Workforce participation of low-skilled women, gender occupational segregation, and male-female earnings gap

Olivier Baguelin*
Centre d’études de l’emploi
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Abstract

We study the male-female earnings gap along the distribution of earnings by percentile. The analysis rests on the organization-based economy model introduced by Garicano and Rossi-Hansberg (2004, 2006). In this economy, agents choose to be self-employed or to produce in a team made up of one (high ability) manager and several (low ability) workers. Producing in a team boosts individual productivity but entails costs. We assume male workers’ aversion to female managers. This translates into higher costs in teams where the manager is a female and workers are males. The direct consequence of this assumption is that labor market is gender-segregated. A second dimension of the analysis is that less skilled women workforce participation is low compared to that of male. This assumption affects the whole distribution of female earnings and occupational choice (worker, self-employed or manager). The paper helps understanding the "glass ceiling" phenomenon as well as the observed profile of male-female earnings differences along the distribution of earnings by percentile.

JEL Classification: J21, J23, J31, J41, J7.

Keywords: sex discrimination; gender-segregation in the labor market; occupational choice; distribution of earnings; continuous assignment problem.

1 Introduction

This article provides an analysis of male-female disparities in the labor market. There indeed remains many gender differences as regards access to labor market opportunities. Women still participate less than men to the labor force and, when they do, they are more often unemployed or employed on a part time basis than men. The distribution of employment by occupation or sector remains very much gender-segregated: the large majority of both women and men are concentrated in a small number of occupations that tend to be either female or male dominated (in particular, women are underrepresented in managerial and top administrative occupations). On average in OECD countries, women still earn

*Centre d’études de l’emploi, 29 promenade Michel Simon, 93166 Noisy-le-Grand Cedex, France. olivier.baguelin@univ-paris1.fr. Fax: +33 (0)1 45926976.
16% less than men per hour worked, and observable characteristics influencing productivity account for little of this gap.\footnote{\textit{See the OECD (2002).}} And yet, the balance in educational attainment between women and men is close to equality in most OECD countries or even in women's favor. The questions, thus, are: where do all these differences come from, and how do they derive the ones from the others?

A first usual interpretation of these facts is that women endure discrimination. They are assumed to have less favourable working terms than men due to some prejudiced employers, customers or co-workers (Becker, 1957). Analyses following this track were often designed to account for male-female wage differences for a given job. Empirical inquiries, however, reveal that pure pay discrimination is not the issue (Cain, 1986). A far more compelling piece of evidence is the "glass ceiling" phenomenon describing the relative absence of women in management ranks (U.S. EEOC, 2004) which usual discrimination-based analytical frameworks are not well equipped to deal with.

A second usual interpretation puts forward male-female differences in human capital accumulation. Because women do the bulk of child rearing, they acquire less experience and fewer job-related ability than do men which results in lower wages (Mincer and Polachek, 1974; Becker, 1985). This scenario has received good empirical support (Gunderson, 1989) which has been recently strengthened by Erosa et alii (2005). They notice, for the U.S. labor market, that the gender gap in hourly wages grows over the life cycle. However, human capital reading does not seem to provide the whole story. Wood et alii (1993) exploit a survey providing, for a population of lawyers, precise measurements of training and work experience as well as extensive information on career interruptions due to child-care responsibilities. They find out that taking time from work to care for children indeed reduces wages significantly but that it still leaves one-fourth to one-third of the earnings gap unexplained.

To better understand male-female disparities, many empirical investigations suggest to focus on vertical occupational segregation (Cain, 1986; Blau and Ferber, 1987; MacPherson and Hirsch, 1995): much of the average difference in pay between men and women is attributable to the fact that women are less likely to be found on higher-paying jobs. This comes close, in particular, to the "glass ceiling" concern: why are women relatively absent in (better-paid) management ranks? Lazear and Rosen (1990) provide a human capital argument considering that "only a tortured taste theory of discrimination" could account for vertical occupational segregation. The idea is that promotion choices hinge on workers' propensity to remain on the job because any firm-specific learning is lost when a worker leaves the firm. The higher propensity of women to quit makes it privately optimal (and socially efficient) to require higher threshold levels of ability for promotion. Women, assumed to have the same ability distribution than men, earn less because of a lower promotion probability. Winter-Ebmer and Zweimüller (1997) test this hypothesis empirically for the Austrian labor market. The hypothesis is not supported: they find that neither the risk of childbearing nor different productive characteristics can explain the crowding of females in lower hierarchical positions. This conclusion suggests that there remains some space for discrimination analyses. Besides, beyond women's underrepresentation in management positions, another striking (although rarely mentioned) feature of gender occupational segregation of the labor market is the overrepresen-
tation of women in the "professionals" category (OECD, 2002; U.S. EEOC, 2004). This is particularly suggestive since occupations in that category require educational attainments comparable to that of the "officials and managers" (without significant management aspects).

In the present paper, we propose a (hopefully, not so tortured) discrimination interpretation of previous empirical evidence with the intention to complement human capital analyses. More precisely, we provide an analysis of the link between gender occupational segregation and earnings differentials which sheds a new light on the "glass ceiling" concern. A special empirical issue related to our analysis, is that the male-female earnings gap widens as we consider higher ranked working persons in the earnings scale. In 1999, for the U.S., the ratio of female-to-male earnings was 0.813 at the 10th earnings percentile but only 0.649 at the 90th earnings percentile. A human capital reading of these figures would be that people in higher percentiles of the earnings distribution are well advanced in their life cycle: gender gap in work experience accumulation would explain the earnings gap. Another (discrimination-based) reading is that people in higher percentiles are well advanced... in job ladders. The remaining of this paper develops this view.

Our analysis lies on the organization-based economy introduced by Garicano and Rossi-Hansberg (2004, 2006). In this economy, agents choose to be self-employed or to produce in a team made up of one (high ability) manager and several (low ability) workers. Producing in a team boosts individual productivity but entails communication costs. We assume male workers' aversion to female managers. This implies higher communication costs in teams where the manager is a female and workers are males. The direct consequence of this assumption is that labor market is gender-segregated. There are only two types of teams: all females or all males. We pursue the analysis by taking into account the particularly low female workforce participation, as compared to that of males, at the bottom of the ability scale. This affects the whole distribution of female earnings and occupational choices (worker, self-employed or manager) and helps understanding the "glass ceiling" phenomenon as well as observed profile of male-female earnings differences across the distribution of earnings by percentile.

The remaining of the paper decomposes into four steps. In the first step, we provide some labor statistics which we believe are related to our analysis. In the second step we present the model. The third step is devoted to the occupational differences between male and female resulting from our assumptions. Finally, the last step addresses the issue of wage differentials.

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2 In the U.S. case, this cell gathers occupations requiring either college graduation or experience of such kind and amount as to provide a comparable background. Includes: accountants and auditors, airplane pilots and navigators, architects, artists, chemists, designers, dietitians, editors, engineers, lawyers, librarians, scientists, registered professional nurses, personnel and labor relations specialists, physicians, teachers, surveyors and kindred workers (see the U.S. EEOC, 2004).

3 See U.S. Census Bureau (2004, p. 15).
2 A few labor statistics

Before presenting the model a few labor statistics related to our analysis deserve attention. When available, we give the statistics for the OECD countries, but we often refer to the U.S. for which we have all the data required here.

2.1 Low-skilled women participation in the labor market is noticeably below that of men.

<table>
<thead>
<tr>
<th>Educational attainment</th>
<th>Female</th>
<th>Male</th>
<th>ratio (F:M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than a high school diploma</td>
<td>49.8%*</td>
<td>75.9%</td>
<td>0.66</td>
</tr>
<tr>
<td>High school graduates, no college</td>
<td>69.9%</td>
<td>85.7%</td>
<td>0.82</td>
</tr>
<tr>
<td>Some college or associate degree</td>
<td>76.3%</td>
<td>88.7%</td>
<td>0.86</td>
</tr>
<tr>
<td>College graduates</td>
<td>79.8%</td>
<td>92.6%</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Table 1 - Labor force as a percent of the population by educational attainment and sex, 2002.4

*Reading: 49.8% of female with less than a high school diploma participate in the labor market.

2.2 Occupational patterns: the differentiated distribution of male and female working persons between job categories.

On average for the OECD countries for which data are available on a harmonised basis:5 women are underrepresented in all three sub-major groups of the administrative and managerial occupations;6 women are overrepresented in the group "Professionals". This holds in all submajors7 except "physical and engineering science professionals" group. The interesting thing is that the two groups require comparable skills levels.

Occupational distribution in the U.S.A.

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5OECD (2002, p. 86) provide an average female representation ratio of female in various occupation group. This ratio is calculated as the female share in the occupational group or sector divided by the female share in total wage and salary employment.
6Major group of "Legislators, senior officials and managers".
7The submajors: physical and engineering science professionals, life science and health professionals, teaching professionals, other professionals.
Table 2 - Occupational Employment in Private industry, 2002.

<table>
<thead>
<tr>
<th>Occupations</th>
<th>White Female</th>
<th>White Male</th>
</tr>
</thead>
<tbody>
<tr>
<td>Officials and managers</td>
<td>9.4%*</td>
<td>16.4%</td>
</tr>
<tr>
<td>Professionals</td>
<td>21.0%</td>
<td>17.5%</td>
</tr>
<tr>
<td>Technicians</td>
<td>6.3%</td>
<td>6.6%</td>
</tr>
<tr>
<td>Sales workers</td>
<td>14.8%</td>
<td>11.0%</td>
</tr>
<tr>
<td>Office and clerical workers</td>
<td>24.1%</td>
<td>5.0%</td>
</tr>
<tr>
<td>Craft workers</td>
<td>2.1%</td>
<td>13.6%</td>
</tr>
<tr>
<td>Operatives</td>
<td>6.2%</td>
<td>16.0%</td>
</tr>
<tr>
<td>Laborers</td>
<td>4.2%</td>
<td>7.0%</td>
</tr>
<tr>
<td>Service workers</td>
<td>11.9%</td>
<td>6.9%</td>
</tr>
</tbody>
</table>

100% 100%


* Reading: 9.4% of female working persons belong to the class Officials and managers.

2.3 Ratio of Women’s earnings to Men’s earnings by earnings percentile: 1999

The gender earnings gap widens along the distribution of earnings by percentiles.

![Graph showing ratio of women's earnings to men's earnings by earnings percentile](http://www.eeoc.gov/stats/jobpat/2002/us.html)

Ratio of Women’s earnings to Men’s earnings by earnings percentile (1999).


3 The model


Following Garicano and Rossi-Hansberg (2004, 2006), we consider an economy in which production results from drawing problems and applying production time to them. Production requires operational time and ability to solve the problems arising in operation. Workers are endowed with some ability level \( z \) which they apply to deal with problems; they draw a particular problem (one per period) and, possibly, solve it, in which case their operational time is useful. The population is described by a given distribution of ability levels, \( G(\cdot) \), with density function \( g(\cdot) \) on \([0,1]\). Problems are distributed on the same segment, problem \( z \) occurring with probability \( f(z) \); \( F(\cdot) \) denotes the associated cumulative distribution function.

An agent of ability level \( \tilde{z} \) can solve all problems in \([0, \tilde{z}]\). This implies that a more able agent can always solve the problems a less able agent can solve.

Agents can form teams, so that whenever they fail to solve a problem on their own they can submit it to more able agents. This allows these agents to specialize on handling some problems and not others. Two assumptions are made about communication between agents. First, it is assumed that higher ability agents spend a fraction \( h < 1 \) of their time communicating their knowledge about each problem submitted to them, irrespectively of whether they can actually solve it or not; solving problems does not take time in itself. Second, an agent asking for help does not know who knows the solution. He first tries to solve the problem himself and, in case of failure, submits it to a more able agent in his team. A worker of ability \( z \), submits a problem with probability \( 1 - F(z) \).

As Garicano and Rossi-Hansberg (2004) do, we focus on the case where organizations only have one or two layers, and agents can choose to work on their own (to be self-employed, the organization has one layer) or work in a team (an organization of two layers). Teams are formed by managers, who use all their time in problem-solving, and workers, who use all their time in production.\(^\text{10}\) Hence, a team of \( n \) workers and 1 manager has \( n \) units of production time available and 1 unit of knowledge communication time. The production of such a team is then simply \( F(z)n \), where \( z \) is the ability of its manager, and is subject to this manager’s time constraint. The span of the manager (number of its subordinates) is limited by workers’ ability. Since each worker of ability \( z^* \) fails to solve his problem with probability \( 1 - F(z^*) \), the managerial time constraint is given by: \((1 - F(z^*)) nh = 1\). Denoting \( w(z^*) \) the wage of an ability \( z^* \) worker, the rents of managers are then given by \( r = F(z)n - w(z^*)n \).

3.2 Male, female, and prejudice

There are two types \( \theta \) of agents: females, indexed by 0, and males, indexed by 1. Agents of both gender are assumed to be identically distributed on the ability levels segment \([0,1]\) i.e. the two populations are fully similar in terms of productivity. Yet, differences exist between the two populations. A first difference concerns the cost of producing in a team: it is assumed that communication between a female manager and a male worker takes more time than in cases where the manager is a male or both agents

\(^{10}\)Garicano (2000) shows that such a specialization pattern is optimal.
are female. If $h^\theta_0$ denotes the fraction of time necessary for a manager of gender $\theta$ to communicate his knowledge to a worker of gender $\theta$, one assumes: $h^\theta_0 = h^\theta_1 = h = h^\theta_1$. A second difference is that less able females may fail to participate in the labor force whereas males always do. Females of ability below some exogenous parameter $z_0 \geq 0$ are assumed not to participate in the labor force. In addition to its recipient’s ability, the wage function may therefore depend on his gender: $w_\theta(,)$. 

3.3 Characterization and properties of the equilibrium

An equilibrium is characterized by: an allocation of agents of each gender to occupations (workers, managers, or self-employed); the ability compositions of teams (i.e. the matching between workers and managers, and spans of control); the gender compositions of teams (i.e. the matching between workers’ and manager’s gender and the representation of workers of each gender within teams); two earnings functions, one for each gender, such that agents do not want to switch either teams or occupations.

Proposition 1 (Garicano and Rossi-Hansberg, 2006) Such an equilibrium exists, is unique, and exhibits positive sorting: higher ability workers are matched to higher ability managers.

It follows that the equilibrium can be characterized by two pairs of thresholds $(\tilde{z}_\theta, \tilde{z}^\theta)_{\theta \in \{0,1\}}$ such that $\tilde{z}_\theta \leq \tilde{z}^\theta$ and, for $\theta \in \{0,1\}$: gender $\theta$ agents of ability $z \leq \tilde{z}_\theta$ become workers, gender $\theta$ agents of ability $z \geq \tilde{z}^\theta$ become managers, and those in between are self-employed.

4 A segregated equilibrium

Suppose that a mass of $1\hat{\theta}$ managers of ability $z$ are matched with a mass $n_0$ of female workers of ability $z_0$, and a mass $n_1$ of male workers of ability $z_1$. This entails devoting a fraction $t^\theta$ of their time to the former, and $1 - t^\theta$ to the latter. For this to be an equilibrium, the assignment must be such that agents do not want to switch either teams or occupations; changing the gender composition of their work team (and thus the allocation of their time). This requires that the assignment solves:

$$\max_{(z_0, n_0), (z_1, n_1), t^\theta} (F(z) - w_0(z_0)) n_0 + (F(z) - w_1(z_1)) n_1,$$

subject to

$$(1 - F(z_0)) h^\theta_0 n_0 = t^\theta,$$

$$(1 - F(z_1)) h^\theta_1 n_1 = 1 - t^\theta,$$

$\quad t^\theta \in [0,1].$$

This rewrites:

$$\max_{z_0, z_1, t^\theta} \frac{F(z) - w_1(z_1)}{(1 - F(z_1)) h^\theta_1} + \left(\frac{F(z) - w_0(z_0)}{(1 - F(z_0)) h^\theta_0} - \frac{F(z) - w_1(z_1)}{(1 - F(z_1)) h^\theta_1}\right) t^\theta, \quad t^\theta \in [0,1]. \tag{1}$$

The proposition below states that if two managers with equal ability but of different gender were to be matched to some workers of a given gender, these workers would be of identical ability.
Lemma 2 Ability assignment functions do not depend on managers’ gender.

Proof. See the appendix.

Ability assignment functions, thus, are only indexed by workers’ gender. Let $m_\theta (.)$ be defined, for any worker’s ability $z$, from the first order optimality condition, that is:

$$w'_\theta (z) = \frac{F(m_\theta (z)) - w_\theta (z)}{1 - F(z)} f(z).$$

In order for labor markets\(^{11}\) to clear, it must be the case that the supply of workers for any measurable set of abilities be equal to the demand for these workers by managers. This labor markets equilibrium condition splits up according to gender as follows: for all $z \in [\bar{z}_0, \bar{z}_0]$,\(^{12}\)

$$\int_{\bar{z}_0}^{z} (1 - F(x)) g(x) dx = \int_{m_0(z)}^{m_0(\bar{z}_0)} \frac{\theta'(x)}{h} g(x) dx + \int_{m_0(z)}^{m_0(\bar{z}_0)} \frac{t'(x)}{h} g(x) dx. \quad (2)$$

while, for all $z \leq \bar{z}_1$,

$$\int_{0}^{z} (1 - F(x)) g(x) dx = \int_{m_1(0)}^{m_1(z)} \frac{1 - t'(x)}{\eta^0} g(x) dx + \int_{m_1(0)}^{m_1(z)} \frac{1 - t'(x)}{\eta^0} g(x) dx. \quad (3)$$

Proposition 3 Equilibrium is gender-segregative: male workers are matched to male managers and female workers to female managers.

Proof. See the appendix.

From previous proposition follows that for all $z \in [\bar{z}_0, 1]$, $t^0 (z) = 1$ whereas, for all $z \in [\bar{z}_1, 1]$, $t^1 (z) = 0$. Therefore, with $h_0^0$ large enough\(^{12}\), labor markets equilibrium conditions simply write: for all $z \in [\bar{z}_0, \bar{z}_0]$,

$$\int_{\bar{z}_0}^{z} (1 - F(x)) g(x) dx = \frac{1}{h} \int_{m_0(\bar{z}_0)}^{m_0(z)} g(x) dx,$$

while, for all $z \leq \bar{z}_1$,

$$\int_{0}^{z} (1 - F(x)) g(x) dx = \frac{1}{h} \int_{m_1(0)}^{m_1(z)} g(x) dx.$$

In the remaining, we focus on the special case where both problems and abilities are uniformly distributed.

5 The properties of the segregated equilibrium

All through this paper, the maximum number of layers is set to 2 which, in the case where abilities and problems are uniformly distributed requires $h \geq 0.75$. The results below requires to build the equilibrium. Note first that, since labor markets equilibrium conditions hold for a continuum of values, we can derive them with respect to $z$. For uniform distributions on $[0,1]$, this leads to the following differential equations:

$$m'_0 (z) = (1 - z) h \text{ for all } z \in [\bar{z}_0, \bar{z}_0],$$

$$m'_1 (z) = (1 - z) h \text{ for all } z \in [0, \bar{z}_1],$$

\(^{11}\)There is one for each pair (gender, ability level).

\(^{12}\)We need that for all $z \in [\bar{z}_0, \bar{z}_1]$. $F(z) \geq r^0 (z)$ i.e. that $h_0^0$ be large enough so that female agents with ability in $[\bar{z}_0, \bar{z}_1]$ prefer to be self-employed than being a manager with male workers.
and therefore:

\[
m_0 (z) = \frac{1}{2} \left( (1 - \tilde{z}_0)^2 - (1 - z)^2 \right) h + \tilde{z}_0^2 \text{ for all } z \in [\tilde{z}_0, \tilde{z}_0],
\]
\[
m_1 (z) = \frac{1}{2} \left( (1 - z)^2 \right) h + \tilde{z}_0^2 \text{ for all } z \in [0, \tilde{z}_1].
\]

We now turn to wage functions. For all \( z \in [\tilde{z}_0, \tilde{z}_0] \),

\[
w'_0 (z) = \frac{F (m_0 (z)) - w_0 (z)}{1 - F (z)} f (z),
\]

that is, for uniform distributions,

\[
w'_0 (z) = \frac{m_0 (z) - w_0 (z)}{1 - z}.
\]

With the expression of \( m_0 (.) \) obtained below, we need to solve the following differential equation:

\[
w'_0 (z) + \frac{w_0 (z)}{1 - z} = \frac{1}{2} \left( (1 - \tilde{z}_0)^2 - (1 - z)^2 \right) h + \tilde{z}_0^2.
\]

The result is, for all \( z \in [\tilde{z}_0, \tilde{z}_0] \):

\[
w_0 (z) = \frac{1}{2} (z - \tilde{z}_0)^2 h + \frac{\tilde{z}_0 - w_0 (\tilde{z}_0)}{1 - \tilde{z}_0} z + \frac{w_0 (\tilde{z}_0) - \tilde{z}_0 \tilde{z}_0}{1 - \tilde{z}_0}.
\]

The determining of \( w_1 (.) \) takes the same stages, leading to:

\[
w_1 (z) = \frac{1}{2} z^2 h - (w_1 (0) - \tilde{z}_1) z + w_1 (0).
\]

To fully characterize equilibrium, it just remains to find \((\tilde{z}_0, \tilde{z}_0), (\tilde{z}_1, \tilde{z}_1), w_1 (0) \) and \( w_0 (\tilde{z}_0) \) such that:

first, \( m_0 (\tilde{z}_0) = 1, w_0 (\tilde{z}_0) = F (\tilde{z}_0), \) and \( r_0 (\tilde{z}_0) = F (\tilde{z}_0) \); second, \( m_1 (\tilde{z}_1) = 1, w_1 (\tilde{z}_1) = F (\tilde{z}_1), \) and \( r_1 (\tilde{z}_1) = F (\tilde{z}_1) \). This leads to:

\[
(\tilde{z}_0, \tilde{z}_0) = \left( 1 - \frac{1}{h} + \rho (h, \tilde{z}_0), \frac{2}{h} - 1 + \tilde{z}_0 - \rho (h, \tilde{z}_0) \right),
\]
\[
w_0 (\tilde{z}_0) = (1 - (1 - \tilde{z}_0) h) \left( \frac{2}{h} - 1 + \tilde{z}_0 - \rho (h, \tilde{z}_0) \right),
\]
\[
(\tilde{z}_1, \tilde{z}_1) = \left( 1 - \frac{1}{h} + \rho (h, 0), \frac{2}{h} - 1 - \rho (h, 0) \right),
\]
\[
w_1 (0) = (1 - h) \left( \frac{2}{h} - 1 - \rho (h, 0) \right).
\]

where \( \rho (h, \tilde{z}_0) = \sqrt{\frac{3}{\pi^2} - \frac{4}{\pi} + 1 + \frac{2}{3} h - \frac{1}{h} + \tilde{z}_0 + \tilde{z}_0^2} \Rightarrow \rho (h, 0) = \sqrt{\frac{3}{\pi^2} - \frac{4}{\pi}}, \)

We now consider some properties of the "uniform case".

### 5.1 Male-female differences in occupational choice

The combination of gender-segregation and low workforce participation of low ability female workers affect the occupational choice of all the female in the labor market.

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\(^{13}\)See the detail in the appendix.
Proposition 4 Let us assume that both abilities and problems are uniformly distributed on $[0, 1]$. Then, $z_0 > 0$ entails:

$$z_0 > z_1, \quad z^0 > z^1.$$ 

Proof. See the appendix. ■

There exists a set of abilities for which women choose to become workers whereas men choose to become self-employed. Stated differently, women remain workers for higher ability.

Proposition 5 Let us assume that both abilities and problems are uniformly distributed on $[0, 1]$. Then, $z_0 > 0$ entails:

$$\frac{z^0 - z_0}{1 - z_0} > z^1 - z_1$$

Proof. See the appendix. ■

The fraction of self-employed is higher for females than for males. We believe that connecting this result to the evidence that women are overrepresented among professionals makes sense. As the self-employed status within Garicano and Rossi-Hansberg’s framework, being a professional allows people to avoid the costs attached to delegating productive tasks, above all, wages.

5.2 Male-female differences in earnings

The narrowing of female workforce, as compared to that of men, entails an adjustment of earnings. Indeed, the combination of gender-segregation and low workforce participation of low ability female workers affect the whole female earnings distributions.

Proposition 6 Let us assume that both abilities and problems are uniformly distributed on $[0, 1]$. Then, $z_0 > 0$ entails: for all $z \in [z_0, \tilde{z}_0]$,

$$w_0(z) > \max\{w_1(z), F(z)\}.$$ 

Proof. See the appendix. ■

Proposition 7 Let us assume that both abilities and problems are uniformly distributed on $[0, 1]$. Then, $z_0 > 0$ entails: for all $z \in [\tilde{z}_1, 1]$,

$$r^1(z) > \max\{r^0(z), F(z)\}.$$ 

Proof. See the appendix. ■

The lack of female workers (labor supply) entails the switching of some virtually self-employed females into workers which supposes higher wages. The virtual excess of female managers (labor demand) is reduced by switching the least able of them into self-employed (which goes with higher wages).
To relevantly assess previous results, one should keep in mind that they are meant to complement the human capital scenario of male-female wage gap as depicted in introduction. Given the gender gap predicted by the human capital theories all through the distribution of agents, previous results indicate how it could be tightened for workers and widened for managers.

5.3 Male-female differences in assignments and spans of control

The next result states that the ability of the manager matched to a given worker is higher when this worker is a man (or, conversely, that the ability of the worker matched to a given manager is higher when this manager is a woman).

**Proposition 8** Let us assume that both abilities and problems are uniformly distributed on $[0, 1]$. Then, $z_0 > 0$ entails:

- from workers perspective, for all $z \in [z_0, \tilde{z}_1]$,
  \[ m_0 (z) < m_1 (z), \]
- or, from managers perspective, for all $z \in [\tilde{z}_0, 1]$,
  \[ z_0 (z) > z_1 (z). \]

**Proof.** See the appendix.
To easily understand the last result, let us consider the case of the most able managers. Garicano and Rossi-Hansberg’s result of positive sorting by ability involves that female managers are matched to the best female workers... whose ability stand higher than that of the best male workers (see the graph).

For \( \theta \in \{0, 1\} \), let \( n_\theta (z) \) denote the mass of (gender \( \theta \)) workers assigned to a (gender \( \theta \)) manager of ability \( z \).

**Corollary 9** Let us assume that both abilities and problems are uniformly distributed on \([0, 1]\). Then, \( z_0 > 0 \) entails: for all \( z \in [\tilde{z}_0, 1] \),

\[
    n_0 (z) > n_1 (z).
\]

This reinforces the "glass-ceiling" aspect of our analysis. All other things equal, bigger female work teams implies less teams... which are supervised by females; thus, less female managers.

### 6 Conclusion

The purpose of this paper was to provide a discrimination-based analysis of male-female disparities coping with the shortcomings of human capital theories. In particular, we wanted to provide an interpretation of the overrepresentation of female among professionals in most OECD countries as well as of the widening earnings gap along the distribution of earnings by percentile. Our interpretation lies on two dimensions: the male-workers’ aversion to female managers, which induces the segregation of the labor market; the relative absence of low ability women from the labor force, which induces the specific female earnings and occupational pattern.

Modelled in this way, the economy exhibits the following features.

- The best female workers are more able than the best male workers;
- The worse female managers are more able than the worse male managers;
- The fraction of self-employed is higher among women than among men;
- For a given ability, female workers obtain higher wage than male workers;
- For a given ability, female managers obtain lower earnings than male managers;
- For a given ability, female managers are matched to more able workers than men managers: consequently, female managers supervise more workers than male and there are less female than male organizations.
References


First order optimality condition writes:

\[ z_\theta^0 (z) = \arg \max_{z_\theta} \frac{F(z) - w_\theta(z)}{(1 - F(z_\theta)) h_\theta}. \]

Proof. For each pair \((\theta, \hat{\theta}) \in \{0, 1\} \times \{0, 1\}\), let’s define functions \( z_\theta^0 (.) \) by:

\[ z_\theta^0 (z) = \arg \max_{z_\theta} \frac{F(z) - w_\theta(z)}{(1 - F(z_\theta)) h_\theta}. \]

First order optimality condition writes:

\[ w'_\theta \left( z_\theta^0 (z) \right) = \frac{F(z) - w_\theta \left( z_\theta^0 (z) \right)}{1 - F \left( z_\theta^0 (z) \right)} f \left( z_\theta^0 (z) \right). \]

It does not depend on \( h_\theta \) and, therefore, on \( \hat{\theta} \): for all \( z \), \( z_\theta^0 (z) = z_\theta^1 (z) \), \( \theta \in \{0, 1\} \).  

7 Appendix

7.1 Ability assignment functions do not depend on managers’ gender

Proof. For each pair \((\theta, \hat{\theta}) \in \{0, 1\} \times \{0, 1\}\), let’s define functions \( z_\theta^0 (.) \) by:

\[ z_\theta^0 (z) = \arg \max_{z_\theta} \frac{F(z) - w_\theta(z)}{(1 - F(z_\theta)) h_\theta}. \]

The employment of a female agent of ability \( z \in [\tilde{z}_0, \tilde{z}_1] \) involves that hiring her is at least beneficial to managers of one gender of ability \( m_0(z) \). Both gender get the same rent hiring a female worker of ability \( z \), \( r_0(m_0(z)) \). With obvious writings, \(^{14}\) it must be that: \( r_0(m_0(z)) \geq \min \{ r_0^1(m_0(z)), r_0^1(m_0(z)) \} = r_0^1(m_0(z)) \). Let us consider a male agent of ability \( z_1(m_0(z)) \) where function \( z_1(.) \) is the inverse function of \( m_1(.) \). \( z \in [\tilde{z}_0, \tilde{z}_1] \Rightarrow m_0(z) \leq 1 \Rightarrow z_1(m_0(z)) \leq \tilde{z}_1 \) i.e. \( z_1(m_0(z)) \) is a worker. This involves:

\[ \max \{ r_0^1(m_1(z_1(m_0(z)))) , r_0^1(m_1(z_1(m_0(z)))) \} \geq r_0(m_1(z_1(m_0(z)))) \Rightarrow \max \{ r_0^1(m_0(z)) , r_0^1(m_0(z)) \} \geq r_0(m_0(z)) \Rightarrow r_0(m_0(z)) \geq r_0(m_0(z)). \]

We thus have, for all \( z \in [\tilde{z}_0, \tilde{z}_1] \), \( r_0^1(m_0(z)) \geq r_0(m_0(z)) \geq r_0^1(m_0(z)) \) i.e. male managers (weakly) prefer male workers whereas, female managers (weakly) prefer female workers. Furthermore, suppose male managers are actually indifferent about workers’ gender, that is \( r_0^1(m_0(z)) = r_0(m_0(z)). \) This entails \( r_0(m_0(z)) > r_0^1(m_0(z)) \) i.e. female managers strictly prefer female workers and are willing to increase female wage to attract them. This leads to \( r_0^1(m_0(z)) = r_0(m_0(z)) \) and male managers strictly prefer male workers. As a consequence, equilibrium is segregative.  

\(^{14}\)For all \( z \in [\tilde{z}_0, 1] \),

\[ r_\theta^0 (z) = \frac{F(z) - w_\theta \left( m_\theta^{-1}(z) \right)}{\left(1 - F \left( m_\theta^{-1}(z) \right) \right)} h_\theta. \]
7.3 Characterizing equilibrium

Characterizing equilibrium requires to solve the two systems:

\[
\begin{align*}
\frac{1}{2} \left( (1 - \tilde{z}_0)^2 - (1 - \tilde{z})^2 \right) h + \tilde{z}^0 &= 1 \\
\frac{1}{2} \left( \tilde{z}_0 - \tilde{z} \right)^2 h + \frac{1 - \tilde{z}_0}{1 - \tilde{z}} w_0 (\tilde{z}_0) + \frac{\tilde{z}_0 - \tilde{z}}{1 - \tilde{z}} z^0 &= \tilde{z}_0 \\
\frac{\tilde{z}^0 - w_0(\tilde{z}_0)}{(1 - \tilde{z})h} &= \tilde{z}^0 \\
\frac{1}{2} \left( 1 - (1 - \tilde{z}_1)^2 \right) h + \tilde{z}_1^1 &= 1 \\
\frac{1}{2} \tilde{z}_1^2 h - (w_1 (0) - \tilde{z}_1^1) \tilde{z}_1 + w_1 (0) &= \tilde{z}_1 \\
\frac{\tilde{z}_1^1 - w_1 (0)}{h} &= \tilde{z}_1^1
\end{align*}
\]

7.4 Occupational choices

**Proof.** For \( \tilde{z}_0 > 0, \rho(h, \tilde{z}_0) > \rho(h, 0) \) which directly induces \( \tilde{z}_0 > \tilde{z}_1 \). Furthermore, given \( \tilde{z}^0 - \tilde{z}_1^1 = \tilde{z}_0 - (\rho(h, \tilde{z}_0) - \rho(h, 0)) \), it is easy to check that \( \arg \max_{h \in [0, 75; 1]} \rho(h, \tilde{z}_0) - \rho(h, 0) = 1 \) so that, for all \( h \in [0, 75; 1] \), \( \rho(h, \tilde{z}_0) - \rho(h, 0) < \rho(1, \tilde{z}_0) - \rho(1, 0) = \tilde{z}_0 \) which ensures that \( \tilde{z}^0 > \tilde{z}_1 \).

**Proof.** One can check that \( \frac{\tilde{z}_0 - \tilde{z}_1}{1 - \tilde{z}_0} > \tilde{z}_1 - \tilde{z}_1^1 \) if and only if \( h < 1 \), which is the case.

7.5 Earnings...

7.5.1 of workers.

**Proof.** For all \( z \in [\tilde{z}_0, \tilde{z}_1] \), earnings are given by

\[
w_0(z) = \frac{1}{2} (z - \tilde{z}_0)^2 h + (\tilde{z}^0 h) z + (1 - h) \tilde{z}^0, \quad \text{and} \quad w_1(z) = \frac{1}{2} z^2 h + (\tilde{z}_1^1 h) z + (1 - h) \tilde{z}_1^1,
\]

so that:

\[
w_0(z) - w_1(z) = \frac{1}{2} (\tilde{z}_0^2 - 2z \tilde{z}_0 h) + ((1 - h) + hp) (\tilde{z}^0 - \tilde{z}_1^1).
\]

\( w_0(z) - w_1(z) > 0 \) if and only if

\[
\tilde{z}^0 - \tilde{z}_1^1 > \frac{1}{2} \frac{2z \tilde{z}_0 - \tilde{z}_0^2}{(1 - h) + h z}.
\]

With \( \tilde{z}^0 - \tilde{z}_1^1 = \tilde{z}_0 - (\rho(h, \tilde{z}_0) - \rho(h, 0)) \), \( w_0(z) - w_1(z) > 0 \) if and only if:

\[
\tilde{z}_0 - \frac{1}{2} \frac{2z \tilde{z}_0 - \tilde{z}_0^2}{(1 - h) + z} > \rho(h, \tilde{z}_0) - \rho(h, 0) (\geq 0),
\]

\[
\frac{(1 - h) \tilde{z}_0 + \frac{1}{2} \tilde{z}_0}{(1 - h) + z} + \rho(h, \tilde{z}_0) - \rho(h, 0) > \tilde{z}_1^1,
\]

\[
\frac{(1 - h) \tilde{z}_0 + \frac{1}{2} \tilde{z}_0}{\rho(h, \tilde{z}_0) - \rho(h, 0)} > \frac{1}{1 - h} + z.
\]

The right term takes its highest value on \([\tilde{z}_0, \tilde{z}_1]\) for \( z = \tilde{z}_1 = 1 - \frac{1}{h} + \rho(h, 0) \). As a consequence,

\[
\frac{(1 - h) \tilde{z}_0 + \frac{1}{2} \tilde{z}_0}{\rho(h, \tilde{z}_0) - \rho(h, 0)} > \frac{1}{1 - h} + \tilde{z}_1 \Rightarrow \frac{(1 - h) \tilde{z}_0 + \frac{1}{2} \tilde{z}_0}{\rho(h, \tilde{z}_0) - \rho(h, 0)} > \frac{1}{1 - h} + z, \quad \text{for all} \ z \in [\tilde{z}_0, \tilde{z}_1].
\]
Let us consider this latter sufficient condition. Given \( \rho(h, z_0) = \sqrt{\frac{1}{h} - \frac{1}{h} + 1 + 2 \frac{1}{h} z_0 + \frac{z_0^2}{2}} \), let \( A = \frac{3}{h^2} - \frac{4}{h} + 1 \) and \( B = \frac{1}{h} z_0 + \frac{z_0^2}{2} \).

\[
\frac{1}{h} - 1 \frac{z_0 + \frac{z_0^2}{2}}{\rho(h, z_0) - \rho(h, 0)} > \rho(h, 0) \iff \frac{B}{\sqrt{A + 2B} - \sqrt{A}} > \sqrt{A} \iff A + B > \sqrt{A^2 + 2AB},
\]

which is indeed the case. For all \( z \in [\tilde{z}_1, z_0] \), female earnings remains wage \( w_0(z) \) whereas male are self-employed and earn \( F(z) \). Yet, by construction, \( w_0(z) > F(z) \) so that female workers earn more than male self-employed.  

7.5.2 of managers.

Proof. For all \( z \in [\tilde{z}^0, 1] \), earnings are given by

\[
r^0(z) = \frac{z - w_0(z_0)}{(1 - z_0(z))h}, \quad r^1(z) = \frac{z - w_1(z_1(z))}{(1 - z_1(z))h},
\]

where \( z_0(.) \) is defined, for all \( z \in [\tilde{z}^0, 1] \), by \( m_\theta(z_0(z)) = z \). That is:\[15,16\]

\[
z_0(z) = 1 - \left(1 - \frac{2}{h} \left( z - z^0 \right) \right) \frac{1}{\frac{1}{h} + \rho(h, z_0)} \quad \text{for all } z \in [\tilde{z}^0, 1],
\]

\[
z_1(z) = 1 - \left(1 - \frac{2}{h} \left( z - z^1 \right) \right) \frac{1}{\frac{1}{h} + \rho(h, 0)} \quad \text{for all } z \in [\tilde{z}^1, 1].
\]

Using the expression of \( \tilde{z}^0 \) and \( \tilde{z}^1 \) one can check that

\[
z_0(z) = 1 - \left(1 - \frac{2}{h} \left( z - z^0 \right) \right) \frac{1}{\frac{1}{h} + \rho(h, z_0)} \quad \text{for all } z \in [\tilde{z}^0, 1],
\]

\[
z_1(z) = 1 - \left(1 - \frac{2}{h} \left( z - z^1 \right) \right) \frac{1}{\frac{1}{h} + \rho(h, 0)} \quad \text{for all } z \in [\tilde{z}^1, 1].
\]

As a consequence

\[
w_0(z_0(z)) = \left(1 - \frac{2}{h} (1 - z) \right) \frac{1}{\frac{1}{h} + \rho(h, z_0)} \frac{1}{2} + \rho(h, z_0) - \frac{2}{h},
\]

\[
w_1(z_1(z)) = \left(1 - \frac{2}{h} (1 - z) \right) \frac{1}{\frac{1}{h} + \rho(h, 0)} \frac{1}{2} + \rho(h, 0) - \frac{2}{h}.
\]

Earnings functions express as:

\[
r^0(z) = \frac{z - 1 - \left(1 - \frac{2}{h} (1 - z) \right) \frac{1}{\frac{1}{h} + \rho(h, z_0)} \frac{1}{2} + \rho(h, z_0)}{\left(1 - \frac{2}{h} (1 - z) \right) \frac{1}{\frac{1}{h} + \rho(h, z_0)} \frac{1}{2} + \rho(h, z_0) + \frac{3}{2h}},
\]

\[
r^1(z) = \frac{z - 1 - \left(1 - \frac{2}{h} (1 - z) \right) \frac{1}{\frac{1}{h} + \rho(h, 0)} \frac{1}{2} + \rho(h, 0)}{\left(1 - \frac{2}{h} (1 - z) \right) \frac{1}{\frac{1}{h} + \rho(h, 0)} \frac{1}{2} + \rho(h, 0) + \frac{3}{2h}}.
\]

\[15\] \( z_0(.) \) is defined for \( z \) such that \( z < \tilde{z}^0 + \frac{1}{h} (1 - z_0^2) \). This condition is always verified for \( z \leq 1 \). Indeed, given \( \rho(h, z_0) = \sqrt{\frac{1}{h} - \frac{1}{h} + 1 + 2 \frac{1}{h} z_0 + \frac{z_0^2}{2}} \), note that \( \tilde{z}^0 + \frac{1}{h} (1 - z_0^2) \geq 1 \iff \left( \rho(h, z_0) - \frac{1}{h} \right)^2 \geq 0 \) which, obviously is true.

\[16\] To check it is the case, read the previous footnote substituting 0 to \( \tilde{z}_0 \).
Let us consider the function $\gamma(.)$ defined by

$$
\gamma(\rho) = \frac{2}{h} - \left( \left( \frac{1}{h} - \rho(h, z_0) \right)^2 + \frac{2}{h} (1 - z) \right)^{\frac{1}{2}} - \rho(h, z_0),
$$

$$
r^0(z) = \frac{2}{h} - \left( \left( \frac{1}{h} - \rho(h, z_0) \right)^2 + \frac{2}{h} (1 - z) \right)^{\frac{1}{2}} - \rho(h, 0),
$$

$$
r^1(z) = \frac{2}{h} - \left( \left( \frac{1}{h} - \rho(h, 0) \right)^2 + \frac{2}{h} (1 - z) \right)^{\frac{1}{2}} - \rho(h, 0).
$$

Let us consider the function $\gamma(.)$ defined by

$$
\gamma(\rho) = \frac{2}{h} - \left( \left( \frac{1}{h} - \rho \right)^2 + \frac{2}{h} (1 - z) \right)^{\frac{1}{2}} - \rho.
$$

Deriving $\gamma(.)$ with respect to $\rho$ leads to

$$
\gamma'(\rho) = \frac{1}{2} \frac{h - \rho}{\left( \left( \frac{1}{h} - \rho \right)^2 + \frac{2}{h} (1 - z) \right)^{\frac{3}{2}}} - 1,
$$

which can be shown to be strictly negative. It follows that for all $z \in [z^0, 1]$ and $z_0 > 0$, $\rho(h, 0) < \rho(h, z_0) \Rightarrow r^1(z) > r^0(z)$. As for $z \in [\bar{z}, z^0]$, the fact that men have chosen to produce as managers rather than self-employed entails $r^1(z) > F(z)$.  

**7.6 Assignment**

**Proof.** Assignment functions express as

$$
m_0(z) = \frac{1}{2} \left( 1 - z \right)^2 (1 - (z - 2)^2) h + \frac{2}{h} (1 - z) - 1 + z_0 - \rho(h, z_0)
$$

for all $z \in [z_0, z_0]$, with

$$
m_1(z) = \frac{1}{2} \left( 1 - (z - 2)^2 \right) h + \frac{2}{h} (1 - z) - 1 - \rho(h, 0)
$$

for all $z \in [0, z_0]$. This condition independant of $z$. This condition rewrites:

$$
2 \left( 1 - \sqrt{h^2 - 4h + 3} \right) > \left( \frac{1}{2} \frac{z^2}{z_0 - z_0} - \rho \right) h^2 + z_0 h.
$$

One can show that for $h \in [0, 75; 1]$, the left side term of this inequality is strictly increasing in $h$ so that $\arg \min_{h \in [0, 75; 1]} 2 \left( 1 - \sqrt{h^2 - 4h + 3} \right) = 0, 75$ and $2 \left( 1 - \sqrt{h^2 - 4h + 3} \right) \geq \frac{1}{2}$. Similarly, for $h \in [0, 75; 1]$, since $z_0 < 1$, the right side term is strictly decreasing in $h$ so that $\arg \max_{h \in [0, 75; 1]} \left( \frac{\sqrt{2} z_0}{z_0} \right) h^2 + z_0 h = 0, 75$ and $\left( \frac{\sqrt{2} z_0}{z_0} \right) h^2 + z_0 h \leq \frac{\sqrt{2} z_0}{z_0} + \frac{z_0 h}{z_0} < \frac{1}{2} + \frac{1}{2} < \frac{1}{2}$. For all $h \in [0, 75; 1]$, thus, the above inequality holds i.e. $m_1(z) > m_0(z)$.

From previous proofs, we have

$$
z_0(z) = 1 - \left( \left( \frac{1}{h} - \rho(h, z_0) \right)^2 + \frac{2}{h} (1 - z) \right)^{\frac{1}{2}}
$$

for all $z \in [z^0, 1]$, with $\rho(h, z_0) > \rho(h, 0)$, which directly entails $z_0(z) > z_1(z)$. 


7.7 Spans of control

Proof. With uniform distributions,

\[
\begin{align*}
n_0(z) &= \frac{1}{(1 - z_0(z)) h} \text{ for all } z \in [\tilde{z}^0, 1], \\
n_1(z) &= \frac{1}{(1 - z_1(z)) h} \text{ for all } z \in [\tilde{z}^1, 1].
\end{align*}
\]

Since, by the previous proof, for \( z \in [\tilde{z}^0, 1] \), \( z_0(z) > z_1(z) \), it is the case that: \( n_0(z) > n_1(z) \). ■