A discussion of statistical methods for estimation of extreme wind speeds

by

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Abstract

Wind speeds in extra tropical latitudes are known to be approximately Weibull distributed. Hence a Weibull distribution fitted to all available data is often used to predict extreme winds. The most extreme values then, however, have little influence on the estimated parent Weibull distribution, and the accuracy of the extreme value predictions obtained in this manner may be questioned. In the present paper such a “Weibull method” is compared to a method based on statistical extreme value theory, “the Annual Maxima method”. The comparison is based on 30 years of 10 minute wind speed averages measured hourly at 12 meteorological stations located at airports in Sweden. Results include that the Weibull method gives incorrect estimates of the tails of the distributions of wind speeds and of the distribution of yearly maximum wind speed, and that serial dependence of individual measurements has to be taken into account. In addition it is inherent in the Weibull method that it does not provide any confidence bounds for the estimates. The Annual Maxima method avoids these problems. The measurements were rounded, first to entire knots, and then to m/s. A further, “technical” result is that if this rounding were disregarded in the estimation procedure, then the computed standard errors of the parameter estimates would be erroneously low. Hence, if rounding is done, it should be taken into account in the estimation procedure. We also believe this to be a clear indication that rounding of the data decreases estimation accuracy.

Keywords and phrases: annual maxima method, building codes, extreme value distribution,
1 Introduction

Wind speeds in extra tropical latitudes are well known to conform closely to
the Weibull distribution. Consequently, methods based on fitting a Weibull
distribution to all available data are often applied to predict sizes of future
extreme winds. This can be done by using the tail of the Weibull distribution
directly – here this will be called “the Weibull method”. However, since the
most extreme values have very little influence on the parameters of the parent
Weibull distribution, there is reason to question the accuracy of the extreme
value predictions obtained in this manner.

An alternative that is sometimes used is to assume that extreme winds
follow a Gumbel distribution with parameters derived from the parent Weibull
distribution (cf. e.g. Davenport 1967, Alexandersson 1979, Bergström 1992).
Since the Gumbel distribution is obtained as the limit of the distribution of
maxima of Weibull variables, this is expected to give similar results to “the
Weibull method”. However, still this “Weibull-Gumbel” method includes one
more, perhaps unnecessary, approximation, and we hence will not study it
further in the body of this paper. Still, a brief discussion is given in an
Appendix.

In the present paper the “Weibull method” is compared to a method based
on statistical extreme value theory, “the Annual Maxima method”. The meth-
ods are briefly introduced in this section, and are discussed more in detail in
Section 3 below. The comparison is aimed at developing methods for con-
struction of Swedish wind standards.

The Weibull method is briefly alluded to in Lundtang Petersen et al.
(1981), with substantial reservations, and Weibull estimation is surveyed in
Conradsen et al. (1984). There are many accounts of the Annual Maxima
method, see e.g. Embrechts et al. (1997, Section 6.3). Teugels (2000) con-
tains an extensive collection of references on the interface between wind speeds
and statistics, and Palutikof et al. (1999) make a very useful survey of methods
to calculate extreme wind speeds.

In discussions of statistical methods for extremes it is convenient to use
the concept of an extremal index. The idea, due to Leadbetter (see Leadbet-
ter et al. (1983)), is standard in statistical extreme value theory and is as
follows. Dependence between consecutive measurements can make extreme
values come in small “clusters”. For instance, a single wind storm often leads

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extreme winds, Weibull method, wind norms, wind standards.

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to several high measured hourly wind speeds. Roughly, the extremal index, \( \theta \), is defined as one divided by the average number of values in such a cluster of large values. Since a cluster must have at least one member, the extremal index can at most be one, and this limiting case corresponds to (approximate) independence where "clusters" in fact only consist of one large value. Under rather general conditions, the maximum of \( n \) dependent observations has approximately the same distribution as the maximum of \( \theta n \) independent observations. Specifically, if the hourly wind measurements have distribution function \( W(x) \) and are mutually independent (so that the extremal index is one) then yearly maxima have distribution function \( W(x)^{365 \times 24} \). However, if the extremal index is not one, but \( \theta \), for some \( \theta \) between zero and one, then the distribution function of yearly maxima instead is \( W(x)^{365 \times 24 \times \theta} \). The Weibull method uses the first of these two formulas for the distribution of the maximum yearly wind speed, and hence implicitly assumes that \( \theta = 1 \). For a more detailed discussion of the concept of an extremal index, and for proofs, see Embrechts et al. (1997, Section 8.1) and Leadbetter et al. (1983, Section 3.7).

A main aim is to estimate 50-year or 100-year winds or, in different terminology, the 50- or 100-year return levels. By definition these are the yearly maximum wind speeds which on the average are exceeded once every 50 or 100 years. Equivalently, the 50- and 100-year winds are the \( 1 - 1/50 \) or \( 1 - 1/100 \) quantiles of the distribution of the annual maximum wind speed. (The 50- or 100-year winds can also be thought of as close approximations to the "characteristic value", i.e. the \( e^{-1} \) quantile of the distribution of the maximum windspeed during a 50 or 100 year period.)

In the Weibull Method a two parameter Weibull distribution is fitted to all available wind speed recordings. The 50- and 100-year winds are then computed from the fitted Weibull distribution by assuming that the recorded wind speeds are serially independent.

Weibull distributions have often been observed to give a good fit to the bulk of wind speed data. However, the Weibull method relies on two further assumptions,

1. a suitable independence of the recorded wind speeds, and
2. that the fit of the Weibull distribution is good also in the extreme right tail of the distribution of wind speeds.

For (1) it should be noted that independence is not necessary for correct estimation of the parameters of the Weibull distribution. It is sufficient that dependence decreases suitably fast with increasing time separation. (Exact conditions for this do not seem to be available, but it is known that they
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would be of very general applicability.)

However, independence is also used in the step where 50- or 100-year winds are computed from the Weibull distribution. Again, full independence is not needed for the calculation to be valid, it is enough that the extreme part of the data is approximately independent. In technical terms it is sufficient that the extremal index equals one, so that there is no clustering of extremes.

Thus Assumption (1) of the Weibull Method is not as restrictive as it would appear at first. Nevertheless, if dependence extends into the extreme right tail of the distribution of the wind speed recordings, then the Weibull Method can give wrong results.

As for (2) there is no physical or theoretical reason that it should be satisfied for wind speeds recordings. If it is not then the Weibull Method, again, can give wrong results.

In the Annual Maxima method, an Extreme Value (EV) distribution (or a Gumbel distribution) is fitted to the annual maximum wind speeds, assuming that maxima from different years are serially independent. The argument for using these distributions comes from statistical extreme value theory (see e.g. Coles (2001), Embrechts et al. (1997), and Rootzén and Tajvidi (1997) for a discussion of this).

Maxima taken over different years are likely to be much more independent than hourly measurements. Thus, the Annual Maxima method circumvents the problematic Assumption (1) by pooling the potentially locally dependent recordings into a single value for each year. Assumption (2) is circumvented, since the statistical fitting only uses extreme values.

There of course is a cost associated with using the Annual Maxima method. This is the very drastic reduction of the number of observations used in the statistical analysis. As a consequence estimates obtained from the Annual Maxima method are much more variable than estimates provided by the Weibull Method. However, if the Weibull Method estimates the wrong quantities, it is not much help if this is done with high precision.

Our comparison of the Weibull and Annual Maxima methods is based on a data set provided by the Swedish Meteorological and Hydrological Institute (SMHI). The data set contains thirty years of hourly windspeed recordings (10 minute averages) for 12 stations in Sweden. We applied both the Weibull and the Annual Maxima method to estimate the distribution functions of annual maximum windspeeds and compared the results to the distribution functions of the observed annual maxima. We also compared the 5-year, 50-year and 100-year winds obtained from the two methods.

The data base is described in Section 2. The Weibull and Annual Maxima
methods and the statistical methods are discussed in Section 3. The next section, Section 4, summarizes the results of the analysis, and a final discussion and conclusions are given in Section 5.

2 The wind data base

The wind database used in this paper was provided by the SMHI. It consists of 10 minute wind speed averages recorded, with some exceptions, at the start of each hour during 1961 – 1990 at 12 synoptic meteorological stations in Sweden (Table 1), and thus contains roughly $30 \times 365 \times 24 \approx 2.6 \times 10^5$ recordings per station. The stations are all located at airports.

WMO regulations for siting of different types of meteorological stations and procedures of observations are given in WMO (1983). In practice, it is not always possible to follow these regulations completely. Therefore, wind speeds in particular should always be regarded as influenced to some degree by local effects. Airports generally present open fetch over the nearest area surrounding the instrument. However, at greater distances (say > 1 km) there may be considerable inhomogeneities also around airports.

All stations had missing observations, ranging from 0.06% for Arlanda to 17% for Karlstad. In addition the entire first year was not included in the database for Arlanda.

Major reasons for “missing observations” was that some airports, in particular minor civil aviation airports, only make observations during hours of operation. Others, notably military airports, make observations every hour during hours of operation but only every 3 hours (the regular synoptic hours of observation) during the remaining part of the day.

Further, a number of data points seemed dubious or plainly wrong. The reasons were recordings at not existing time points, forgotten conversions from knots to m/s, and measurements which were very out of line with adjacent measurements, such as one very high wind speed recorded when all the measurements around it were very low. We deleted or corrected these data points manually. Only recordings with wind speeds in excess of 14 m/s were corrected, since errors in lower wind speeds were deemed to have a negligible influence on the results.

All synoptic observations were made in knots. The data was then converted to m/s and rounded off to the nearest m/s by the SMHI when the data was stored in the historical database.

All the stations in Table 1 (except perhaps Bredåkra) are situated in landscape with rather gentle topography. Their elevation above sea level is rather
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Table 1: The recording sites

<table>
<thead>
<tr>
<th>site</th>
<th>missing</th>
<th>deleted</th>
<th>corrected</th>
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</thead>
<tbody>
<tr>
<td>Arlanda</td>
<td>145</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Barkära</td>
<td>1,999</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Bredära</td>
<td>1,996</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>Bromma</td>
<td>20,339</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Karlstad</td>
<td>44,916</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>Ljungbyhed</td>
<td>2,014</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Norrköping-Sörby</td>
<td>2,153</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>Säve</td>
<td>3,400</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Söderhamn</td>
<td>2,093</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>Uppsala</td>
<td>2,166</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>Västerås-Hässlö</td>
<td>12,782</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>Visby</td>
<td>8,090</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

It is likely that over a 30-year period the landscape around the recording instruments have changed enough to affect the measurements. A particular problem arises at near urban sites, where urban-industrial growth causes gradual changes in fetch conditions. Arlanda airport is an example of this sort. This civil aviation airport has grown extremely over the past decades and today presents a much more urbanised landscape with significantly higher surface roughness than during the 1960-ies. Most likely this has caused a general reduction in wind speed. It is also likely that such an effect is more pronounced for some wind directions than for others. In particular topographical changes could lead to spurious time trends or correlations. We have not had the possibility to correct for this possibility.

3 The Weibull and the Annual Maxima methods

The analysis was performed separately and in the same way for each of the recording sites. To describe it, denote the (rounded) hourly wind speed recordings at the site by \( \{x_i\} \), and let \( \{m_j\} \) be the yearly maxima of these (rounded) recordings. Further, let \( F \) be the distribution of the yearly maximum wind
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speed. Then the $T$-year wind $u_T$ is obtained as a solution of

$$T(1 - F(u_T)) = 1. \quad (1)$$

(For a discussion of slightly different versions of this, see Rasmussen and Rosbjerg (1991).)

The Weibull cumulative distribution for the $x_i$’s is $W(x) = 1 - \exp\{- (x/\sigma)^k\}$

with density function

$$w(x) = \frac{k}{\sigma} \left(\frac{x}{\sigma}\right)^{k-1} \exp\{- (x/\sigma)^k\}. \quad (2)$$

The distribution function $F$ needed for the computation of return levels in Equation (1) is then computed as if the observations were independent, that is as

$$u_T = \sigma(- \ln(1 - (1 - 1/T)^{1/(365 \times 24)}))^{1/k}. \quad (3)$$

Finally, the parameters have to be estimated from the data. This is here done by the so called maximum likeness method, for grouped samples, which is to use the values of $\sigma$ and $k$ which maximize

$$\sum_i n_i \ln(W(x_i + 1/2) - W(x_i - 1/2)), \quad (4)$$

for $n_i = \text{the number of recordings with wind speed } x_i \text{ m/s}$. (In the standard Weibull method the rounding often is not taken into account. This, however, would change the estimates only marginally.) The estimates are the same as would be obtained by ordinary Maximum Likelihood estimation if the hourly observations were assumed to be independent (for studies of estimation for rounded data, see Kulldorf (1961), Sen and Singer (1993)). The Weibull method does not provide any confidence intervals for the estimated return values.

In the Annual Maxima method, the yearly maxima, $m_j$, are assumed to be independent and to have the Extreme Value (EV) distribution function $G(x) = \exp\{-(1 + \gamma \frac{x - \mu}{\sigma})_+^{-1/\gamma}\}$, where $\mu$ is a location parameter, $\sigma$ is a scale parameter, $\gamma$ is a shape parameter and $+$ signifies “positive part”. That is, for $x$-values which make the expression in the inner parenthesis negative, the expression should be replaced by zero. The density of the EV distribution is

$$g(x) = \frac{1}{\sigma}(1 + \gamma \frac{x - \mu}{\sigma})_+^{-1/\gamma} \exp\{-(1 + \gamma \frac{x - \mu}{\sigma})_+^{-1/\gamma}\}. \quad (5)$$
The case $\gamma = 0$ is defined by continuity, that is for $\gamma = 0$ the EV distribution has the Gumbel distribution function $G_0(x) = \exp(-\exp(-\frac{x-\mu}{\sigma}))$ with density
\begin{equation}
    g_0(x) = \frac{1}{\sigma} \exp(-\frac{x-\mu}{\sigma}) \exp(-\exp(-\frac{x-\mu}{\sigma})).
\end{equation}

The distribution $G_0$ occupies a special place in extreme value theory, both for theoretical reasons and because it has been found to fit many data sets well.

Finally, the distribution function $F$ in Equation (1) of course is replaced by $G$ (or $G_0$) in the Annual Maxima method, and hence
\begin{equation}
    u_T = \mu + \frac{\sigma}{\gamma} \{(-\ln(1 - \frac{1}{T}))^{-\gamma} - 1\}
\end{equation}

or, for $\gamma = 0$,
\begin{equation}
    u_T = \mu - \sigma \ln(-\ln(1 - \frac{1}{T})).
\end{equation}

The maximum likelihood estimates, if the rounding of the data is disregarded, are the values of the parameters which maximise the log likelihood function
\begin{equation}
    \sum_j \ln(g(m_j)),
\end{equation}
where the summation is over the number of data points. If the rounding is taken into account the log likelihood function instead becomes
\begin{equation}
    \sum_j n_j \ln\{G(m_j + 1/2) - G(m_j - 1/2)\},
\end{equation}
where summation again is over groups, and then the maximum likelihood estimates are obtained by maximising (10).

For likelihood computations without taking rounding into account we used a set of S+ routines provided by Coles (http://www.stats.bris.ac.uk/ masgc/) and for the likelihood computations with the rounding taken into account we wrote new S+ routines. Unlike Coles we computed the information matrix analytically and used expected information instead of observed.

We studied two different methods to compute confidence intervals. The first one used the inverse of the observed information together with the delta method. (The observed information matrix is minus the Hessian of the log likelihood function evaluated at the maximum: see e.g. Coles (2001)). The second one was to compute profile likelihood confidence intervals, again, see Coles (2001). The calculations with information are simpler, but profile likelihood intervals are expected to be superior when the distribution of the estimates
are skewed. The two methods yield similar results if the distribution of the estimate is reasonably symmetric.

The calculations described above were repeated with $G$ replaced by the Gumbel distribution function $G_0$. The hypothesis that $\gamma = 0$ was tested using a likelihood ratio test (Coles (2001)) at the 5% significance level.

The goodness-of-fit of the estimated distributions of the annual maxima were checked by Chi-Square tests and graphically. For the Chi-Square test we used 3 - 5 groups and computed the degrees of freedom for the Weibull method as the number of groups - 1 and for the Annual maxima method as the number of groups - the number of estimated parameters - 1. The reason we did not subtract the number of estimated parameters in the Weibull method was that for this method the parameters were not estimated from the yearly maxima but from the (very many) hourly observations.

To check the fit we also used standard empirical distribution function and quantile-quantile plots, and plots of the return levels. The empirical distribution function plot for $G$ consists of a “staircase” through the points $(m(j), N_j/(n + 1) : j = 1, \ldots, K)$ and the quantile-quantile plot shows the points $(m(j), G^{-1}(N_j/(n+1)) : j = 1, \ldots, K)$ where $m(1) < m(2) < \ldots < m(K)$ are the ordered (rounded) yearly maxima, $N_j$ is the number of yearly maxima less than or equal to $m(j)$, $K$ is the number of (rounded) values which were actually observed, and $n = 30$ (for Arlanda 29) is the number of years of observation. The empirical distribution function plots in addition show the estimated distribution function and the quantile-quantile plots include an $y = x$ line. Plots for $G_0$ were made similarly.

If $G$ (or $G_0$) fits the data, the empirical and estimated distribution functions should be close and the quantile plot should consist of points close to the $y = x$ line. Return level plots show the empirical return levels $m(j)$ plotted against $(-1/\ln(N_j/(n + 1)))$ and the $T$-year return levels estimated from the model plotted against $-1/\ln(1 - 1/T)$. (It is customary to use $-1/\ln(1 - 1/T)$ instead of $T$. For large values of $T$ this does not make any difference. For small values, the return period can be obtained from the equality $T = 1/(1 - \exp\{1/(1/\ln(1 - 1/T))\})$ if the model fits the data, the estimated and empirical return levels should be close together. Further the $x$-axis is on a log scale, which produces a straight line for the Gumbel distribution. For further discussion of these plots, see e.g. Coles (2001).

To summarize the annual maxima method we now give a step by step description of the calculations for one of the stations, Arlanda. In the example we take the rounding into account. The 29 observed annual maxima for this station are given in Table 3.
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Table 2: Maximum annual wind speeds at Arlanda, 1962 - 1990, rounded to entire m/s.

<table>
<thead>
<tr>
<th>wind speed</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>21</th>
</tr>
</thead>
<tbody>
<tr>
<td>no. of occurrences</td>
<td>1</td>
<td>5</td>
<td>10</td>
<td>7</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

The log likelihood function $\ell(\mu, \sigma, \gamma)$ with the EV distribution hence was

$$
\ell = \ln\{G(12 + .5) - G(12 - .5)\} + 5 \ln\{G(13 + .5) - G(13 - .5)\} + \ldots + \ln\{G(21 + 1/2) - G(21 - 1/2)\},
$$

with $G(x) = \exp\{- (1+\gamma \frac{z-\mu}{\sigma})_{+}^{-1/\gamma}\}$. The corresponding log likelihood function $\ell_0(\mu, \sigma)$ for the Gumbel distribution is obtained by replacing $G(x)$ in (12) by $G_0(x) = \exp(- \exp(- \frac{x-\mu}{\sigma}))$.

Step 1, Computation of the maximum likelihood estimates: These were found numerically as the values of $\mu, \sigma, \gamma$ which maximized the log likelihood function (12). The resulting estimates were $\hat{\mu} = 13.9$, $\hat{\sigma} = 1.25$, $\hat{\gamma} = 0.08$.

Step 2, model control: This was done by producing empirical distribution function plots, quantile-quantile plots and return level plots, as described above. The plots for Arlanda are show in Figure 3. One way of getting a feeling for how large deviations from the lines one should expect is by simulation: simulate a number of new samples of size 29 from an EV distribution with parameters $\mu = 13.9$, $\sigma = 1.25$, $\gamma = 0.08$, and produce the same plots from the simulated samples. One then knows that for the simulated samples the model fits perfectly and that deviations only are due to randomness. If the deviations from the line for the real measurements are similar to the deviations for the simulated samples, the model would seem reasonable.

Step 3, computation of T-year winds: These were simply obtained by substituting the estimated parameters into (7). Thus, e.g. the estimated 50-year wind using the EV distribution was obtained as $\hat{u}_{50} = 13.9 + \frac{1.25}{\gamma} \{(- \ln(1 - \frac{1}{50}))^{-0.08} - 1\} = 19.6$

Step 4, computation of confidence intervals for the T-year winds: Here we only consider profile likelihood intervals. For a complete readable account of these, see Coles (2001). Very briefly, a profile confidence interval for, say, $u_{50}$ using e.g. the Gumbel distribution is obtained as follows. First one of the parameters in the model, e.g. $\mu$ is by means of (8) expressed in terms of $u_{50}$ and the remaining parameter $\sigma$, and this expression for $\mu$ is inserted into the log likelihood function (12), making it a function of $u_{50}$ and $\sigma$. The
profile likelihood function is a function of \( u_{50} \) only, and for each value of \( u_{50} \) is obtained as the maximum the log likelihood function over the remaining variable \( \sigma \). Finally, the 95% profile likelihood confidence interval consists of those values of \( u_{50} \) for which the profile likelihood function is within 1.91 of its global maximum. For Arlanda, the 95% Gumbel profile likelihood confidence interval for the 50 year return level, was calculated to be (17.48, 20.94). This concludes the step by step example.

The final ingredient of the present discussion of the Weibull and Annual Maxima methods is the extremal index, which we estimated using a number of different thresholds, where the threshold specifies what is meant by a “large value”, and also affects what is meant by a cluster. The estimate of the extremal index for a threshold of, say, 14 m/s, was computed as the number of “clusters” divided by the total number of recorded wind speeds strictly larger than 14 m/s. A cluster of exceedances of 14 m/s was defined as follows: the recordings were divided into blocks of size \( p \times 24 \) (we tried with \( p = 3, 6, 9 \) and 12); if a block contained at least one record strictly larger than 14 m/s then this block was considered to be a cluster. If one instead of the threshold 14 m/s uses the threshold 15 m/s one of course gets a different estimated value for the extremal index, and the threshold 16 m/s gives a further different estimated value, and so on. Similarly different block sizes give different values of the estimates. The choice of threshold and blocksize is an important and difficult ingredient in the estimation of the extremal index.

4 Results

"Technical” results: In the Annual Maxima method the differences between the estimates obtained from the likelihood function (10) with rounding taken into account and the likelihood function (9) were quite small for the location parameter, less (usually much less) than 33% of the estimated standard error for the scale parameter and less than 19% of the estimated standard error for the shape parameter, for all sites. The estimated standard errors for the location parameter were very similar regardless of whether rounding was taken into account or not, while the estimated standard errors for the scale parameter were larger (4% to 62%) if rounding was taken into account, and this difference was even larger for the shape parameter (10% to 53%).

Confidence intervals for return levels based on observed information and on profile likelihood were rather similar. For instance, for the 50-year winds, with one exception, the profile likelihood intervals typically were slightly shorter, and shifted roughly 0.5m/s towards larger values. The exception was Säve
where the observed information confidence interval was (23.1, 27.0) and the profile likelihood interval was (23.9, 28.3).

The Weibull Method: The Weibull distribution often underestimated the tail of the distribution of the 10 minute wind speeds by about 1 m/s, see Figure 1.

Estimation of the extremal index is well known to be sensitive to the choice of threshold. This was the case also for the present data. However, nevertheless, the extremal index seemed clearly less than one for all sites, see Table 3.

The distribution for yearly maxima obtained by the Weibull Method were very strongly rejected by the Chi square test. The p-value was 0.02 for Norrköping-Sörby, 0.007 for Söderhamn and less than $10^{-5}$ for the other sites.

That the Weibull distribution for yearly maxima did not fit was also very evident from plots of estimated distribution functions (Figures 3, 4).

The Annual Maxima method: Plots of autocorrelation’s (Figure 2) indicated little serial dependence of the annual maxima. Formal portmanteau tests (Brockwell and Davis (1996)) showed a small deviation from independence for the station Bredåkra, but not for the other stations. Likelihood ratio tests at the 5% level rejected the hypothesis that the shape parameter $\gamma$ in the EV distribution was zero for three sites, Bredåkra, Säve and Visby (Figure 4). The p-values were 0.04, 0.02, and 0.02, respectively. The estimates of the shape parameters for these three sites were negative, corresponding to somewhat smaller extremes than for $\gamma = 0$. The Chi square goodness of fit tests did not reject the EV distribution assumption for any of the sites, and did not reject the Double Exponential distribution for any of the 9 sites where $\gamma = 0$ was not rejected by the likelihood ratio test.

Comparison of results: The quality of the fit for the two methods and return levels for other periods than 5, 50 and 100 years are illustrated in Figures 3 and 4. The sites shown in the figure were chosen as follows: Barkåkra is a typical site, Arlanda is the site where the Gumbel distribution has the visually poorest fit although it nevertheless was not rejected by the likelihood ratio test, and Bredåkra, Säve and Visby are the three sites were the hypothesis that $\gamma = 0$ was rejected. The 50 year return levels estimated by the Weibull Method and by the Annual Maxima method were the same for 4 sites, 1 or 2 m/s larger with the Weibull Method for 4 sites, 4 m/s larger with the Weibull Method for Säve, 5 m/s larger with the Weibull Method for Ljungbyhed and 2 m/s smaller for Norrköping-Sörby and Söderhamn (Table 3). For the 2 sites Ljungbyhed and Säve, the 50 and 100 year Weibull estimates were outside of the Annual Maxima profile likelihood confidence intervals (Table 4).
Table 3: Return levels (in m/s) and extremal index. W = Weibull, EV = Annual Maxima with EV distribution and likelihood with rounding, G = Annual Maxima with Gumbel distribution and likelihood with rounding, * = the Gumbel distribution was rejected at the 5% level, and EI = the extremal index estimated with the characteristic value, i.e. the 37% quantile of the yearly maxima, as threshold and with block size 9 days.

<table>
<thead>
<tr>
<th>period (ys)</th>
<th>5</th>
<th>50</th>
<th>100</th>
<th>W</th>
<th>EV</th>
<th>G</th>
<th>W</th>
<th>EV</th>
<th>G</th>
<th>EI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arlanda</td>
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Table 4: Profile likelihood 95% confidence intervals for return levels. The likelihood function with rounding taken into account and with the Gumbel distribution for annual maxima was used except where marked by *, where the EV distribution was used.

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5 Discussion and conclusions

The goal of this paper was to compare the Weibull Method to compute return levels with methods based on extreme value theory. The comparison was based on 30 years of hourly wind speed recordings from 12 sites in Sweden.

The main conclusions were that the Weibull Method

• inherently does not provide confidence bounds for the estimates,
• gave incorrect estimates of the distribution of yearly maximum wind speeds,
• used an incorrect form of the tail of the marginal distribution of wind speeds, and
• did not take into account that the serial dependence of hourly wind measurements extended into the tail of the distribution.

Here the second item is based on Chi-square goodness of fit tests and on graphical analysis (Section 4, Figures 3 and 4), the third on comparison of fitted and empirical tail functions (Figure 1) and the fourth on estimates of the extremal index of the wind measurements (Table 3).

The Annual Maxima method with EV distributions gives credible confidence intervals, a good fit to the tail of the distribution of extreme wind speeds and seems to avoid the problem of serial dependence connected with hourly measurements (Figure 2). The different versions of the Annual Maxima method gave rather similar results, and in particular likelihood ratio tests of the assumption that the shape parameter of the EV distribution was zero, or equivalently that the annual maxima followed a Gumbel distribution, was only rejected for three sites, and then only with a small margin. Estimates and confidence intervals obtained assuming an EV distribution and assuming a Gumbel distribution were quite similar. We had expected that the point estimates would not change, but had expected a clear shortening of the confidence intervals when the Gumbel distribution was used, and hence the latter finding was a surprise.

The results in Table 4 show higher wind speeds at more south-westerly locations (with the exception of Norrköping) and similar winds speeds at nearby locations such as Arlanda, Bromma, and Säve, as could be expected. Visby which is located at an island in the Baltic, also had higher wind speeds. Confidence intervals with approximately the same center also were approximately equally wide, and confidence intervals which included lower wind speeds were shorter than those with higher wind speeds, again as expected.
In this paper we have used calendar years. It may be somewhat preferable to have different storm seasons in separate years, that is to let years start, say, at July 1 instead. Cook (1982, 1985) finds that dynamic pressure leads to a better fit of extreme winds to the Gumbel distribution. We have not investigated if this is the case also for the present data set.

For the Weibull Method the 50 year return level estimates were close to those given by the Annual Maxima method for some of the sites, but still rather different for others. For other return periods the Weibull results differed from those obtained by the Annual Maxima method.

On a more technical level, taking into account that the observations were rounded to entire m/s changed the parameter estimates rather little in the Annual Maxima analysis. However, still

- when the rounding to m/s was taken into account, computed standard errors of the parameter estimates increased substantially in many cases.
  
  Hence such rounding must be taken into account. We also believe this to be a clear indication that rounding decreases the accuracy of estimated extreme wind speeds.

Specifically, as discussed at the beginning of Section 4, rounding made the standard errors of the estimates of the scale and shape parameters substantially larger, and also made confidence intervals for the return levels markedly wider. Both results were as expected.

Confidence intervals obtained using profile likelihood and observed information were rather similar. Since the shape parameters of the fitted EV distribution were close to zero this was as expected. The sample sizes are rather small, and it would be interesting to check the accuracy of the confidence intervals by simulation.

In conclusion, it is our opinion that the Weibull Method should not be used for estimation of extreme winds.

A simple alternative is the Annual Maxima method assuming a Gumbel distribution, disregarding the rounding of the observations and using maximum likelihood estimation. This may give slightly conservative estimates, but should still provide reasonably reliable results. However, if confidence intervals are computed, they will show a too optimistic view of the accuracy of the estimates.

The preferred alternative, however, is the Annual Maxima method with the EV distribution and with rounding taken into account in the likelihood functions, and to use profile likelihood confidence intervals. If not rejected by a likelihood ratio test the Gumbel distribution could alternatively be used. The quality of the fit should always be checked graphically.
The Peaks over Thresholds method is expected to produce similar results to the Annual Maxima method but has a potential to be slightly more accurate, at the expense of the arbitrariness involved in the choice of storm events, or more or less equivalently, in the choice of threshold and in the declustering. We have not investigated this for the present data set.

A further problem is the influence of inhomogeneous site conditions (topography and land use) at meteorological stations and the effects of temporal changes in site conditions or instruments at meteorological stations. This might in principle be approached by careful adjustments for inhomogeneities and temporal changes.

It is an interesting future task to investigate the spatial dependence of measurements taken at different recording stations, and how it can be used to improve estimation of extreme winds.

Acknowledgment: We are grateful to Stuart Coles for providing the S+ functions for the (continuous) likelihood method and for inspiration on how to adapt these functions to rounded data. We also want to thank two referees for detailed comments which led to a considerable improvement of the paper.

References


Appendix: a comparison with the Weibull-Gumbel method

In connection with producing wind norms for Sweden, the SMHI made a comparison of different approaches to extreme wind estimation. In this appendix
we compare results from the present paper with a part of the SMHI results on “the Weibull-Gumbel approach”. In this approach, a Weibull distribution is fitted to all wind speed data, and maxima of wind speeds are then assumed to follow a Gumbel distribution with parameters derived from the parent Weibull distribution.

Table 5: 50-year return levels. W = Weibull, EV = Annual Maxima with EV distribution and likelihood with rounding, G = Annual Maxima with Gumbel distribution and likelihood with rounding, * = the Gumbel distribution was rejected at the 5 % level, and WG = the Weibull-Gumbel method.

<table>
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Specifically, if wind measurements were made every hour for thirty years, then the T-year wind is assumed to have the Gumbel distribution $G_0(u) = \exp(-\exp(-\frac{u-\mu_T}{\sigma_T}))$ where $\mu_T$ and $\sigma_T$ are called the modal value and dispersion factor, respectively, and are obtained from the Weibull parameters $\mu$, $\sigma$ as

$$\mu_T = \sigma (\log(N))^{1/k}$$
$$\sigma_T = \frac{\sigma}{k} (\log(N))^{1/k - 1}$$

for $N = 24 \times 365 \times 30 \times T$. Variants of this method include adjustment for dependence in the data, similar to the use of the extremal index. The SMHI results for the Weibull-Gumbel method are given in Table 5. The consistently lower values for the Weibull-Gumbel method as compared to the Weibull Method is due to adjustments for dependence in the version of the former method which is used by the SMHI.
Figure 1: Tail of qq-plot of the empirical quantiles of the hourly 10 minute windspeed measurements against quantiles of fitted Weibull distribution (⋯) and the x=y line (——)
Extreme winds in Sweden

Figure 2: Autocorrelation functions for annual maximum wind speeds
Figure 3: Top Barkåkra (a typical station), bottom Arlanda (worst fit of stations with $\gamma = 0$ not rejected). In each of the plots the solid lines are the estimated values and 95% confidence bounds for the Annual Maxima estimates. The dotted lines are the Weibull estimates.
Figure 4: Stations with $\gamma = 0$ rejected. Top Bredåkra, middle Säve, bottom Visby. In each of the plots the solid lines are the estimated values and 95% confidence bounds for the Annual Maxima estimates. The dotted lines are the Weibull estimates.