M1 TUTORIAL:
EXERCISE ON CHAPTER III:
ENVIRONMENTAL ECONOMICS

Consider a population of households of total mass N uniformly distributed over the segment $[0, N]$. The households are endowed with one unit of time. Time can be spent at work and let l_i be the work time for a household i. Thus the leisure of the household is given by $1 - l_i$ and $l_i \in [0, 1]$. The households derive their welfare both from the consumption (let q_i be the household consumption level) and leisure. Let $U(q_i, l_i) \equiv u(q_i) + v(1 - l_i)$ be their individual utility function, the same for all households. Let $q \equiv \int_0^N q_i di$ be the aggregate consumption level. Similarly, let $l \equiv \int_0^N l_i di$ be the aggregate work time of the society. The consumption good is produced from labor through the technical relationship: $q \leq f(l)$. The consumption activity of the households generates pollution. Let $D(q)$ be the damage function in welfare terms induced by an aggregate consumption level q. Assume that the functions U, f and D satisfy the following set of assumptions:

Assumption A. 1

- *$u(q_i)$ is a class $\mathcal{C}^2$ function, $u' > 0$, $u'' < 0$ and $\lim_{q_i \downarrow 0} u'(q_i) = +\infty$;*
- *$v(1-l_i)$ is a class $\mathcal{C}^2$ function, $v' > 0$, $v'' < 0$ and $\lim_{l_i \uparrow 1} v'(1-l_i) = +\infty$;*
- *$f(l)$ is a class $\mathcal{C}^2$ function, $f' > 0$, $f'' < 0$ and $\lim_{l \downarrow 0} f'(l) = +\infty$*
- *$D(q)$ is a class $\mathcal{C}^2$ function, $D' > 0$, $D'' > 0$ and $\lim_{q \downarrow 0} D'(q) = 0$*

1 Part I : Pollution control in an egalitarian society

Q 1 : Since the households share the same welfare characteristics and suffer identically from pollution, they decide to share equally between them the consumption good and their participation to the work force. Write down the social utility function for an egalitarian society (Hint: since q_i is the same for all households, $q_i = q/N$, the same applies to labor force participation). Check that this function is only depending upon q and l.

Q 2 : Derive the corresponding social welfare maximization under the production constraint. Write down the associated Lagrangian and compute the first order conditions for a maximum.

Q 3 : Show that the optimal levels of q and l are linked through a monotonous relationship. Let $l^*(q)$ denote this relation and show that $dl^*(q)/dq < 0$. You will denote by $\bar{q}$ the level of q such that $u'(q) = D'(q)$. Making use of a graph in the space (q, l), illustrate graphically the determination of the optimal pair (q^*, l^*).

Q 4 : Assume that the households decide to ignore the pollution problem and try to maximize social welfare without taking the environmental damage into account. How the optimality conditions would be modified ? Show that in this case q and l are linked by another relation, $l = \hat{l}(q)$ also decreasing in q.

Q 5 : Compare $\hat{l}^*(q)$ and $\hat{l}(q)$ for a given q. Using the previous graph, picture the curve $\hat{l}(q)$ and show the pair $(\hat{q}, \hat{l})$ solving this constrained optimization program. What do you observe concerning the comparison between $\hat{q}$ and q^*, and the comparison between $\hat{l}$ and l^*?

Q 6 : Assume now that the households are embedded into a perfectly competitive market economy. The consumption good is produced by a large number of identical and competitive firms. π, the aggregate industry profit is shared equally between the households, that is each household receives a share π/N of the total profit. Maximizing the individual household utility denoting by p the price of the consumption good and by w the wage rate derive the first order conditions defining labor supply by the household and its consumption good demand. Since all households behave symmetrically, deduce the aggregate consumption good demand function and the aggregate labor supply function. Maximizing the industry profit, determine similarly the aggregate supply function of the consumption good and the aggregate demand function for labor.

Q 7 : Since the consumption good market and the labor market should clear at the equilibrium, check that the equilibrium pair (q^e, l^e) is in fact identical to $(\hat{q}, \hat{l})$ determined in question $Q4$. What do you deduce with respect to the Pareto optimality of the free market allocation?

Q 8 : The society decides to put in place an environmental tax over consumption to deal with the pollution externality. Let τ be the unit tax per consumption unit. The product of the tax: $t \equiv \tau q$ is equally distributed to the households as a lump sum transfer. Compute the level of the tax which would restore the optimality of the market economy. Using Walras Law you will assume that the price of the consumption good is normalized to one, that is the consumption good is the *numéraire* of the economy.

Q 9 : Instead of putting in place an environmental tax, the society decides to design a labor tax to deal with the pollution problem. let a be the tax per labor hour. Compute the level of the tax which would restore the optimality of the economy. The product of the tax al is assumed to be redistributed equally to the households. Compare the lump sum transfer to the households in the two systems.