Stabilizing Fiscal Policy with Capital Market Imperfections *

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Abstract

We analyse how fiscal policy can affect aggregate volatility and growth in economies subject to capital market imperfections. Within a model featuring both frictions on the capital market and unequal access to investment opportunities among individuals, we show that appropriate fiscal policy parameters are able to rule out the occurrence of slump regimes, and immunize the economy against endogenous fluctuations in GDP, investment and interest rates. For given levels of the credit market development, we provide specific fiscal parameters able to insulate the economy from crises, fuel its long-run growth rate and place it on a permanent boom dynamic path. We analyse how conditions on the stabilizing fiscal parameters are modified when frictions in the economy evolve. Eventually, we study how the tax system impacts the economy’s response to temporary and permanent productivity shocks.

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1 Introduction

The growth and amplification effects of capital market frictions have been the subject of a large literature. At the aggregate level, a number of studies have highlighted the importance of credit constraints in explaining fluctuations in activity (see, in particular, Bernanke [10], Eckstein and Sinai [18], and Friedman [24]). Bernanke and Gertler [11] develop a simple neoclassical model of the business cycle in which the condition of borrowers' balance sheets is a source of output dynamics. Higher borrower net worth reduces the agency costs of financing real capital investments. Business upturns improve net worth, lower agency costs, and increase investment, which amplifies the upturn; vice versa, for downturns. Shocks that affect net worth (as in a debt-deflation) can initiate fluctuations. Kiyotaki and Moore [36] construct a model of a dynamic economy in which lenders cannot force borrowers to repay their debts
unless the debts are secured. The dynamic interaction between credit limits and asset prices turns out to be a powerful transmission mechanism by which the effects of shocks persist, amplify, and spill over to other sectors. They show that small, temporary shocks to technology or income distribution can generate large, persistent fluctuations in output and asset prices. Holmstrom and Tirole [30] study an incentive model of financial intermediation in which firms as well as intermediaries are capital constrained. They show that all forms of capital tightening (a credit crunch, a collateral squeeze, or a savings squeeze) hit poorly capitalized firms the hardest, but that interest rate effects and the intensity of monitoring will depend on relative changes in the various components of capital. Bernanke, Gertler and Gilchrist [13] develop a dynamic general equilibrium model intended to help clarify the role of credit market frictions in business fluctuations, from both a qualitative and a quantitative standpoint. The framework exhibits a "financial accelerator", i.e. endogenous developments in credit markets work to amplify and propagate shocks to the macroeconomy. Their use of money and price stickyness allows them to study how credit market frictions may influence the transmission of the monetary policy. In addition, they allow for lags in investment which enables the model to generate both hump-shaped output dynamics and a lead-lag relation between asset prices and investment, as is consistent with the data. Finally, they allow for heterogeneity among firms to capture the fact that borrowers have differential access to capital markets. Under reasonable parametrizations of the model, the financial accelerator has a significant influence on business cycle dynamics.

The purpose of this paper is to analyse how fiscal policy can affect aggregate volatility and growth in economies subject to capital market imperfections. The model is based upon Aghion, Banerjee and Piketty [3], in which the combination of frictions on the capital market and unequal access to investment opportunities among individuals can generate endogenous and permanent fluctuations in aggregate GDP, investment and interest rates. In this setup, savers and investors are separated along two dimensions: first, a pure simple physical separation, since many people who save are in no position to invest directly in physical capital (as distinct from financial capital); and a more market based separation embodied in the constraints on the amounts investors can borrow from savers. Aghion et al. [3] show that when the credit market development is high enough, and the separation between savers and investors is low enough, the economy can stay in a permanent boom regime. In contrast, a high degree of such separation leads the economy to fluctuate around its steady-state growth path. More specifically, it is shown that under a relatively high degree of physical separation of savers and investors and a poorly functioning capital market, the economy will always converge to a cycle around its trend growth path, unless the capital market frictions are so high that the economy falls into a
permanent slump regime. Economies with less developed financial markets and a sharper physical separation between savers and investors will then tend to be more volatile and grow more slowly. For a number of obvious reasons, both of these dimensions of separation are likely to be greater in emerging market economies. However, there is at least some evidence that this kind of mechanism based on the functioning of the credit market is also relevant for understanding the business cycle properties of more developed economies. For instance, this type of analysis may shed some light on the case of advanced market economies such as Finland, where financial development is still lagging behind and which has experienced high macroeconomic volatility over the past decade (see Honkapohja and Koskela [31]). Moreover, even in financially developed economies like the United States, the analysis remains relevant for the case of small investors whose investments turn out to be significantly correlated with current cash flows.

The contribution of the paper is to show that appropriate fiscal policy parameters are able to rule out the occurrence of slump regimes, and immunize the economy against endogenous and permanent fluctuations in GDP, investment and interest rates. For given levels of the credit market development in the economy, we provide specific fiscal parameters able to insulate the economy from crises, fuel its long-run growth rate and place it on a permanent boom dynamic path.

The main mechanism driving our result is the following. The fiscal policy we analyse, introducing a tax on savers' labor income and redistributing the proceeds into investors' wealth, is tantamount to an increase in the fraction of the labor force having direct access to capital investment opportunities (and therefore to a decrease in the fraction of agents unable to invest directly in the production process). More precisely, a structural policy that would remove institutional obstacles and rigidities separating savers and investors to promote growth, stability and equity at the same time, would presumably have a similar effect on the economy's dynamics.

We analyse how the conditions on the stabilizing fiscal parameters are modified when frictions in the economy evolve. Eventually, we study how the tax system impacts the response of the economy to temporary and permanent productivity shocks. Typically, aside from its direct growth-enhancing effects, it is shown that this type of fiscal policy moderates the wealth distribution effects following a productivity shock in a slump episode.

Our findings complement the conclusions of Aghion et al. [3]. In the last part of their paper, they suggest a government could absorb idle savings in the economy by public debt issuance, and finance investment subsidies so as to counteract the limited debt capacity of investors in slumps. However, in many actual economies, the public debt option is a constrained policy tool. In contrast, labor income taxation is a feature shared by the quasi totality of modern
economies. It is then interesting to see in our setup that basic income taxation can also have stabilizing properties in economies where capital markets are subject to frictions. Another dimension in which our paper departs from Aghion et al. [3] is that we provide an exhaustive analytic characterization of dynamic regimes possibilities, depending on the tax rate level and the friction parameters values.

Our results can be linked to a parallel strand of the macroeconomic literature, investigating the stabilizing properties of fiscal policies on local indeterminacy and belief-driven endogenous fluctuations. Many papers in that field (e.g., among others, Dromel and Pintus [16]) show that flat rate taxation is not efficient in stabilizing business fluctuations. In contrast here, a standard linear tax does have an effect on dynamics, and can effectively isolate the economy from (another type of) endogenous cycles (than those analyzed in the sunspot literature).

The remainder of this paper proceeds as follows. The next section lays out the model, while section 3 analyses dynamics and regime change conditions, showing how fiscal policy may help getting out of slumps, cycles, and fuel the growth rate. Section 4 discusses the comparative statics properties of the suggested stabilizing and growth-enhancing fiscal policy. Section 5 investigates how this policy changes the response of the economy to productivity shocks. Section 6 briefly discusses the question of Welfare in this setup, and announces directions for further research. Some concluding remarks are gathered in section 7.

2 The Economy

This paper introduces fiscal policy, through labor-income taxation and lump-sum transfers, into the positive long-run growth AK model with capital market imperfections studied in Aghion et al. [3]. For ease of comparison with this benchmark model, we keep the same notations and dynamic analysis methods.

2.1 Production Technology

An homogeneous good is produced and serves both as capital and as a consumption commodity. In each period $t \in \mathbb{N}$, agents are endowed with one unit of time.
The good is produced according to the technology: \( F(K, L) = AK^\beta L^{1-\beta} = Y \). We assume the growth rate of the workforce to be at least equal to the one of the capital stock. Then, all agents are willing to work at a wage greater than or equal to one, so that the equilibrium labor price can be set to unity.

**Assumption 2.1.**

\[
\frac{\partial F}{\partial L} = 1 \Rightarrow L = ((1-\beta)A)^{1/\beta} K \Rightarrow Y = \sigma K \text{ with } \sigma = A((1-\beta)A)^{(1-\beta)/\beta}
\]

The parameter \( \beta \) stands for the capital share in final output, whereas \((1 - \beta)\) denotes the labor share.

### 2.2 Dualism

The economy is physically split into two categories of agents. Only a fraction \( 0 \leq \mu \leq 1 \) of the workforce (called the *productive investors*) can directly invest in physical capital. The other individuals (called the *savers*), can either lend their savings to the productive investors at current interest rate \( r \), or invest in a low-yield asset with a return \( \sigma_2 < \sigma_1 = \beta \sigma \). When \( \mu \) increases from 0 to 1, the separation between savers and investors becomes thinner.

Due to asymmetric information issues (moral hazard), capital market is subject to a borrowing constraint. There is a constant \( 0 \leq \nu \leq 1 \) such that anyone who wants to invest an amount \( I \) must have assets of at least \( \nu I \). In other words, \( 1/\nu \) is nothing else than a credit multiplier. Indeed, when \( \nu \) decreases from 1 to 0, credit market development improves.

### 2.3 Interest rate setting

The AK type technology used in this framework implies that the equilibrium interest rate will take two possible values. When investment exceeds savings (i.e., demand for savings is (very) large), the gross interest rate will take its "high" value \( \sigma_1 = \beta \sigma \). In contrast, when savings are in excess supply, the gross equilibrium interest rate will drop to \( \sigma_2 < \sigma_1 \).
2.4 Government

The government chooses tax policy and balances the budget at each point in time. The public authority is assumed to care about the level of productive investment in the economy. To this end, linear taxes are applied on savers’ labor income, to finance a transfer than can be thought of as an investment subsidy.\(^1\)

2.5 Timing of the model

In the beginning of a period (say, in the morning), the respective amounts of planned investment and available savings in the economy are compared. Depending on their relative magnitude, the interest rate is set. If investment runs ahead of savings, the higher interest rate prevails, and the non-investors are willing to lend all their savings to the productive investors. Thus, during these boom episodes, all available savings in the economy will be invested in the high-yield activity. In contrast, if investment plans are not large enough to absorb all available savings, the interest rate will be set at its low value \(\sigma_2\). Then savers will be indifferent between issuing low-return loans, or investing in the low-yield asset. At the end of the day, returns to investment are realized, borrowers pay back their debt to lenders, wages are paid. Taxes are also levied and redistribution occurs. Consumption finally takes place, from the net resources of the day. For sake of simplicity, and to ease comparisons with the Aghion et al. [3] benchmark model, a linear savings rate is assumed, as in the case of a standard logarithmic utility function. The non-consumed part of the day’s net resources constitutes the amount of available savings in the next morning.

2.6 Agents’ Wealth Accumulation

Let \(W^t_B\) and \(W^t_L\) respectively represent the wealth levels of the borrowers (productive investors) and of the lenders (savers) in the morning of period \((t+1)\). We denote by \(S_t\) the total amount of savings: \(S_t = W^t_B + W^t_L\), and \(I^t_{t+1} = W^t_B/\nu\) the total planned investment in the morning of period \((t + 1)\).

In a boom, the investment capacity of investors is higher than the available amount of savings \((I^t_{t+1} \geq S_t)\). The prevailing interest rate is \(\sigma_1 = \beta\sigma\), such that all aggregate savings \((W^t_B + W^t_L)\) are invested in the high-yield activity.\(^1\)

\(^1\)Even though we choose to present the model with an exceedingly pared-down fiscal structure, the following results are robust to the introduction of a richer tax structure, as long as the fiscal scheme is tantamount to a net transfer to investors.
The wealth accumulation of borrowers and lenders can be summarized as follows:

**BOOM**

\[
W^{t+1}_B = (1 - \alpha)\left[\mu(1 - \beta)\sigma(W^t_B + W^t_L) + \beta\sigma(W^t_B + W^t_L) - \beta\sigma W^t_L + T^t\right] \\
W^{t+1}_L = (1 - \alpha)\left[(1 - \tau)(1 - \mu)(1 - \beta)\sigma(W^t_B + W^t_L) + \beta\sigma W^t_L\right]
\]

The total return of the high-yield activity \(\sigma(W^t_B + W^t_L)\), is shared between labor income with a fraction \((1 - \beta)\), and capital income. Productive investors (resp. savers) represent a fraction \(\mu\) (resp. \(1 - \mu\)) of the total labor share in output. Only borrowers take advantage of the return \(\beta\sigma\) on physical capital investment, but have to refund and pay the high level interest rate on the amount they borrowed (the whole \(W^t_L\), since at the high interest rate \(\sigma_1 = \beta\sigma\), investing in the low-yield asset is a dominated strategy for lenders). To support productive investment, the government operates a transfer of resources by taxing saver’s labor income at rate \(0 < \tau < 1\) and redistributing the proceeds \(T^t\) in the form of an investment subsidy.\(^2\)

As public budget is balanced in each period:

\[
T^t = \tau(1 - \mu)(1 - \beta)\sigma(W^t_B + W^t_L)
\]

Hence, we can re-write the borrowers’ wealth motion equation in a boom as:

\[
W^{t+1}_B = (1 - \alpha)\left[\mu + \tau(1 - \mu)(1 - \beta)\sigma(W^t_B + W^t_L) + \beta\sigma W^t_L\right]
\]

In a slump, the investment capacity of productive investors is lower than the level of aggregate savings \((I^t_{t+1} < S_t)\). The prevailing interest rate is then \(\sigma_2 < \beta\sigma\), such that only \(\frac{W^t_B}{\nu}\) can be invested in the high-yield activity, generating a total revenue equal to \(\sigma\frac{W^t_L}{\nu}\).

**SLUMP**

\[
W^{t+1}_B = (1 - \alpha)\left[\mu(1 - \beta)\frac{1}{\nu}W^t_B + \beta\frac{1}{\nu}W^t_B - \sigma_2(\frac{1}{\nu} - 1)W^t_B + T^t\right] \\
W^{t+1}_L = (1 - \alpha)\left[(1 - \tau)(1 - \mu)(1 - \beta)\frac{1}{\nu}W^t_B + \sigma_2(\frac{1}{\nu} - 1)W^t_B + \sigma_2(W^t_L - (\frac{1}{\nu} - 1)W^t_B)\right]
\]

\(^2\)Let us notice at this point that borrowers’ labor and capital incomes could also be taxed, and lenders could as well benefit from some kind of transfers. But as the main following results rely on a positive tax net of transfers levied on savers, and a positive transfer net of taxes granted to borrowers, we choose to be parcimonious and keep the exposition of the fiscal scheme as simple as possible.
The total revenue $\sigma \frac{W_t}{\nu}$ remunerates labor up to a fraction $(1 - \beta)$, with borrowers (resp. lenders) getting a share $\mu$ (resp. $1 - \mu$) of that wage income. Only the borrowers get the fraction $\beta$ of the total revenue, remunerating physical capital investment. As productive investors have actually borrowed $W_B^t/\nu - W_B^t$, they repay this amount to the lenders with the interest $\sigma_2$ prevailing in a slump period. Aside from this repayment, lenders get also the return $\sigma_2$ from investing the rest of their savings in the low-yield activity. Once again, the government operates a transfer of resources by taxing saver’s labor income and redistributing the proceeds in the form of an investment subsidy.

As the public budget is balanced, we can re-write the borrowers’ wealth motion equation in a slump as:

$$W_B^{t+1} = (1 - \alpha) \{\mu + \tau(1 - \mu)\} (1 - \beta) \sigma_1 \nu W_B^t + \beta \sigma_2 W_L^t$$

3 Analysis of the Dynamics

Defining $q_t = \frac{S_t}{I_{t+1}} = \frac{W_B^t + W_L^t}{W_B^t} \nu$ as the ratio of aggregate savings over investment plans in the high-yield activity in the morning of period $(t+1)$, we can obtain from the previous wealth motion laws the two following difference equations, allowing the global dynamics analysis of this economy.

When at the beginning of period $t+1$ planned investment runs ahead of savings ($q_t \leq 1$), the economy is in a boom:

$$\frac{1}{q_t^{t+1}} = \frac{\mu + \tau(1 - \mu)}{\nu} (1 - \beta) + \frac{\beta}{q_t^t}$$

In contrast, when $q_t > 1$ the economy is experiencing a slump:

$$q_t^{t+1} = \frac{[(\sigma - \sigma_2) + \sigma_2 q_t^t]}{(\mu + \tau(1 - \mu)(1 - \beta)\sigma/\nu) + \beta \sigma / \nu - (\frac{1}{\nu} - 1)\sigma_2}$$

It is worth noticing that if we set $\tau$ to zero, we recover the benchmark model of Aghion et al. [3].

These two difference equations behavior can be studied graphically in the $(q_t, q_t^{t+1})$ plane. It is straightforward to show (cf. Appendix A) that (BB) is monotonic, increasing and concave while (SS) is linearly increasing. As shown by Aghion et al. [3], there are only three dynamic regimes the economy can actually experience, corresponding to the three possible rankings between 1, b and s, where s and b are steady-state values of the savings to planned investment ratio, respectively determined by the intersections between (SS) and (BB) with the 45° line. When $q_t \leq 1$ (i.e., when
the planned investment volume is higher than the aggregate savings amount), only the (BB) curve is relevant, while if 
\( q^t > 1 \), only the (SS) locus prevails.

The steady-state savings to planned investment ratio in a boom writes as 
\[ b = \frac{\nu}{\mu + \tau(1-\mu)}. \]
As soon as \( b \leq 1 \), the economy is in a permanent boom.

— Figure 1 about here —

From any initial \( q^t < 1 \), the economy will converge to \( b \), and the long-run growth rate is nothing else than the Harrod-Domar one, that is the product of the savings rate by the average productivity of capital \( g^* = (1-\alpha)\sigma \) (cf. Appendix B). The condition for a permanent boom can also be written in terms of the fiscal parameter \( \tau \): the economy will experience a permanent boom regime if and only if

\[ \tau \geq \frac{\nu - \mu}{1 - \mu} = \tau_b \]

Increasing \( \tau \) lowers the level of (BB) in the plane and the savings to planned investment steady-state level \( b \). The ordinate to origin of (BB) \( q^{t+1}|_{q^t=0} = 0 \) remains equal to zero, whatever the tax rate.

The steady-state savings to planned investment ratio in a slump writes as 
\[ s = \frac{(\sigma - \sigma_2)\nu}{\mu + \tau(1-\mu)(1-\beta)\sigma + \beta \sigma - \sigma_2}. \]
As soon as \( s > 1 \), the economy will go through a permanent slump.

— Figure 2 about here —

The condition for the permanent slump can be also stated as function of the fiscal parameter \( \tau \):

\[ \tau < \frac{\nu(\sigma - \sigma_2) + \sigma_2 - \beta \sigma - \mu(1-\beta)\sigma}{(1-\beta)\sigma(1-\mu)} = \tau_s \]

Increasing \( \tau \) lowers the level of (SS) in the plane and the steady-state savings to planned investment ratio \( s \). The ordinate to origin of (SS) drops when the tax rate is increased since \( \frac{\partial}{\partial \tau} (q^{t+1}|_{q^t=0}) < 0 \). The long-run growth rate in a slump can be written as 
\[ g_s = \frac{(1-\alpha)}{\nu} \{[\mu + \tau(1-\mu)](1-\beta)\sigma + \beta \sigma - \sigma_2(1-\nu)} = (1-\alpha)\{\frac{\sigma}{\nu} + (1-\frac{1}{\nu})\sigma_2\}, \]
and clearly depends positively on the fiscal parameter \( \tau \). Since, \( b-s = \frac{\nu(\beta \sigma - \sigma_2)(1-\tau)(1-\mu)}{[\mu + \tau(1-\mu)](1-\beta)\sigma + \beta \sigma - \sigma_2} > 0 \), we know that \( b \) is
always greater than \( s \). However, the distance between the two steady-state savings to planned investment ratios shrinks as soon as \( \tau \) increases from 0 to 1.

The remaining case corresponds to the intermediary situation where \( \tau_s < \tau < \tau_b \). In that situation, a cyclical regime prevails.

--- Figure 3 about here ---

The economy will then keep back and forth between episodes of booms and periods of slumps, and will eventually converge to a limit cycle, which periodicity depends upon some deep parameters of the model. The logic behind the cycles is the following. Periods of slow growth are periods when savings are plentiful relative to the limited debt capacity of potential investors, which implies a low demand for savings and therefore low equilibrium interest rates. This in turn implies that the investors can retain a high proportion of their profits (since the interest rate and hence the debt burden on investors is low), which allows them to rebuild their reserves and debt capacity and expand their investment. This in turn, generates more profits and more investment until, eventually, planned investment runs ahead of savings forcing the interest rates to rise. Then, the debt burden on the investors will to be higher, retained earnings will be lower, and investment will collapse, taking the economy back to a period of slower growth.

We can summarize the different dynamic regimes by the following proposition:

**Proposition 3.1** (Stabilizing and Growth-Enhancing Fiscal Policy).

1. When \( \nu < \mu \), the economy is in a permanent boom whatever \( \tau \). This case is covered in Aghion et al. [3]

2. When \( \mu < \nu \), the economy

   1. is in a permanent boom if \( \tau > \frac{\nu - \mu}{1 - \mu} = \tau_b \Leftrightarrow b < 1 \)
   
   2. is in a permanent slump if \( \tau < \frac{\nu(\sigma - \sigma_2) + \beta(\sigma - \sigma_0)(1 - \beta)\sigma}{(1 - \beta)\sigma(1 - \mu)} = \tau_s \Leftrightarrow s > 1 \).
   
   3. cycles if \( \tau_s < \tau < \tau_b \Leftrightarrow s < 1 < b \).

Let \( \nu = \nu(\mu) = \frac{(1 - \beta)\sigma + \beta\sigma - \sigma_2}{\sigma - \sigma_2} \Leftrightarrow \tau_s = 0 \). We will have \( \tau_b > 0 \Leftrightarrow \nu > \mu \) and \( \tau_s > 0 \Leftrightarrow \nu > \nu(\mu) \).
• For any \( \mu < \nu < \bar{\nu} \) : if \( 0 < \tau < \tau_b \) then the economy experiences a cyclical regime, alternating between phases of expansion and downturns; if \( \tau_b < \tau \), the economy is in a permanent boom, which long-run growth rate is the Harrod-Domar one.

• For any \( \bar{\nu} < \nu < 1 \) : if \( 0 < \tau < \tau_s \), the economy is trapped into a permanent slump regime; if \( \tau_s < \tau < \tau_b \) then the economy experiences a cyclical regime alternating between phases of expansion and downturns; if \( \tau_b < \tau \), the economy is in a permanent boom, which long-run growth rate is the Harrod-Domar one.

Similarly, let \( \mu = \bar{\mu}(\nu) = \frac{\nu(\sigma - \sigma_2) + \sigma - \beta \sigma}{(1 - \beta)\sigma} \Leftrightarrow \tau_s = 0 \). We will have \( \tau_b > 0 \Leftrightarrow \nu > \mu \) and \( \tau_s > 0 \Leftrightarrow \mu < \bar{\mu}(\nu) \).

• For any \( \bar{\mu}(\nu) < \mu < \nu \) : if \( 0 < \tau < \tau_b \) then the economy experiences a cyclical regime, alternating between phases of expansion and downturns; if \( \tau_b < \tau \), the economy is in a permanent boom, which long-run growth rate is the Harrod-Domar one.

• For any \( 0 < \mu < \bar{\mu}(\nu) \) : if \( 0 < \tau < \tau_s \), the economy is trapped into a permanent slump regime; if \( \tau_s < \tau < \tau_b \) then the economy experiences a cyclical regime alternating between phases of expansion and downturns; if \( \tau_b < \tau \), the economy is in a permanent boom, which long-run growth rate is the Harrod-Domar one.

Nota : \( \bar{\mu}(\nu) = \frac{[\bar{\nu}(\mu) + \nu(\sigma - \sigma_2)] - \mu(1 - \beta)\sigma - 2(\beta \sigma - \sigma_2)}{(1 - \beta)\sigma} \).

If the tax rate initially set to \( \tau < \tau_s \) is raised to a value \( \tau_s < \tau' < \tau_b \), the dynamic regime goes from a permanent slump to a cyclical motion (cf. Fig. 4). Starting from the previous permanent slump steady-state level of the savings to planned investment ratio, the economy hits the new (SS') locus, and enters a cycle, alternating between temporary booms and slumps.

— Figure 4 about here —

If the tax rate initially set to \( \tau_s < \tau < \tau_b \) is raised to a value \( \tau' = \tau_b \), the dynamic regime goes from a cycle to a permanent boom (cf. Fig. 5). In that case, \( b \) is set equal to 1, so that the economy can not go back to the slump zone, since when \( q \leq 1 \), only (BB) is relevant.
Eventually, if the tax rate initially set to $\tau < \tau_s$ is raised to a value $\tau' = \tau_b$, the dynamic regime goes from a permanent slump to a permanent boom (cf. Fig. 6). In that case, $b$ is set equal to 1, so that the economy can not go back to the slump zone, since when $q \leq 1$, only (BB) is relevant.

### 3.1 Intuition

To gain insight into the mechanism that drives our result, it is useful to analyse how the introduction of this fiscal scheme affects the fractions of the labor share going respectively to investors and to savers. Actually, introducing a tax on savers’ labor income and redistributing the proceeds into the investors’ wealth is tantamount to an increase in the fraction of the labor force having direct access to capital investment opportunities. As a matter of fact, the original setup studied in Aghion et al. [3] is modified up to the following parameter changes: $\mu$ becomes $\mu + (1 - \mu)\tau$, which is increasing in $\tau$, and $(1 - \mu)$ becomes $(1 - \mu)(1 - \tau)$, decreasing in $\tau$.

In other words fiscal policy, usually mobilized as a conventional countercyclical tool, affects here the economy in the same way as a structural reform would do. More precisely, a structural policy that would remove institutional obstacles and rigidities separating savers and investors to promote growth, stability and equity at the same time, would presumably have a similar effect on the economy dynamics. In general, such structural policies may be difficult to implement (especially in the short-run), and are in some cases just not feasible; governments cannot simply decide that access to credit and investment opportunities should be extended. Interestingly, with a very basic setup, the fiscal policy we feature can impact the dynamics as if a structural policy had managed to improve the access to productive investment opportunities.
4 Comparative Statics

In the following section, we assess how $\tau_b$ and $\tau_s$ behave when one of the friction parameters $\nu$ and $\mu$ is made to vary.

4.1 Comparative Statics Properties of $\tau_b$ and $\tau_s$ with respect to $\nu$

Let $\tau_{b,\nu}$ and $\tau_{s,\nu}$ be the geometrical loci respectively depicting the sensitivity of $\tau_b$ and $\tau_s$ with respect to $\nu$, *caeteris paribus*.

$\tau_{b,\nu}$ and $\tau_{s,\nu}$ are linear. Since $0 < \frac{\partial \tau_b}{\partial \nu} < \frac{\partial \tau_s}{\partial \nu}$, both are upward sloping, but $\tau_{s,\nu}$ is steeper than $\tau_{b,\nu}$. It can be easily shown that $0 < \frac{\partial \tau_b}{\partial \nu} < \frac{\partial \tau_s}{\partial \nu}$, so that $\tau_{s,\nu}$ hits the abscissa axis for a higher value of $\nu$ than $\tau_{b,\nu}$ does. As $\tau_{b} - \tau_{s} > 0$, $\tau_{b,\nu}$ is always "higher" than $\tau_{s,\nu}$ in the plane, for any value of $\nu > \mu$. The intersection of $\tau_{b,\nu}$ and $\tau_{s,\nu}$ eventually occurs at $\nu = 1$.

Let us now turn to the effect of a variation in $\mu$ (i.e., a change in the access to productive investment opportunities) on the respective properties of $\tau_{b,\nu}$ and $\tau_{s,\nu}$. We suppose $\mu$ goes from $\mu_1$ to $\mu_2 > \mu_1$, i.e. the separation between savers and productive investors is smaller.

Since $0 < \frac{\partial^2 \tau_b}{\partial \mu \partial \nu} < \frac{\partial^2 \tau_s}{\partial \mu \partial \nu}$, when $\mu$ increases, both $\tau_{b,\nu}$ and $\tau_{s,\nu}$ become steeper, but the steepness rise is higher for $\tau_{s,\nu}$. Moreover, as $0 < \frac{\partial \tau}{\partial \mu} < \frac{\partial \tau}{\partial \nu}(\tau_{s,\nu}) = 1$, both abscissa to origin values of $\tau_{b,\nu}$ and $\tau_{s,\nu}$ increase following a rise in $\mu$, but the abscissa to origin value of $\tau_{b,\nu}$ reacts more. Hence, when the degree of separation between savers and investors decreases ($\mu$ goes up from $\mu_1$ to $\mu_2$), the permanent boom likelihood is increased for any $\nu \geq \mu_1$ (permanent boom can be achieved with a lower $\tau$) and the permanent slump likelihood reduces for any $\nu \geq \hat{\nu}(\mu_1)$ (we can get out of slumps with a lower $\tau$). The cycles likelihood reduces for any $\mu_1 < \nu < \hat{\nu}(\mu_1)$ and expands for any $\nu > \hat{\nu}(\mu_1)$. 

— Figure 7 about here —

— Figure 8 about here —
We also notice that \( \frac{\partial}{\partial \nu} (\tau_b - \tau_s) > 0 \). Very intuitively, improving the access to investment opportunities facilitates the conditions needed to reach a permanent boom, or to get out from a permanent slump trap.

Besides, \( \tau_{b,\nu} \) and \( \tau_{s,\nu} \) can also be affected by a productivity shock (namely, a rise in \( \sigma \), from \( \sigma_1 \) to \( \sigma_2 > \sigma_1 \)). Since

\[
0 = \frac{\partial^2 \tau_b}{\partial \sigma \partial \nu} < \frac{\partial^2 \tau_s}{\partial \sigma \partial \nu} \quad \text{and} \quad 0 = \frac{\partial}{\partial \sigma} (\nu|_{\tau_b=0}) < \frac{\partial}{\partial \sigma} \bar{\nu}(\mu),
\]

both the slope and the abscissa to origin value of \( \tau_{s,\nu} \) will go up, whereas \( \tau_{b,\nu} \) will remain unchanged. Hence, if a productivity shock occurs, the likelihood of permanent booms will remain unchanged for any \( 0 < \nu < 1 \), while the permanent slump likelihood will reduce and the cycles likelihood will increase for any \( \nu > \bar{\nu}(\mu)|_{\sigma=\sigma_1} \).

### 4.2 Comparative Statics Properties of \( \tau_b \) and \( \tau_s \) with respect to \( \mu \)

Let \( \tau_{b,\mu} \) and \( \tau_{s,\mu} \) be the geometrical loci respectively depicting the sensitivity of \( \tau_b \) and \( \tau_s \) with respect to \( \mu \), caeteris paribus.

--- FIGURE 9 ABOUT HERE ---

Since \( \frac{\partial \tau_s}{\partial \mu} < \frac{\partial \tau_b}{\partial \mu} < 0 \) and \( \frac{\partial^2 \tau_s}{\partial \mu^2} < \frac{\partial^2 \tau_b}{\partial \mu^2} < 0 \), both \( \tau_{b,\mu} \) and \( \tau_{s,\mu} \) are decreasing and concave, but \( \tau_{s,\mu} \) is more concave. The ordinate and abscissa to origin values of \( \tau_{b,\mu} \) are the same. The locus \( \tau_{s,\mu} \) shares the same property, such that:

\[
0 < \tau_s|_{\mu=0} = \mu|_{\tau_s=0} = \bar{\mu}(\nu) < \mu|_{\tau_b=0} = \tau_b|_{\mu=0} = \nu < 1.
\]

Let us now turn to the effect of a variation in \( \nu \), (i.e. a change in the credit market development), on the properties of \( \tau_{b,\mu} \) and \( \tau_{s,\mu} \). We suppose \( \nu \) goes from \( \nu_1 \) to \( \nu_2 > \nu_1 \), i.e. the credit market development gets poorer.

--- FIGURE 10 ABOUT HERE ---

Since \( 0 < \frac{\partial^2 \tau_s}{\partial \nu \partial \mu} < \frac{\partial^2 \tau_b}{\partial \nu \partial \mu} \), and \( 0 < \frac{\partial}{\partial \nu} (\mu|_{\tau_b=0}) = \frac{\partial}{\partial \nu} (\tau_b|_{\mu=0}) < \frac{\partial}{\partial \nu} (\tau_s|_{\mu=0}) = \frac{\partial}{\partial \nu} \bar{\mu}(\nu) \), the upward shift and the concavity reduction of \( \tau_{s,\mu} \) following a rise in \( \nu \) is stronger than the reaction of \( \tau_{b,\mu} \). Hence, when conditions on the credit market deteriorate (\( \nu \) increases from \( \nu_1 \) to \( \nu_2 \)), the permanent slump likelihood increases for any \( 0 < \mu < \bar{\mu}(\nu_2) \).
(we need a higher $\tau$ to get out from the permanent slump) and the permanent boom likelihood decreases for any $0 < \mu < \nu_2$. The cycles likelihood reduces for any $0 < \mu < \bar{\mu}(\nu_2)$, but increases for any $\bar{\mu}(\nu_2) < \mu < \nu_2$. We also notice that $\frac{\partial}{\partial \sigma}(\tau_b - \tau_s) < 0$. Very intuitively, a deterioration in the credit market development makes stronger the conditions needed to reach a permanent boom, or to get out from a permanent slump.

Besides, $\tau_{b,\mu}$ and $\tau_{s,\mu}$ can also be affected following a productivity shock (namely, a rise in $\sigma$ from $\sigma_1$ to $\sigma_2 > \sigma_1$). Since $\frac{\partial^2 \tau_s}{\partial \sigma \partial \mu} < \frac{\partial^2 \tau_b}{\partial \sigma \partial \mu} = 0$ and $\frac{\partial}{\partial \mu}(\bar{\mu} - \tau_s) = 0$, we know that both the slope, the abscissa to origin and the ordinate to origin values of $\tau_{s,\mu}$ will decrease following a productivity shock, whereas $\tau_{b,\mu}$ will remain unchanged. Hence, for any $\mu < \bar{\mu}(\nu_2)$, the likelihood of permanent slumps will reduce and the likelihood of cycles will increase. However, the likelihood of permanent boom will remain unchanged for any $0 < \mu < 1$.

5 Response to Shocks

Is the fiscal structure featured in this economy able to affect its reaction to productivity shocks (such as shocks on $\sigma$)? As in the benchmark case of Aghion et al. [3], $\sigma$ does not appear in the expression of (BB). It can be easily shown in a boom that, following a shock on $\sigma$ (may it be permanent or temporary), loans repayments and investment returns vary in the exact same proportion, so that the distribution of wealth between savers and investors remains unchanged (indeed, $q$ provides a direct measure of any evolution in this repartition). A productivity shock during a boom does affect the Harrod-Domar growth rate $g^*$. But as $q$ is not affected by any variation in the productivity level, all the shock will be registered instantly in $g^*$ (there will be no indirect effects due to a change in wealth distribution). Put differently, the tax schedule has no effect whatsoever on the way $g^*$ reacts to $\sigma$.

In contrast, during slumps, (SS) does react to any variation in $\sigma$ (Cf. Appendix B and Appendix D). Since $\frac{\partial^2 q^t+1}{\partial \sigma \partial q^t} < 0$ a positive productivity shock on $\sigma$ decreases the slope of the (SS) curve. We can see from $\frac{\partial^2 q^t+1}{\partial \tau \partial \sigma q^t} > 0$ that increasing $\tau$ makes $\frac{\partial^2 q^t+1}{\partial \sigma q^t}$ less negative. In other words, the reduction in the slope of (SS) due to a productivity shock is lower when the tax rate is high. Moreover, following a productivity shock, the ordinate to origin of (SS) goes up since, $\frac{\partial}{\partial \sigma}(q^t+1 | q^t=0) > 0$. Increasing $\tau$ moderates this increase, as $\frac{\partial}{\partial \tau}[\frac{\partial}{\partial \sigma}(q^t+1 | q^t=0)] < 0$.  

— Figure 11 about here —
Following a permanent productivity shock (cf. Fig. 11), the steady-state savings to planned investment ratio in a slump $s$ decreases, since $\frac{\partial s}{\partial \sigma} < 0$. Hence, aside from the direct effect of $\sigma$ on $g_s$, $\frac{\partial g_s}{\partial \sigma} > 0$, the drop in $s$ adds an indirect effect on growth, via the shift in wealth distribution in favor of the productive investors as $q$ goes down. However, when the tax rate is increased, $s$ still decreases following a productivity shock, but to a lower extent, since: $\frac{\partial^2 s}{\partial \tau \partial \sigma} > 0$.

Hence, although $\frac{\partial^2 g_s}{\partial \tau \partial \sigma} > 0$ which basically says that $\tau$ reinforces the direct positive effect on $g_s$ of a positive shock on $\sigma$, the indirect growth effects linked to the convergence to the new steady-state savings to planned investment ratio will be smaller when taxes are increased.

When the productivity shock is only temporary (cf. Fig. 12), the steady-state savings to planned investment ratio does not change. When $\tau$ is increased, both the short-run and the long-run convergence path will be shorter, also meaning smaller indirect wealth distribution effects.

--- Figure 12 about here ---

### 6 Welfare Analysis: an Avenue for Further Research

A crucial dimension that we plan to seriously investigate in the soon future is the issue of Welfare. Although this model is not particularly adequate for a rigorous, well-founded Welfare analysis, we can provide however some intuitions based upon the evolution of agents’ consumption through time, whether the fiscal policy we analysed is implemented or not. Let us compare the situations when $\tau$ is set to $\tau_b$ (ensuring permanent boom), and when $\tau$ is zero.

It is obvious that the borrowers (cf. Fig. 13) will favor the type of fiscal policy featured in this paper. Their wealth and consumption is instantaneously increased at soon as the tax schedule is implemented. Moreover, the indirect positive effects on labor and capital income of such a growth-enhancing policy will make them even more well-off.

--- Figure 13 about here ---
In contrast, although they will undoubtedly benefit from future increases in labor and interest income of this growth-promoting policy, lenders have to incur today the whole cost of this transfer mechanism (cf. Fig. 14). The extent to which they accept such a fiscal scheme depends on two crucial parameters: their degree of impatience, and the speed of convergence.

— Figure 14 about here —

In a one period environment, the degree of patience is directly linked to the notion of altruism. The parameter $\alpha$ captures these notions in the mobilized setup. The shift from $\tau = 0$ to $\tau_b$ can be Pareto improving if and only if lenders are patient or altruistic enough (that is, if $\alpha$ is low enough). In other words, $\tau_b$ will be effectively implemented as long as there exists a level of altruism for lenders such that $\tau_b$ is preferred to $\tau = 0$, despite the loss in consumption they incur at the date of the fiscal change.

A more precise characterization of preferences, notably about the intertemporal elasticity of substitution in consumption, and an endogenous savings behavior would certainly provide new answers to the questions about welfare emerging from this setup. This is an issue we are currently working on.

7 Conclusion

The purpose of this paper is to analyse how fiscal policy can affect aggregate volatility and growth in economies subject to capital market imperfections. Within a model featuring both frictions on the capital market and unequal access to investment opportunities among individuals, we have shown that appropriate fiscal policy parameters are able to rule out the occurrence of slump regimes, and immunize the economy against endogenous fluctuations in GDP, investment and interest rates. For given levels of the credit market development, we provide specific fiscal parameters able to insulate the economy from crises, fuel its long-run growth rate and place it on a permanent boom dynamic path.

The main mechanism driving our result is the following. The fiscal policy we analyse, introducing a tax on savers’ labor income and redistributing the proceeds into the investors’ wealth, is tantamount to an increase in the fraction
of the labor force having direct access to capital investment opportunities (and therefore to a decrease in the fraction of agents unable to invest directly in the production process). We analyse how conditions on the stabilizing fiscal parameters are modified when frictions in the economy evolve. Eventually, we study how the tax system impacts the economy’s response to temporary and permanent productivity shocks. Typically, aside from its direct growth-enhancing effects, it is shown that this type of fiscal policy moderates the wealth distribution effects following a productivity shock in a slump episode.

These findings complement the conclusions of Aghion et al. [3]. Abstracting from the utilization of public debt issuance, which can be a constrained instrument in many modern economies, we show that labor income taxation (which is a widely available instrument) has also some stabilizing properties in economies where capital markets are subject to frictions.

Some directions for further research naturally follow. We have seen in this economy that the shift from the situation without taxes to the tax rate ensuring a permanent boom can be Pareto-improving if and only if lenders are patient or altruistic enough. Augmenting the micro-foundations of the model, through a more precise characterization of preferences, notably about the intertemporal elasticity of substitution in consumption, and an endogenous savings behavior would certainly provide new answers to the questions about welfare emerging from this setup. Besides, an analysis of "optimal fiscal rules", aimed at achieving both stabilization and inequality reduction in this economy would certainly be of interest.
Appendix

A Properties of the BB and SS loci

A.1 The BB locus

Since \( \frac{\partial q^{t+1}}{\partial q} = \frac{\beta}{\mu + \tau(1 - \mu)} > 0 \), and \( \frac{\partial^2 q^{t+1}}{\partial (q^t)^2} = -\frac{2\beta(1 - \beta)(\mu + \tau(1 - \mu))}{\nu([\mu + \tau(1 - \mu)](1 - \beta)^2 + \beta^3)} < 0 \), (BB) is positively sloped and concave.

Moreover when \( \tau \) is increased, (BB) moves downwards: \( \frac{\partial^2 q^{t+1}}{\partial \tau \partial q^t} = -\frac{2\beta(1 - \beta)(1 - \mu)}{\nu([\mu + \tau(1 - \mu)](1 - \beta)} < 0 \).

The steady-state level in a boom writes as: \( b = \frac{\nu(\mu + \tau(1 - \mu))}{\mu + \tau(1 - \mu)} \).

Since \( \frac{\partial b}{\partial \tau} = -\frac{\nu(1 - \mu)}{\mu + \tau(1 - \mu)} < 0 \), the steady-state level \( b \) decreases when \( \tau \) increases.

The ordinate to origin \( q^{t+1}|_{q^t=0} = 0 \) remains zero, whatever the tax rate.

A.2 The SS locus

Since \( \frac{\partial q^{t+1}}{\partial q} = \frac{\sigma^2 \nu}{\mu + \tau(1 - \mu)(1 - \beta)(1 - \beta)} > 0 \), (SS) is linear and positively sloped.

The ordinate to origin of (SS) writes as \( q^{t+1}|_{q^t=0} = \frac{(\sigma - \sigma^2)(\mu + \tau(1 - \mu))}{\nu([\mu + \tau(1 - \mu)](1 - \beta)(1 - \beta)} < 0 \), increasing \( \tau \) lowers the ordinate to origin.

Moreover, since \( \frac{\partial^2 q^{t+1}}{\partial \tau \partial q^t} = -\frac{\sigma^2(\mu + \tau(1 - \mu))}{([\mu + \tau(1 - \mu)](1 - \beta)(1 - \beta)} < 0 \), increasing \( \tau \) decreases the slope of (SS).

The steady-state level in a slump writes as: \( q^{t+1} = s = \frac{\sigma - \sigma^2}{\nu([\mu + \tau(1 - \mu)](1 - \beta)} < 0 \) the steady-state level \( s \) is lower when \( \tau \) increases.

Nota: the (BB) curve always lies above the (SS) locus at \( q^t = 1 \).
\[ q^{t+1}(q^t = 1, r_{t+1} = \sigma_1) = \left\{ \frac{[\mu + \tau(1-\mu)][1-\beta]}{\nu} + \beta \right\}^{-1} \] > \[ q^{t+1}(q^t = 1, r_{t+1} = \sigma_2) = \left\{ [\mu + \tau](1-\beta)/\nu + (\beta/\nu) - [(1/\nu - 1)](\sigma_2/\sigma) \right\}^{-1} \]

### A.3 Steady state levels

Since \( b - s = \frac{\nu(\beta \sigma - \sigma_2)(1-\tau)(1-\mu)}{[\mu + \tau(1-\mu)][(1-\beta)\sigma + \beta \sigma - \sigma_2]} > 0 \), we know that \( b \) is always greater than \( s \).

Since \( \frac{\partial}{\partial \tau}(b - s) = \frac{\nu(\beta \sigma - \sigma_2)(1-\mu)(\mu + \tau(1-\mu))(1-\beta)\sigma + (1-\mu)[(1-\beta)\sigma + \beta \sigma - \sigma_2]}{\nu(\beta \sigma - \sigma_2)(1-\mu)(1-\beta)\sigma + \beta \sigma - \sigma_2} < 0 \), we know that increasing \( \tau \) reduces the gap between \( b \) and \( s \), from \( (b - s)|_{\tau=0} = \frac{\nu(\beta \sigma - \sigma_2)(1-\mu)}{\mu(\beta \sigma + \beta \sigma - \sigma_2)} > 0 \) to \( (b - s)|_{\tau=1} = 0 \).

### B Growth rates

In a boom \( I_{t+1}^d \geq S_t \), the actual investment in the high-yield activity is \( \min(S_t, I_{t+1}^d) = S_t \).

The boom growth rate can then be measured by:

\[ \frac{Y_{t+2}}{Y_{t+1}} = \frac{\sigma K_{t+2}}{\sigma K_{t+1}} = \frac{\sigma S_{t+1}}{\sigma S_t} = (1 - \alpha) \sigma = g^* \]

In a slump, \( I_{t+1}^d < S_t \), the actual investment in the high-yield activity is \( \min(S_t, I_{t+1}^d) = I_{t+1}^d \).

The slump growth rate will be therefore written as:

\[ \frac{K_{t+2}}{K_{t+1}} = \frac{W_{t+1}^{t+1}/\nu}{W_t^t/\nu} = \frac{(1-\alpha)}{\nu} \left\{ [\mu + \tau(1-\mu)](1-\beta)\sigma + \beta \sigma - \sigma_2(1-\nu) \right\} = g_s = (1 - \alpha)\{ \frac{\sigma}{\sigma^*} + (1 - \frac{1}{\nu})\sigma_2 \} \]

Since \( \frac{d g_s}{d \tau} = \frac{(1-\alpha)}{\nu}(1-\mu)(1-\beta)\sigma > 0 \), raising \( \tau \) increases long-run growth during slumps.

Indeed \( g_s|_{\tau=0} = \frac{(1-\alpha)}{\nu}[\mu(1-\beta)\sigma + \beta \sigma - \sigma_2(1-\nu)] < g_s|_{\tau>0} \).

Very intuitively, a positive productivity shock on \( \sigma \) affects positively the growth rate, both during booms and during slumps: \( \frac{d g}{d \sigma} = (1 - \alpha) > 0 \) and \( \frac{d g}{d \sigma} = \frac{(1-\alpha)}{\nu}[(\mu + \tau(1-\mu))(1-\beta) + \beta] > 0 \). Since \( \frac{d^2 g}{d \sigma^2} = \frac{(1-\alpha)}{\nu}(1-\mu)(1-\beta) > 0 \), we know that increasing \( \tau \) reinforces the positive effect on \( g_s \) of a positive shock on \( \sigma \).
C Comparative Statics (Detailed Expressions)

C.1 Comparative Statics Properties of \( \tau_b \) and \( \tau_s \) with respect to \( \nu \)

Let \( \tau_{b,\nu} \) and \( \tau_{s,\nu} \) be the geometrical loci respectively depicting the sensitivity of \( \tau_b \) and \( \tau_s \) with respect to \( \nu \), *caeteris paribus*.

\( \tau_{b,\nu} \) and \( \tau_{s,\nu} \) are linear. Since \( 0 < \frac{1}{(1-\nu)^2} = \frac{\partial \tau_b}{\partial \nu} < \frac{\sigma - \sigma_2}{(1-\nu)\sigma} \frac{\partial \tau_b}{\partial \nu} = \frac{\partial \tau_s}{\partial \nu} \) both are upward sloping, but \( \tau_{s,\nu} \) is steeper than \( \tau_{b,\nu} \). Since \( 0 < \mu = \nu|_{\tau_s = 0} < \nu|_{\tau_s = 0} = \bar{\nu}(\mu) = \frac{\mu(1-\beta)\sigma + \beta\sigma - \sigma_2}{\sigma - \sigma_2} < 1 \), \( \tau_{s,\nu} \) hits the abscissa axis for a higher value of \( \nu \) than \( \tau_{b,\nu} \) does.

Let us now turn to the effect of a variation in \( \mu \) (i.e. a change in the access to productive investment opportunities) on the properties of \( \tau_{b,\nu} \) and \( \tau_{s,\nu} \). We suppose \( \mu \) goes from \( \mu_1 \) to \( \mu_2 > \mu_1 \), i.e. the separation between savers and productive investors is smaller.

Since \( 0 < \frac{1}{(1-\nu)^2} = \frac{\partial^2 \tau_b}{\partial \mu \partial \nu} < \frac{\sigma - \sigma_2}{(1-\nu)^2} \frac{\partial^2 \tau_b}{\partial \mu \partial \nu} = \frac{\partial^2 \tau_s}{\partial \mu \partial \nu} \) when \( \mu \) increases, \( \tau_{b,\nu} \) and \( \tau_{s,\nu} \) become steeper, but the steepness rise is higher for \( \tau_{s,\nu} \). Moreover as \( 0 < \frac{\partial}{\partial \mu} \bar{\nu}(\mu) = \frac{\partial}{\partial \mu} (\nu|_{\tau_s = 0}) = \frac{(1-\beta)\sigma}{\sigma - \sigma_2} < \frac{\partial}{\partial \mu} (\nu|_{\tau_b = 0}) = 1 \), both abscissa to origin values of \( \tau_{b,\nu} \) and \( \tau_{s,\nu} \) increase following a rise in \( \mu \), but abscissa to origin value of \( \tau_{b,\nu} \) reacts more. Hence, when the degree of separation between savers and investors decreases (\( \mu \) goes up from \( \mu_1 \) to \( \mu_2 \)), the permanent boom likelihood is increased for any \( \nu \geq \mu_1 \) (permanent boom can be achieved with a lower \( \tau \)) and the permanent slump likelihood reduces for any \( \nu \geq \bar{\nu}(\mu_1) \) (we can get out of slumps with a lower \( \tau \)). The cycles likelihood reduces for any \( \mu_1 < \nu < \bar{\nu}(\mu_1) \) and expands for any \( \nu > \bar{\nu}(\mu_1) \). We also notice that \( \frac{\partial}{\partial \nu} (\tau_b - \tau_s) = \frac{(\beta\sigma - \sigma_2)(1-\nu)}{(1-\beta)\sigma(1-\mu)} > 0 \). Very intuitively, improving the access to investment opportunities facilitates the conditions needed to reach a permanent boom, or to get out from a permanent slump trap.

Besides, \( \tau_{b,\nu} \) and \( \tau_{s,\nu} \) can also be affected by a productivity shock (namely, a rise in \( \sigma \), from \( \sigma_1 \) to \( \sigma_2 > \sigma_1 \)). Since \( 0 = \frac{\partial^2 \tau_b}{\partial \sigma \partial \nu} < \frac{\partial^2 \tau_s}{\partial \sigma \partial \nu} = \frac{\sigma_2}{(1-\beta)\sigma^2(1-\mu)} \) and \( 0 = \frac{\partial}{\partial \sigma} (\nu|_{\tau_s = 0}) < \frac{\partial}{\partial \sigma} (\nu|_{\tau_s = 0}) = \frac{\partial}{\partial \sigma} \bar{\nu}(\mu) = \frac{\sigma_2(1-\beta)(1-\nu)}{(\sigma - \sigma_2)^2} \), both the slope and the abscissa to origin value of \( \tau_{s,\nu} \) will go up, whereas \( \tau_{b,\nu} \) will remain unchanged. Hence, if a productivity shock occurs, the likelihood of permanent booms will remain unchanged for any \( 0 < \nu < 1 \), while the permanent slump likelihood
reduces and the cycles likelihood increases for any $\nu > \nu(\mu)|_{\sigma=\sigma_1}$.

C.2 Comparative Statics Properties of $\tau_b$ and $\tau_s$ with respect to $\mu$

Let $\tau_{b,\mu}$ and $\tau_{s,\mu}$ be the geometrical loci respectively depicting the sensitivity of $\tau_b$ and $\tau_s$ with respect to $\mu$, caeteris paribus.

Since $\frac{\partial \tau_s}{\partial \nu} = \frac{\sigma - \sigma_2}{(1-\beta)\sigma} \frac{\partial \tau_s}{\partial \mu} < \frac{\sigma - \sigma_2}{(1-\beta)\sigma} \frac{\partial \tau_s}{\partial \mu} = 0$ and $\frac{\partial^2 \tau_s}{\partial \mu^2} = \frac{\sigma - \sigma_2}{(1-\beta)\sigma} \frac{\partial^2 \tau_s}{\partial \mu^2} < \frac{\sigma - \sigma_2}{(1-\beta)\sigma} \frac{\partial^2 \tau_s}{\partial \mu^2} = \frac{2(1-\nu)}{(1-\mu)^2} < 0$, both $\tau_{b,\mu}$ and $\tau_{s,\mu}$ are decreasing and concave, but $\tau_{s,\mu}$ is more concave. The ordinate and abscissa to origin values of $\tau_{s,\mu}$ are the same. The locus $\tau_{s,\mu}$ shares the same property, such that: $0 < \nu(\sigma - \sigma_2 + \sigma_2 - \beta) = \tau_s|_{\mu=0} = \mu|_{\tau_s=0} = \bar{\mu}(\nu) < \mu|_{\tau_s=0} = \bar{\mu}|_{\mu=0} = \nu < 1$.

Let us now turn to the effect of a variation in $\nu$, (i.e. a change in the credit market development), on the properties of $\tau_{b,\mu}$ and $\tau_{s,\mu}$. We suppose $\nu$ goes from $\nu_1$ to $\nu_2 > \nu_1$, i.e. the credit market development gets poorer.

Since $0 < \frac{1}{(1-\mu)^2} = \frac{\partial^2 \tau_s}{\partial \nu \partial \mu} < \frac{\sigma - \sigma_2}{(1-\beta)\sigma} \frac{\partial^2 \tau_s}{\partial \mu^2} = \frac{\partial^2 \tau_s}{\partial \mu^2}$, and $0 < 1 = \frac{\partial}{\partial \nu}(\tau_b|_{\mu=0}) = \frac{\partial}{\partial \nu}(\mu|_{\tau_b=0}) < \frac{\partial}{\partial \nu}(\tau_s|_{\mu=0}) = \frac{\partial}{\partial \nu}(\mu|_{\tau_s=0}) = \frac{\partial}{\partial \nu}(\bar{\mu}(\nu)) = \frac{\sigma - \sigma_2}{(1-\beta)\sigma}$, the upward shift and the concavity reduction of $\tau_{s,\mu}$ following a rise in $\nu$ is stronger than the reaction of $\tau_{b,\mu}$. Hence, when conditions on the credit market deteriorate ($\nu$ increases from $\nu_1$ to $\nu_2$), the permanent slump likelihood increases for any $0 < \mu < \bar{\mu}(\nu_2)$ (we need a higher $\tau$ to get out from the permanent slump) and the permanent boom likelihood decreases for any $0 < \mu < \nu_2$). The cycles likelihood reduces for any $0 < \mu < \bar{\mu}(\nu_2)$, but increases for any $\bar{\mu}(\nu_2) < \mu < \nu_2$. We also notice that $\frac{\partial}{\partial \nu}(\tau_b - \tau_s) = -\frac{\sigma(\sigma - \sigma_2)}{(1-\beta)\sigma(1-\mu)} < 0$. Very intuitively, a deterioration in the credit market development makes stronger the conditions needed to reach a permanent boom, or get out from a permanent slump.

Besides, $\tau_{b,\mu}$ and $\tau_{s,\mu}$ can also be affected following a productivity shock (namely, a rise in $\sigma$ from $\sigma_1$ to $\sigma_2 > \sigma_1$). Since $\frac{\sigma(1-\nu)}{(1-\beta)\sigma(1-\mu)^2} = \frac{\partial^2 \tau_s}{\partial \sigma \partial \mu} = \frac{\partial^2 \tau_s}{\partial \sigma \partial \mu} = 0$ and $\frac{\sigma(1-\nu)}{(1-\beta)\sigma^2} = \frac{\partial}{\partial \sigma}(\mu|_{\tau_s=0}) = \frac{\partial}{\partial \sigma}(\bar{\mu}(\nu)) = \frac{\partial}{\partial \sigma}(\tau_s|_{\mu=0}) < \frac{\partial}{\partial \sigma}(\mu|_{\tau_s=0}) = \frac{\partial}{\partial \sigma}(\tau_b|_{\mu=0}) = 0$, both the slope, the abscissa to origin and the ordinate to origin values of $\tau_{s,\mu}$ will decrease following a productivity shock, whereas $\tau_{b,\mu}$ will remain unchanged. Hence, for any $\mu < \bar{\mu}(\nu)|_{\sigma=\sigma_1}$ the likelihood of permanent slumps will reduce and the likelihood of cycles will increase. However, the likelihood of permanent boom will remain unchanged for any $0 < \mu < 1$. 

D Response to Productivity Shocks during Slumps

Since \( \frac{\partial^2 q_{t+1}}{\partial \sigma \partial q_t} = -\frac{\nu \sigma_2 ((1-\beta)(1-\mu)) + \beta}{(1-\beta)\sigma + \beta \sigma - \sigma_2(1-\nu)} < 0 \) a positive productivity shock on \( \sigma \) decreases the slope of the (SS) curve. We can see from \( \frac{\partial^2 q_{t+1}}{\partial \sigma \partial q_t} = \frac{\nu \sigma_2 ((1-\beta)(1-\mu)) (1-\beta)\sigma + \beta \sigma - \sigma_2(1-\nu)}{(1-\beta)\sigma + \beta \sigma - \sigma_2(1-\nu)} > 0 \) that increasing \( \tau \) makes \( \frac{\partial^2 q_{t+1}}{\partial \sigma \partial q_t} \) less negative. In other words, the reduction in the slope of (SS) due to a productivity shock is lower when the tax rate is high.

Moreover, following a productivity shock, the ordinate to origin of (SS) goes up since:

\[
\frac{\partial}{\partial \sigma} (q_{t+1}^i | q_{t+1} = 0) = \frac{\nu \sigma_2 ((1-\beta)(1-\mu)) (1-\beta)\sigma + \beta \sigma - \sigma_2(1-\nu)}{(1-\beta)\sigma + \beta \sigma - \sigma_2(1-\nu)} > 0.
\]

Increasing \( \tau \) moderates this increase in the ordinate to origin, since:

\[
\frac{\partial^2}{\partial \sigma \partial q_t} (q_{t+1}^i | q_{t+1} = 0) = \frac{\nu \sigma_2 ((1-\beta)(1-\mu)) (1-\beta)\sigma + \beta \sigma - \sigma_2(1-\nu)}{(1-\beta)\sigma + \beta \sigma - \sigma_2(1-\nu)} < 0.
\]

Following a productivity shock, the steady-state level in a slump \( s \) decreases, since \( \frac{\partial s}{\partial \sigma} = \frac{-\nu \sigma_2 ((1-\beta)(1-\mu)) (1-\beta)\sigma + \beta \sigma - \sigma_2(1-\nu)}{(1-\beta)\sigma + \beta \sigma - \sigma_2(1-\nu)} < 0 \). However, when the tax is increased, \( s \) still decreases following a productivity shock, but in a lower extent, since:

\[
\frac{\partial^2 s}{\partial \sigma \partial q_t} = \frac{-\nu \sigma_2 ((1-\beta)(1-\mu)) (1-\beta)\sigma + \beta \sigma - \sigma_2(1-\nu)}{(1-\beta)\sigma + \beta \sigma - \sigma_2(1-\nu)} > 0.
\]
References


[34] S. M. Kanbur, Of risk taking and the personal distribution of income, J. Pol. Econ. 87 (1979), 769-97.


[38] T. Piketty, The dynamics of the wealth distribution and interest rate with credit rationing, Rev. Econ. Stud. 64 (1997) 173-89.

Figure 1: The Permanent Boom

Figure 2: The Permanent Slump
Figure 3: The Cyclical Regime

Figure 4: From a Permanent Slump to a Cycle
Figure 5: From a Cycle to a Permanent Boom

Figure 6: From a Permanent Slump to a Permanent Boom
Figure 7: Comparative Statics Properties of $\tau_b$ and $\tau_s$ with respect to $\nu$

\[ \frac{\mu(1-\beta)\sigma + \beta\sigma - \sigma_1}{\sigma - \sigma_1} = \mathcal{V}(\mu) \]

Figure 8: Comparative Statics Properties of $\tau_b$ and $\tau_s$ with respect to $\nu$, when $\mu$ rises

\[ \frac{\mu_1(1-\beta)\sigma + \beta\sigma - \sigma_1}{\sigma - \sigma_1} = \mathcal{V}(\mu_1) \]
Figure 9: Comparative Statics Properties of $\tau_b$ and $\tau_s$ with respect to $\mu$

$$\mathbb{P}(V) = \frac{\nu(\sigma - \sigma_s) + \sigma_s - \beta\sigma}{(1 - \beta)\sigma}$$

Figure 10: Comparative Statics Properties of $\tau_b$ and $\tau_s$ with respect to $\mu$, when $\nu$ rises

$$\Delta V > 0$$

$$\mathbb{P}(V) = \frac{\nu(\sigma - \sigma_s) + \sigma_s - \beta\sigma}{(1 - \beta)\sigma}$$
Figure 11: Effect of $\tau$ on the Response to a Permanent Productivity Shock in a Slump

Figure 12: Effect of $\tau$ on the Response to a Temporary Productivity Shock in a Slump
Figure 13: Effect of the $\tau = \tau_b$ Fiscal Policy on Borrowers’ Consumption Growth

Figure 14: Effect of the $\tau = \tau_b$ Fiscal Policy on Lenders’ Consumption Growth