801</td>
</tr>
<tr>
<td>$\rho = 10$</td>
<td>6.5045</td>
<td>6.8602</td>
<td>6.6500</td>
</tr>
<tr>
<td>$\alpha = 0.35$</td>
<td>5.2922</td>
<td>5.6824</td>
<td>5.4178</td>
</tr>
<tr>
<td>$\alpha = 0.4$</td>
<td>3.4709</td>
<td>3.6023</td>
<td>3.5312</td>
</tr>
<tr>
<td>$\mu = 2$</td>
<td>7.5613</td>
<td>11.7500</td>
<td>8.1712</td>
</tr>
<tr>
<td>$\mu = 10$</td>
<td>7.1861</td>
<td>7.2351</td>
<td>7.3073</td>
</tr>
</tbody>
</table>
Appendix to "Fiscal Policy in an Estimated Model of the EMU"

A Price Dispersion in Calvo Price Setting Models

Following Galí and Monacelli [2005], the variations of Price Dispersion are of second order. We begin with a second order approximation of,

$$\left[ \frac{P_{i,t}(k)}{P_{i,t}} \right]^{1-\theta} \approx 1 + \frac{(1 - \theta)^2}{2} \widehat{p}_{i,t}(k)^2 - (1 - \theta) \widehat{p}_{i,t}(k),$$

with $\widehat{p}_{i,t}(k) = p_{i,t} - p_{i,t}(k)$. Integrating among firms implies that,

$$\int_0^1 \left[ \frac{P_{i,t}(k)}{P_{i,t}} \right]^{1-\theta} dk \approx 1 + \frac{(1 - \theta)^2}{2} E_k \left[ \widehat{p}_{i,t}(k)^2 \right] + (1 - \theta) E_k \left[ \widehat{p}_{i,t}(k) \right] = 1.$$

Thus,

$$\frac{(1 - \theta)^2}{2} E_k \left[ \widehat{p}_{i,t}(k)^2 \right] = (\theta - 1) E_k \left[ \widehat{p}_{i,t}(k) \right].$$

A second order approximation of Price Dispersion yields,

$$\left[ \frac{P_{i,t}(k)}{P_{i,t}} \right]^{-\theta} \approx e^{-\theta \widehat{p}_{i,t}(k)} \approx 1 + \left[ \frac{\theta^2}{2} \widehat{p}_{i,t}(k)^2 - \theta \widehat{p}_{i,t}(k) \right].$$

integrating among firms, we get,

$$\int_0^1 \left[ \frac{P_{i,t}(k)}{P_{i,t}} \right]^{-\theta} dk \approx 1 + \frac{\theta^2}{2} E_k \left[ \widehat{p}_{i,t}(k)^2 \right] - \theta E_k \left[ \widehat{p}_{i,t}(k) \right],$$

Thanks to (23), we get,

$$\int_0^1 \left[ \frac{P_{i,t}(k)}{P_{i,t}} \right]^{-\theta} dk \approx 1 + \frac{\theta}{2} E_k \left[ \widehat{p}_{i,t}(k)^2 \right].$$
which according to the law of large numbers can be approximated by,

$$D P_{i,t} = \int_0^1 \left[ \frac{P_{i,t}(k)}{P_{i,t}} \right]^{-\theta} \, dk \simeq 1 + \frac{\theta}{2} \text{var}_k \left[ p_{i,t} \right].$$

Thus, the first order approximation of Price Dispersion is,

$$d p_{i,t} = 0,$$

B The Natural Equilibrium

We get most of natural expression with $\eta^h = \eta^f = 0$. It implies, $mc_t^h - p_{h,t} = 0$ and $mc_t^f - p_{f,t} = 0$. Adding, we get

$$w_t^u - p_t^u = a_t^u.$$

where $x_t^u = \frac{1}{2} \left[ x_t^h + x_t^f \right]$. The union leisure-consumption arbitrage is,

$$\psi n_t^u + \rho c_t^u = w_t^u - p_t^u.$$

The union production function is,

$$y_t^u = a_t^u + n_t^u,$$

and union final goods markets equilibrium is,

$$y_t^u = (1 - \kappa)c_t^u + \kappa g_t^u.$$

Combining these relations, with,

$$\tilde{g}_t^i = -\frac{\psi}{\rho_g} n_t^i,$$
we get,

\[
\begin{align*}
\tilde{y}^\mu_t &= \frac{\rho_g (1 - \kappa) (1 + \psi) + \rho \kappa \psi}{\rho_g [\rho + \psi (1 - \kappa)] + \rho \kappa \psi} a^\mu_t, \\
\tilde{g}^\mu_t &= \frac{\psi [\rho - (1 - \kappa)]}{\rho_g [\rho + \psi (1 - \kappa)] + \rho \kappa \psi} a^\mu_t, \\
\tilde{c}^\mu_t &= \frac{\rho_g (1 + \psi + \kappa)}{\rho_g [\rho + (1 - \kappa) \psi] + \rho \kappa \psi} a^\mu_t.
\end{align*}
\]

In the natural equilibrium, the financial market is complete, implying,

\[2 \rho \tilde{c}^\tau_t = -(1 - 2 \alpha) \tilde{s}_t.\]

Using the equilibrium conditions on final goods markets, we get,

\[\tilde{y}^\tau_t = -\omega_\alpha \tilde{s}_t - \frac{\kappa \psi}{\rho_g} \tilde{n}_t^\tau,\]

where \(\omega_\alpha = \mu \alpha (1 - \alpha)\) and \(\tilde{\omega}_\alpha = \frac{(1 - \kappa)((1 - 2 \alpha)^2 + 4 \rho \kappa \alpha)}{2 \rho}\). Now, the definition of final terms of trade implies,

\[\tilde{s}_t = \tilde{p}_{f,t} - \tilde{p}_{h,t} = 2 \tilde{m} c^\tau_t.\]

Since,

\[\tilde{m} c^\tau_t = \tilde{w}_t^\tau - a^\tau_t,\]

we get,

\[\tilde{s}_t = 2 (\tilde{w}_t^\tau - a^\tau_t) .\]

Using the relative version of consumption-leisure arbitrage conditions

\[\psi \tilde{n}_t^\tau = \tilde{w}_t^\tau = \frac{1}{2} \tilde{s}_t + a^\tau_t,\]

Combining the relative production function with the above expression of \(\tilde{y}^\tau_t\), we get,

\[\tilde{n}_t^\tau = \tilde{y}_t^\tau - a^\tau_t = -\omega_\alpha \tilde{s}_t - \frac{\kappa \psi}{\rho_g} \tilde{n}_t^\tau - a^\tau_t,\]

36
implying,
\[ \tilde{n}_t^r = - \frac{\rho_g \psi}{\rho_g + \kappa \psi} \tilde{s}_t - \frac{\rho_g}{\rho_g + \kappa \psi} a_t^r \]

Combining with,
\[ \psi \tilde{n}_t^r = \frac{1}{2} \tilde{s}_t + a_t^r, \]
we finally get,
\[ \tilde{s}_t = - \left[ \frac{2 \rho_g (1 + \psi) + 2 \kappa \psi}{\kappa \psi + \rho_g (1 + 2 \psi \omega_\alpha)} \right] a_t^r, \]
\[ \tilde{c}_t, \tilde{n}_t^r \text{ and } \tilde{g}_t^r \text{ finally write,} \]
\[ \tilde{s}_t = - \frac{2 \rho_g (1 + \psi) + 2 \kappa \psi}{\kappa \psi + \rho_g (1 + 2 \psi \omega_\alpha)} a_t^r, \]
\[ \tilde{c}_t = \frac{\rho_g (1 + \psi)(1 - 2\alpha) + \kappa \psi (1 - 2\alpha)}{\rho \left[ \kappa \psi + \rho_g (1 + 2 \psi \omega_\alpha) \right]} a_t^r, \]
\[ \tilde{n}_t^r = \frac{\rho_g [2 \omega_\alpha - 1]}{\kappa \psi + \rho_g (1 + 2 \psi \omega_\alpha)} a_t^r, \]
\[ \tilde{g}_t^r = - \frac{\psi [2 \omega_\alpha - 1]}{\kappa \psi + \rho_g (1 + 2 \psi \omega_\alpha)} a_t^r. \]