Nonparametric Test of Congestion

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Abstract

In this paper we show how the Färe, Grosskopf and Lovell (1985) input congestion nonparametric test can be performed to reach a statistical conclusion. To achieve this goal, we use Simar and Wilson (1998, 2000) radial distance functions bootstrap technique. An empirical illustration on airports congestion is provided. The results show that the technology is globally congested and that nine of the forty airports are input congested.

Keywords: DEA, Distance Functions, Congestion, Bootstrap, Nonparametric Test.
Introduction

The concept of congestion is often used in the common language and more specifically in the transports sector. For instance, a traffic congestion implies that the road network use can not be increased without decreasing the traffic speed. This concept can be more generally used in economics. Some researchers linked the concept of congestion with the one of Turgot's law of "variable proportions" (see Cherchye, Kuosmanen and Post 2001). This law implies that the more a production factor is increased, the less the production increases. In fact, congestion is a special case of this law. The increase of one input keeping the other inputs constant does not increase the production and in some cases lowers it or even brings it to zero.

Färe and Svensson (1980) defined different degrees of economic congestion. Later, Färe and Grosskopf (1983) and Färe, Grosskopf and Lovell (1985) suggested a measure of input congestion which is the ratio of two Farrell technical efficiency measures: one measure is computed under weak disposability of inputs, the other one under strong disposability of inputs. We will denote this test "FGL". The FGL test has been widely used in the economic literature. Moreover, though it dates back to the eighties, it was also recently discussed by many papers in the journals "Socio-Economic Planning Sciences" and "European Journal of Operational Research".

However, so far, the FGL test does not allow for any general statistical conclusion. It only allows one to conclude whether a production plan is input congested relatively to the DEA-estimated technology which is not the true technology. The main goal of this paper is to show how to perform the FGL test and reach some general statistical conclusions.

To achieve this goal, we use Simar and Wilson (1998,2000) bootstrap technique. This technique was designed to obtain confidence intervals and to
perform tests on radial distance functions estimated through DEA methods. Simar and Wilson (2002) further used this technique to perform tests of returns to scale.

The paper is organized as follows. The second section presents the notations, assumptions and basic concepts while the third one focuses on the measure of input congestion. The fourth section establishes how to use the bootstrap method to test for input congestion. The fifth section presents an empirical illustration using data on 40 Italian and Portuguese airports. The last section concludes.

1 Notations, Assumptions and Basic Concepts

Let $x \in \mathbb{R}^+_N$ and $y \in \mathbb{R}^+_M$ be respectively some vectors representing the quantities of inputs and outputs. Define the technology by its input sets

$$L(y) = \{ x \in \mathbb{R}^+_N : x \text{ can produce } y \} \quad (1.1)$$

Let us assume the following properties of the technology:

L.1: $0_N \in L(0_M)$ and $y > 0_M \Rightarrow 0_N \notin L(y)$.

L.2: If, $y_n, n \geq 0$ is such that $\lim_{n \to \infty} y_n = \infty$, then $\bigcap_{n \geq 0} L(y_n) = \emptyset$.

L.3: $L(y)$ is a closed set.

L.4: $L(y)$ is convex for all $y \in \mathbb{R}^+_N$.

L.5: $y' \geq y \Rightarrow L(y') \subseteq L(y)$.

Note that though we assume strong disposability of outputs, at this point, we do not make any assumption on the disposability of inputs. Now, let us
define the set

\[ L^{sd}(y) = L(y) + \mathbb{R}^+_M. \]  

From the latter definition, it is obvious that

\[ L(y) \subseteq L^{sd}(y). \]  

Furthermore, the technology exhibits strong disposability of inputs if and only if,

\[ L(y) = L^{sd}(y). \]  

Alternatively, the technology does not exhibit strong disposability of inputs but only weak disposability if and only if,

\[ L(y) \subset L^{sd}(y). \]  

Strong disposability of inputs implies weak disposability but the converse is not true. As stated in Färe, Grosskopf and Lovell (1994) and very well illustrated in Kerstens (1996), when the technology exhibits weak disposability but not strong disposability, some production points of the input sets are input congested. Let us define the isoquant \( IsoqL(y) \) as

\[ IsoqL(y) = \{ x \in \mathbb{R}^+_N : x \in L(y), \lambda x \notin L(y) \forall \lambda < 1 \}. \]

\( IsoqL(y) \) is the weakly efficient set. It is the frontier of the input set \( L(y) \). A point of \( IsoqL(y) \) is input congested if the marginal productivity of any of the inputs is negative. In other words, increasing the congested input does not
allow the same production but a smaller one. Formally, for any \( x \in IsoqL(y) \) if there exists an input \( i \) for which any \( x' \) such that \( x'_i \geq x_i \) and \( x'_j = x_j, j \neq i \) implies that \( x' \notin L(y) \), then the input \( i \) is congested. Hence, any backward bending part of \( IsoqL(y) \) is a congested subset and a technology which exhibits weak disposability but not strong disposability of inputs is input congested. The following figure illustrates a congested technology.

For this technology, the inputs are weakly but not strongly disposable. It exhibits input congestion. For instance, \( x' \) is such that \( x' > x \) with \( x' \in \theta L(y), 0 \leq \theta < 1 \) but \( x' \notin L(y) \). Consequently, the point \((x, y)\) is input congested.

Now, for any point \( x \in L(y) \), the Shephard input distance function gives the following technical efficiency measure:

\[
D_i(y, x) = \sup \{ \lambda > 0 : x/\lambda \in L(y) \}.
\]

Clearly, if \( D_i(y, x) > 1 \), the inputs can be decreased proportionally while keeping the same input level and when \( D_i(y, x) = 1 \), the input level can not be decreased and the point is said to be technically efficient. Moreover, another distance function can be defined in regard with the strongly disposable input set \( L^{sd}(y) \):

\[
D_i^{sd}(y, x) = \sup \{ \lambda > 0 : x/\lambda \in L^{sd}(y) \}.
\]

From \((??)\) and for any \( x \in L(y) \), we obviously have

\[
D_i^{sd}(y, x) \geq D_i(y, x) \geq 1.
\]
Along the lines of Färe, Grosskopf and Lovell (1985), these distance functions allow to compute for any point the input congestion index

\begin{equation}
ICI(y, x) = \frac{D_i(y, x)}{D_i^{sd}(y, x)}.
\end{equation}

Obviously, \( ICI(y, x) \leq 1 \) and when the technology is not output congested at the point \((x, y)\), \( ICI(y, x) = 1 \).

\section{Estimating the input congestion index}

However, the true congestion index can not be observed for even the true distances used in the index remain unknown. They can only be estimated using a set of observations \( S_K = \{x_k, y_k\}_{k=1}^K \) which gives the estimator \( \hat{L}(y) \) of \( L(y) \) defined by

\begin{equation}
\hat{L}(y) = \{ x \in \mathbb{R}^N_+ : \sum_{k=1}^K \lambda_k y_k \geq y, \sum_{k=1}^K \lambda_k x_k = \gamma x, \sum_{k=1}^K \lambda_k = 1, \gamma \in [0, 1] \}. \end{equation}

Analogously, the strongly free disposable set \( L^{sd}(x) \) can be estimated by

\begin{equation}
\hat{L}^{sd}(y) = \{ x \in \mathbb{R}^N_+ : \sum_{k=1}^K \lambda_k y_k \geq y, \sum_{k=1}^K \lambda_k x_k \leq x, \sum_{k=1}^K \lambda_k = 1 \}.
\end{equation}

As for the true sets, the strongly disposable estimated set contains the weakly disposable one: \( \hat{L}(y) \subseteq \hat{L}^{sd}(y) \). These estimators are consistent estimators of \( L(y) \) and \( L^{sd}(y) \) (see Kneip et al., 1998 and Korostelev et al., 1995). Some consistent estimations of the true distance functions can be computed from these sets. Formally, the weakly disposable distance function estimator can
be defined as

\[(2.3) \quad \hat{D}_i(y_0, x_0) = \sup \{\delta > 0 : x_0/\delta \in \hat{L}(y_0)\}\]

In practice, when the technology is one of variable returns to scale (VRS), this distance function can be computed using the following linear program:

\[(2.4) \quad \hat{D}_i(y_0, x_0)^{-1} = \min \{\delta : \sum_{k=1}^{K} \lambda_k y_k \geq y_0, \sum_{k=1}^{K} \lambda_k x_k = \delta x_0, \sum_{k=1}^{K} \lambda_k = 1\}.\]

Along this line, the strongly disposable output distance function can be written

\[(2.5) \quad \hat{D}^{sd}_i(y_0, x_0) = \sup \{\delta > 0 : x_0/\delta \in \hat{L}^{sd}(y_0)\},\]

and it can be computed using the VRS linear program

\[(2.6) \quad \hat{D}^{sd}_i(y_0, x_0)^{-1} = \min \{\delta : \sum_{k=1}^{K} \lambda_k y_k \geq y_0, \sum_{k=1}^{K} \lambda_k x_k \leq \delta x_0, \sum_{k=1}^{K} \lambda_k = 1\}.\]

Hence, the FGL input congestion measure (see Färe, Grosskopf and Lovell, 1985) can be estimated for any point \((x_0, y_0)\) by

\[(2.7) \quad \hat{ICI}(y_0, x_0) = \hat{D}^{sd}_i(y_0, x_0)^{-1}/\hat{D}_i(y_0, x_0)^{-1} = \hat{D}_i(y_0, x_0)/\hat{D}^{sd}_i(y_0, x_0) \leq 1.\]

3 Testing for congestion

Though this estimator was computed in numerous papers for some specific production plans, it is difficult to conclude that the technology is input congested for a given point because of the lack of knowledge about the estimator's distribution. The only definitive conclusion that can be made is whether the
estimated input sets are congested or not. But nothing can be straightly inferred for the real input sets. However, a statistical conclusion about the real sets would be helpful for several reasons. First, it is obviously important for the production manager: if the production plan is input congested, it is necessary to change the input mix to reach a non congested area before any increase of the congested input. Second, the estimated technical efficiency measures will differ when a congested production plan is assessed. Moreover, in this case, the strong disposability distance function estimator will be inconsistent. On the contrary, when the technology is not input congested, the strong disposability distance function estimator will converge at a higher rate than the weak disposability distance function estimator. This raises the question of whether the technology is globally congested rather than whether a single production plan is congested. A distance function estimator depends on the whole observations set and not on a single point. Hence, the input congestion should be measured for a single point only when the technology exhibits congested areas.

This is why we intend to test whether a technology exhibits congested areas. Since $L^{sd}(y)$ can bring non consistent distance function estimators when the technology is congested, we deduce that the strong disposability estimator is more restrictive than the weak disposability one. Thus, we want to test:

$$H_0 : \ L(y) \text{ is not congested for all } y \in \mathbb{R}^M_+$$

versus

$$H_1 : \ L(y) \text{ is congested for some } y \in \mathbb{R}^M_+$$

Different test statistics can be built from the individual $\hat{ICI}(y_k,x_k)$. The
more natural one is the mean,

\[
\hat{ICI}_K = \frac{1}{K} \sum_{k=1}^{K} \hat{D}_i(y_k, x_k) / \hat{D}_{sd}^i(y_k, x_k),
\]

which is an estimator of \( ICI_K = \frac{1}{K} \sum_{k=1}^{K} D_i(y_k, x_k) / D_{sd}^i(y_k, x_k) \). When \( H_0 \) is true, \( D_i(y_k, x_k) = D_{sd}^i(y_k, x_k) \) for all \( k = 1, 2, \ldots, K \) and \( ICI_K = 1 \). When \( H_0 \) is wrong, \( D_i(y_k, x_k) < D_{sd}^i(y_k, x_k) \) for some \( k \) and \( ICI_K < 1 \).

Now, to perform our test, we need to obtain the critical values \( c_\alpha \) such that \( Pr(\hat{ICI}_K \leq ICI_K - c_\alpha | H_0) = \alpha \) for a test of size \( \alpha \). But the estimator’s distribution remains unknown. However, the test statistic is a ratio of distance functions and the technique for bootstrapping distance functions developed by Simar and Wilson (1998, 2000) offers a solution to obtain the critical values.

This technique is adapted to the asymmetric problem of estimating production frontiers. The production plans are reflected through the frontier during the bootstrapping process to restore the symmetry. Once the process is over, the unobservable points (those who are beyond the frontier) are reflected back through the frontier. To obtain the bootstrap samples, the data generation process (DGP) must be replicated \( B \) times under the null hypothesis which is the non congestion assumption in our case. Hence, the distance function estimators that must be used during the bootstrapping process are the strongly disposable distance function estimates, \( \hat{D}_{sd}^i(y, x) \).

Once they are available, each bootstrap sample defines a new production frontier. Hence, the strongly and weakly disposable distance function estimates of each of the original production plans can be computed relatively to these new frontiers. This brings \( B \) estimates of each distance function for each production point, \( \hat{D}_{i}^{s}(y_k, x_k) \) and \( \hat{D}_{i}^{sd}(y_k, x_k) \). From these estimates, \( B \)
statistics $\hat{ICI}(y_k, x_k), k = 1, \ldots, K$ and $\hat{ICI}_K^*$ can be deduced. These bootstrap values can be used as estimates of the real distributions. For instance, for the global congestion test, $\hat{ICI}_K$, if the DGP was well replicated we have:

\begin{equation}
(\hat{ICI}_K - 1)|H_0 \sim (\hat{ICI}_K^* - \hat{ICI}_K)|H_0, S_K.
\end{equation}

Now, we want to obtain the p-value, $p = Pr(\hat{ICI}_K \leq \hat{ICI}_K^{obs}|H_0)$ where $\hat{ICI}_K^{obs}$ is the observed value for the original sample. Under $H_0$, this is equivalent to $p = Pr(\hat{ICI}_K - 1 \leq \hat{ICI}_K^{obs} - 1|H_0)$. Using (3.2), this can be approximated by $\hat{p} = Pr(\hat{ICI}_K^* - \hat{ICI}_K \leq \hat{ICI}_K^{obs} - 1|H_0)$. Finally, since conditionally on $S_K$, $\hat{ICI}_K = \hat{ICI}_K^{obs}$, we obtain $\hat{p} = Pr(\hat{ICI}_K^* \leq 2\hat{ICI}_K^{obs} - 1|H_0, S_K)$.

4 Empirical illustration

The word ”congestion” is most commonly used in the transports sector. In the airplane transports sector, though the airliners flight routes are pretty free of congestion, the airports are well known to be congested, especially for the main cities. It is often said that the airport traffic is congested due to the lack of landing tracks and a strong increase of the number of flights since the main airports were built. But is this true and if so, to which extent and for which cities? The congestion test that we designed is particularly adapted to answer these questions.

Hence, in order to illustrate the way to perform the input congestion test, we use a database on airplane transports. The data on 31 Italian and 9 Portuguese airports were collected for the years 2001 and 2002. The technology is a two-input, two-output one. The inputs are the operational costs (in millions of Euro) and the capital (in millions of Euro) while the outputs are the number of transported passengers and the total cargo (in tons).
The use of the input congestion test will allow to conclude whether this technology is globally input congested. If the answer to this question is yes, then it will be possible to identify which of these airports are input congested. In particular, it is common knowledge that many airports have reached the maximum use of their landing tracks (represented by the input "capital"). Hence increasing the operational costs without any increase of the capital might result in a decrease of the number of planes or transported passengers or cargo rather than an increase. The test will allow us to check whether the airports are really capital congested and should increase their capacity by modifying the input mix with a capital increase. On the other hand, though it does not appear today as an important matter for the airports, some of them may use too much capital for their operational costs level. In other words, they may be oversized. Our technique will allow us to detect these problems if there are any.

Since it encompasses the case of constant returns to scale, we assume that the technology is one of variable returns to scale\(^2\). Instead of using Simar and Wilson’s (2000) method to compute the optimal bandwidth used in the kernel density estimation, we use the bandwidth that usually minimizes the asymptotic mean integrated square error (AMISE):

\[
\hat{h} = \left( \frac{4}{p + q + 2} \right)^{1/(p+q+4)} n^{-1/(p+q+4)}
\]

Unfortunately, the data might not be normally distributed and this value of \(\hat{h}\) might not minimize the AMISE. Hence, we also compute all the results for \(h = \hat{h}/2\) and \(h = 2\hat{h}\) to control for the sensitivity to the bandwidth.

The following table presents the probability \(p\) for the test of global congestion \((H_0: \text{the technology is not input congested})\) for the two years and the
different bandwidths:

[Insert table 1 here]

The global non congestion assumption is rejected at 10% for both years and any bandwidth. The maximal p-value equals 6.2%. However, the technology appears to be more congested for the year 2001 since no p-value exceeds 3.6%. The p-values are sensitive to the bandwidth but it does not change the final result.

An important consequence of these results is that contrary to what is usually done, the distance function that must be used to measure airports efficiency is the weakly disposable one ($\hat{D}_i$) rather than the strongly disposable one ($\hat{D}^{sd}_i$). If the latter distance function is used, the efficiency measures will be over-estimated.

Now, that global congestion is established, we can turn to the individual congestion tests. We test for individual congestion using the FGL test and the bootstrap samples to approximate the distribution of the test statistic. We used the traditional bandwidth and set the type I error at 10%. For the congested airports, we then determined which input is the source of congestion: capital or operational costs. The following table presents the individual results for the year 2001. The third column shows the FGL statistic while the last one exhibits the congested input.

[Insert table 2 here]

The FGL statistic shows that the difference between the weakly and the strongly disposable distance functions can be important. For instance, Porto airport would be efficient with the strongly disposable one while its weakly disposable equivalent equals $1.0/0.37 = 2.7$. Ten airports are input efficient: five Italian and five Portuguese airports.
Among the 40 airports, 9 are input congested. One the one hand, seven of these airports are capital congested: Bologna, Firenze, Napoli, Torino, Venezia, Porto and Faro. The operational costs of these airports are too important compared to their capital level. An increase in operational cost would not increase the output level. Consequently, they should change their input mix by increasing the capital/operational costs ratio. This does not mean that the other airports are not congested in the common sense. The supply might be lower than the demand in other areas. However, the input mix is better balanced in the other airports. On the other hand, 2 of these airports, Bari and Olbia, are operational costs congested. The airport structure (capital) is too important compared to the operation costs level.

Finally, the results show that the use of the bootstrap technique to perform a statistical test brings a real improvement. For instance, without the use of Simar and Wilson’s technique, Parma airport would be considered as congested. It has a FGL statistic of 0.75 which is much lower than that of Olbia or Napoli which are congested. However, Parma is not significantly congested. Its p-value equals 32.3%!

5 Concluding Comments

In this paper, we showed how well Simar and Wilson’s technique to bootstrap distance functions is adapted to perform a nonparametric test of congestion. The empirical results prove that the test outcomes are valuable for different reasons. First, it allows to determine which of the two disposability assumptions is to be used to estimate the distance functions. Second, it brings a statistical answer to the question ”is the technology globally congested ?”. Finally, if the answer is yes, it establishes which of the decision making units
is congested and which are the congested inputs.
Notes

1 The following linear programs can easily be transposed to the non increasing or constant returns to scale cases.

2 We could have used Simar and Wilson’s (2002) method to determine the returns to scale.

3 There are probably different explanations of the congestion type in these airports. For example, one can cite the size of the city, tourism seasonality... However, such arguments require more research and it is not the purpose of this paper. They can be explored in future research with appropriate transportation problematic.
References


